

From Differentiated Matter to Metric Exhaustion

Finite source reduction, trapped interiors, and invariant limits of metric continuation

Abstract

Can decreasing matter differentiation brake contraction inside a black hole, and can that physical evolution establish the exhaustion of metric support? We separate these questions within a conditional canonical-scalar realization on a licensed metric domain. Exact paired branches and an open local potential class provide finite inward response reduction, conservation and explicit preparation bounds. Different fixed-potential preparations attain braking entry, a positive-field turn and exit; conserved components and signed spherical shear change the event order. A smooth nonnegative completion receives four certified open Kantowski–Sachs preparation boxes inside one-ended asymptotically flat black-hole developments. An explicit allowed exterior completion fixes an initial event-horizon area and calibrates all twenty recorded events. This normalization is demonstrably completion dependent. The same completed source attains a finite-time scalar-curvature end in its homogeneous evolution. An optical estimate receives the entire approach in the unchanged global spacetime for both positive-shear boxes. A local diameter argument excludes any added limit point for a sequence approaching the end whose longitudinal coordinate converges strictly inside the central strip in a same-dimensional continuous nondegenerate Lorentzian extension through a smooth isometric embedding; neither symmetry, field equations nor a scalar extension is assumed. Independent curvature and scalar arguments also prove timelike incompleteness, a C^2 metric obstruction and failure of a finite continuous original-scalar endpoint. The fixed-horizon area ratio tends to zero from the regular side after braking has ended. Exact families nevertheless have identical complete future exteriors and different original interior source readouts, while declared preparation data can restore reconstruction on narrower families. A compact representation preserves every original regular-side operation without restoring the prohibited endpoint. These constructions yield precise failures of specified original uses, not full residual-support exhaustion: the complete witness-subtracted black-hole burden and a universal endpoint duty remain to be received. No inverse reheating, complete bearer recomposition, physical GUT threshold, cross-boundary energy ledger or intrinsic AMetric geometry is inferred. The local uncertain-potential results, global completed-source results and residual-duty conditions retain distinct assumptions throughout.

Keywords: AASC; persistence bearer; canonical scalar field; contraction; trapped surfaces; black-hole interior; invariant singularity; continuous metric extension; metric applicability.

1. The physical question and the route to an endpoint

The possibility motivating this paper is an inward counterpart to differentiation: as a black-hole interior contracts, might its source lose distinctions associated with particle, gauge, mass and record roles, recover an inflation-like pressure response, and eventually reach the boundary of metric application? This question contains three separate demands. There must be an actual conserved source whose changing responses justify the proposed loss of differentiation; a received black-hole geometry must contain the evolution; and any claimed loss of metric support must concern the full original use and its admissible realizations. A large density, a shrinking sphere or a formal reversal of an inflationary equation does not establish all three.

We construct a conditional physical model and follow its consequences through those demands. A prepared inward scalar response can brake contraction for a finite interval. Spherical curvature and signed shear leave the spheres trapped and distinguish mean from areal braking. A global completion receives the finite experiment and supplies an actual event-horizon reference. For two open positive-shear families, the same global development also receives a later finite-time invariant end. A local diameter argument excludes a continuous nondegenerate Lorentzian endpoint for every approach in those families whose longitudinal coordinate tends to a point strictly inside the central strip, with no extension symmetry or field-equation requirement. Independent curvature and scalar arguments retain their own continuation consequences. We then test what this failure says about reconstruction and the larger residual-support claim. Exact exterior ambiguity and exact regular-side reconstruction show why the final inference requires the original burden and successor duties, rather than a replacement definition of exhaustion.

The resulting arc is constructive but not a claim that every theorem describes successive stages of one trajectory. The open uncertain-potential halving theorem and the fixed-potential entry/exit theorem use different preparations. The completed real-scalar spherical family carries the global horizon and endpoint results. Their hypotheses are kept distinct so that a stronger conclusion in one family cannot silently enlarge another.

1.1. Structural motivation and the persistence-bearer question

The AASC dependence order runs from premetric constraints and gravity through comparison and bounded transport to a common distributed persistence bearer, joint role-forming responses, differentiated sectors and metric realizations. These arrows express dependence, not a temporal fragmentation of an inert substance. The bearer already carries locality, identity, comparison, restoration, boundary behavior and transport [A2, Proposition 1.2 and §6; A3, Theorems 8.1–9.1]. Field regimes without a stable localized particle class are allowed by the particle analysis [A4, §8]. They motivate a question about changing role realization without identifying every such change with complete bearer return.

The inflation analysis establishes a structural role for early transport closure under its original witness, lineage and continuation hypotheses. It does not derive accelerated expansion from an undifferentiated bearer; a nonaccelerating mechanism can occupy that role [A1, §§17–25, 38]. Its reheating map takes field/vacuum closure into particle, gauge, mass and record roles while preserving standing, carriage and source accounting [A1, §§26–28, 46]. No inverse map is established there. Closure stress is successor thinness rather than physical stress-energy, and the nonempty onset-thin successor family differs from terminal exhaustion [A1, §§9–11, 44]. The source-response postulates below therefore add effective physical content openly.

The black-hole sequence permits a proposed pre-endpoint reversal as interior dynamics while separating extreme curvature, residual-support exhaustion and full AMetric closure [A6, Theorem 8.7 and §9.2]. Governance depth concerns successor support; it is not by definition radius, proper time, curvature or energy. Energy and temperature require licensed roles, and standing loss alone neither annihilates residual source load nor derives a discontinuous stress tensor [A5, PDF pp.35–43]. A horizon is not an AMetric boundary. The

parent energy ledger does not cross such a boundary by default [A7, §13.4]. These distinctions guide the physical calculation and its stopping point.

In particular, increasing local density does not itself reverse structural order or select a GUT threshold. The Higgs analysis admits large local energy densities while preserving structural envelope order; physical scale identification needs its own map [A8, §9.2, Theorem 10.4 and §12.2]. We introduce no universal density for recomposition. The model's scales and proper time remain quantities of its metric-side realization.

We distinguish designated differentiated response roles, $\mathcal{F}_D$, from the metric, clock and stress-readout roles, $\mathcal{F}_g$, retained to describe their evolution. Their joint incidences belong to the original larger target. The effective scalar matches selected changes in $\mathcal{F}_D$ while $\mathcal{F}_g$ continues to apply. The later invariant obstruction concerns definite operations in that retained description. Neither part identifies a finite set of source insertions with all original roles or proves the joint withdrawal of mass, energy, space and time.

This is also distinct from heat death. Loss of free-energy gradients is a physical condition within an ongoing description. The proposed terminal question concerns the conditions for that description's uses. Our energy account remains the conserved source ledger on the metric side. Negative pressure can lower the energy of a contracting material cell through work without drawing energy from a separately defined metric reservoir.

1.2. Result roadmap and model dependencies

The first part develops the physical response before any global interpretation. Sections 2–4 state the matching and source postulates, construct exact inward and outward branches, and prove an open-potential full-halving corridor with explicit response and preparation bounds. Sections 5–7 establish attained entry, turn and exit for a different preparation; add conserved component and charge competition; and retain fixed response protocols through a turn. Sections 8–9 then construct and globally receive the trapped spherical experiments. The remaining sections follow that one completed real-scalar family from horizon calibration to its invariant end and reconstruction limits.

Table 1. Model classes and the scope of the proved results.

Result family	Fixed source and preparation	Proved scope and dependency
Exact and profile-driven response	Flat canonical scalar with general normalization and stated profile inequalities	Finite inward response, pressure work and acceleration bounds
Robust full halving	$\phi = 2 M d$, open local potentials on $[\cdot 2475, \cdot 505]$, constrained data near $d_i = .5$	Full halving, decreasing independent insertions and controlled response error
Attained entry and competing exits	Fixed local $W_0 = 3 - 8 d^2 + 3 d^4$; separate real-scalar, component and charged preparations	Actual event sequences in each declared local model, not one concatenated history
Trapped and globally received experiments	Completed nonnegative real-scalar potential, four KS boxes and a smooth global constraint construction	All recorded central experiments lie in a one-ended asymptotically flat black-hole development
Fixed horizon calibration	An explicit subclass of that same allowed exterior completion	Exact initial horizon, twenty event calibrations and constructive nonuniqueness under changed completions

Result family	Fixed source and preparation	Proved scope and dependency
Invariant future end	The same completed source and archived finite records	Finite-time homogeneous end for all four boxes; entire approach received in the unchanged global spacetime for the two positive-shear boxes, with a local continuous-metric endpoint obstruction for their central strip
Exterior reconstruction and regular-side representation	Fixed preparation family, marking, completion and declared witness contract	Exact all-readers exterior ambiguity; preparation-conditional inverse; faithful representation on original domains
Residual-support exhaustion	Independently fixed original BH burden, witness subtraction and complete successor duties	Specific endpoint uses are excluded; the universal common-duty bridge and full AMetric closure remain unproved

The full-halving and loop preparations must not be concatenated. The corridor requires $|Q_i - (1 - d_i^2)| < .0009$ near $d_i = .5$. On the prepared inward loop, $\Delta = Q - (1 - d^2)$ starts below $-.005$ and decreases until the turn. That loop does not enter the corridor's preparation strip. The certified spherical records stay above $d = .33230$ and do not execute the exact reference segment from $.5$ to $.25$. Their shared constitutive mechanism supports comparison, not a certified chronological identification.

The global completion agrees with W_0 throughout the recorded local experiment and changes it only outside the specified field window. The later endpoint depends on that completed potential; extending the uncompleted polynomial would be a different problem. The completion does not receive every profile, uncertain potential, fluid or charged construction by implication. Finite-dimensional openness of homogeneous data also does not assert stability under arbitrary spatial or nonspherical perturbations.

The horizon calibration is likewise a theorem about the completed classical model. Its denominator is the area of a fixed initial event-horizon cut in each solution; distinct completions of the same local experiment can give different values. The zero area ratio at the received invariant end is a one-sided metric limit. It assigns no radius, area, volume, clock or energy to an AMetric regime.

1.3. Proof status and the final inference

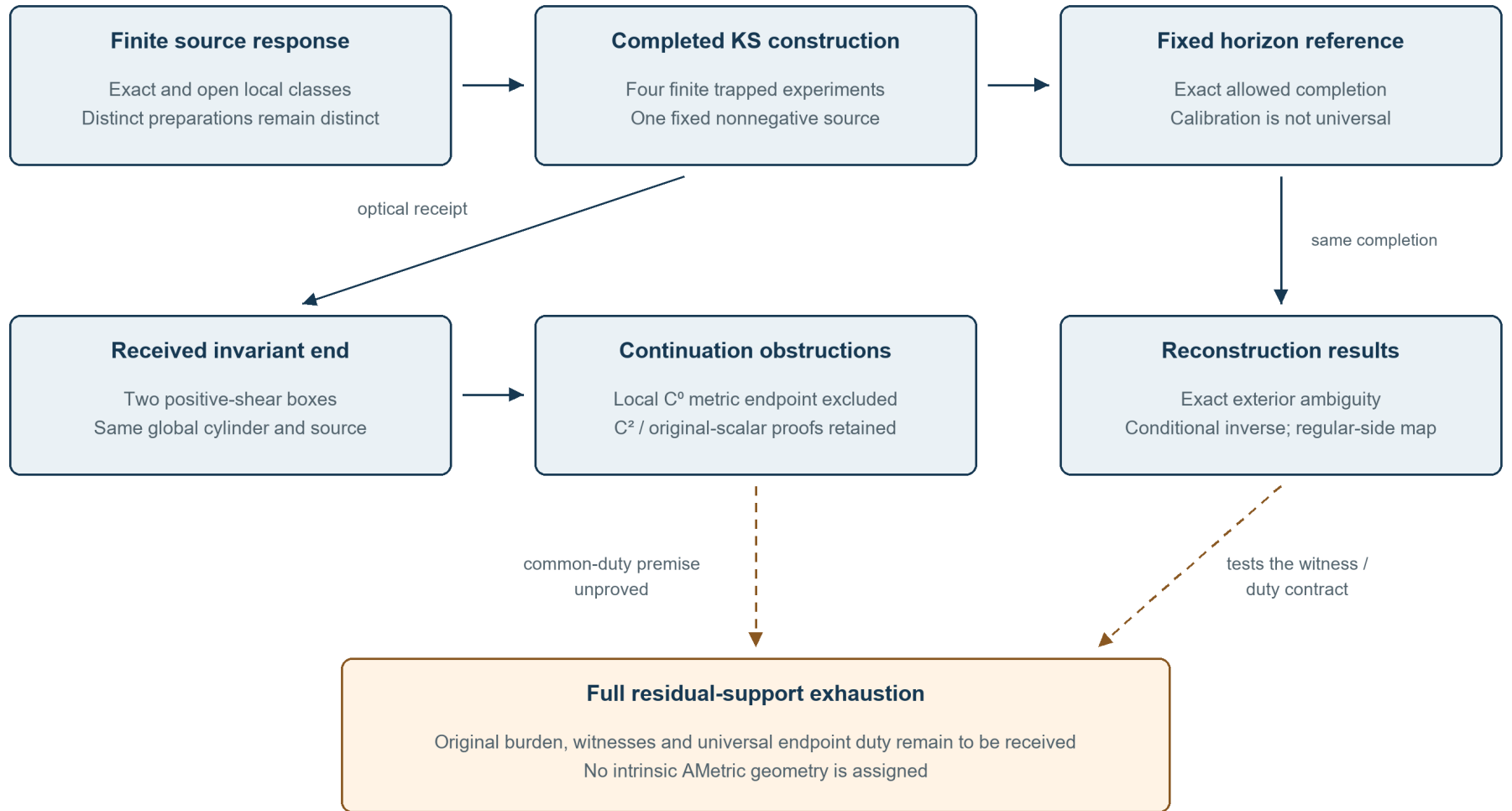
The analytic statements below include their proofs. The finite component and KS event statements use continuous-step interval enclosures under their recorded arithmetic premises. Ordinary numerical center values illustrate those results but do not supply whole-box bounds. The later positive-box endpoint receipt is derived from analytic tail estimates and the existing enclosure records; it is not inferred from an unvalidated numerical continuation. Appendix C distinguishes these evidence types and identifies the reproducible records.

An invariantly divergent scalar curvature rules out a nondegenerate $\mathcal{C}^2$ extension giving this approach a regular endpoint. Divergence of the original scalar separately rules out an endpoint on which that field is finite and continuous, even with a weaker metric. Those two arguments alone do not decide metric-only $\mathcal{C}^0$ or $\mathcal{C}^1$ extendibility. Theorem 27 supplies the further geometric argument: in either received positive-shear family, no smooth isometric embedding into a same-dimensional continuous nondegenerate Lorentzian manifold can give an added limit point to a sequence approaching the finite end with its longitudinal coordinate tending to an interior point of the central strip. Accessibility is derived from the proposed limit point. The proof uses the globally hyperbolic original development, shrinking causal travel and divergent longitudinal distance; it imposes neither symmetry nor Einstein or scalar equations on the extension [P11, Proposition 2.2 and local interior argument; P12, Proposition 4]. It does not classify every boundary of the entire spacetime or receive

the negative-shear futures. The original $\mathcal{C}^2$ and scalar proofs remain useful because they isolate distinct curvature and source duties without the additional diameter argument.

The final problem is semantic as well as physical. A reconstruction from the complete future exterior must distinguish all permitted interiors, and a faithful replacement must preserve every original question, operation domain, incidence and supplied input. We prove exact exterior collisions and construct exact original-domain inverse readers to test those requirements. They do not certify a surviving residual successor. Conversely, failure of a particular endpoint operation excludes every faithful realization of that duty without proving that all residual successors have the duty. Section 14 fixes the direction and remaining hypotheses of that universal bridge under the complete Kernel, reconstruction and exhaustion machinery [A10–A16].

The paper thus answers the motivating question in stages: a finite, conserved pressure response is physically realizable under declared matching assumptions; a received black-hole family has a genuine later invariant end and a proved local continuous-metric obstruction; and the passage from that end to loss of all residual metric support remains a specific original-duty theorem to be established. No finite audit, label change, absent counterexample or forward extrapolation beyond a lost application supplies that theorem.



Solid arrows: proved dependencies within stated classes. Dashed arrows: remaining receiving obligations.

Figure 1. The article follows a specified completed KS branch from finite source response to globally received invariant failure and a local continuous-metric endpoint obstruction. Different local source and preparation classes are not one trajectory. Solid arrows show dependencies within the stated classes. Dashed arrows mark the still-unproved receipt of the original burden, witnesses and universal endpoint duty; they do not depict a physical continuation beyond metric license.

2. Model postulates and operational meaning

2.1. A differentiation coordinate

Postulate D — source-response matching. Fix, before selecting a history, a family of source-native preparations and response comparisons associated with designated distinctions in $\mathcal{F}_D$. The family may span joint response, charge or gauge, deformation and sector roles; it is not restricted to particle counts or a final numerical projection. Let $C_\alpha \geq 0$ be their dimensionless separation readouts, defined on their original common-input domains and invariant under the faithful redescriptions admitted for those uses. On the proposed effective family, assume

$$C_\alpha(d) = c_\alpha d^{p_\alpha}, c_\alpha > 0, p_\alpha > 0, d > 0. (1)$$

One calibrated readout fixes d ; the others give substantive cross-response predictions. For example, a calibration with $p_* = 1$ gives $d = C_*/c_*$. The remaining relations then cannot be recovered merely by renaming the coordinate. Their domains, preparations and comparison rules stay fixed as the state changes. If two originally distinct roles have become lawfully identified, the relevant receiving map must preserve their operation meanings and shared-input incidences; equality of two displayed numbers is not sufficient.

Equation (1) is a postulate about a candidate matched family, not a demonstrated microscopic map for all AASC roles. It represents the coordinated loss of designated contrasts effectively. Their actual common realization, and the treatment of any application that withdraws before a common limit, belong to the source matching. This matching retains the bearer's underlying standing, identity, structural comparison and transport, and preserves earlier valid occurrences and records. It does not treat the two families as independent Cartesian factors. A finite list of probes cannot establish coverage of the entire original role family. These limitations specify what is modeled; they do not invalidate the conditional experiment.

For $p_\alpha = 1$, halving d halves every designated contrast. For general exponents it multiplies each by 2^{-p_α} . Pointwise disappearance as $d \rightarrow 0$ follows for each designated readout if the matching remains valid throughout that approach; a finite-window match alone does not authorize that extrapolation. Uniform disappearance over an infinite family requires additional uniform control. A defined zero response is not, by itself, the loss of that response's application.

2.2. An explicit effective source

Postulate E — local Einstein-scalar closure. On a licensed interval, assume a spatially flat homogeneous metric

$$ds^2 = -dt^2 + a(t)^2 dx^2,$$

and a single canonical scalar $\phi = \mu d$, with $\mu > 0$. Units are $c = \hbar = 1$; the reduced Planck mass satisfies $M^2 = (8\pi G)^{-1}$. The effective action is

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} \mathcal{R} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]. (2)$$

Here $\mathcal{R}$ is the spacetime Ricci scalar. There is no additional independent matter component, phenomenological energy drain or separately adjustable cosmological constant in this closure. The potential includes the full effective potential energy. The matching assumption is that the retained microscopic response channels are represented by this collective degree of freedom and its effective coefficients on the stated interval; no omitted independently evolving stress component is appreciable there. If a receiving source

requires another active field or fluid, its stress and exchange terms must be included and the present one-field solution no longer applies unchanged.

This is a changed, openly declared effective theory. It is not asserted to be the complete CSP source with selected gauge, Yukawa or fermionic terms deleted. The probe relations (1) do not add independent reservoirs of energy on top of (2). They are constitutive readouts of the same effective organization. A microscopic derivation of that closure would be a further application of the model.

Postulate V — a local differentiation potential. Choose $h > 0$, $\gamma > 0$, set $\mu = 2M/\sqrt{\gamma}$, and define

$$V(d) = M^2 h^2 [3(1 - d^2)^2 - 2\gamma d^2]. \quad (3)$$

The scale h is an inverse proper time within the licensed domain. Its identification with a measured source scale is part of matching; it is not set by governance depth. The coefficient γ controls the rate of change of the differentiation amplitude. We use (3) on a stated local window $0 < d < d_{\max} < 1$, chosen so that the potential is positive and the desired transition is included. No claim about a global vacuum or reheating completion is made from the polynomial outside that window.

2.3. Why a scalar construction is appropriate here

A canonical scalar supplies a minimal conserved source that can interpolate between potential-dominated and more differentiated effective states while retaining a local dynamical equation. Vacuum-like stress as a driver of expansion is established inflationary physics [P4], and the Hamilton–Jacobi method for constructing scalar solutions is standard [P1]. The new proposal is the conditional interpretation and matching of its collective coordinate to designated role differentiation, together with examination of the inward branch and its possible support-interface termination. No priority is claimed for the underlying Einstein-scalar mathematics.

Spatially flat homogeneity supplies a controlled reference problem. It is not the generic geometry of a black-hole interior, and the flat results alone establish neither a trapped surface nor matching to an exterior black-hole solution. Application to a particular interior requires an actual receiving patch, its initial data and any required anisotropy or inhomogeneity. The trapped-spherical and global-reception sections below supply that geometry and receipt for a specified completed real-scalar family; they do not receive every general profile or uncertain potential considered here. The flat reference model asks what response a patch could support if the stated approximation and role matching hold.

3. Exact source response and the finite inward reference

3.1. Conservation and the paired exact solutions

The equations of (2) are

$$\begin{aligned} 3M^2 H^2 = \rho = \frac{1}{2} \dot{\phi}^2 + V, \quad \dot{H} = -\frac{\dot{\phi}^2}{2M^2}, \quad (4) \\ \ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0, \end{aligned}$$

where $H = \dot{a}/a$. The pressure is $p = 1/2 \dot{\phi}^2 - V$. All equations concern the same field and state.

Proposition 1 — closed stress and pressure work. On solutions of (4),

$$\dot{\rho} + 3H(\rho + p) = 0. \quad (5)$$

For a fixed comoving cell of physical volume $\mathcal{V} = \mathcal{V}_0 a^3$, its matter energy $E = \rho \mathcal{V}$ obeys

$$\dot{E} = -p \dot{\mathcal{V}}. \quad (6)$$

Proof. Differentiate $\rho = \dot{\phi}^2/2 + V$ and substitute the scalar equation. Since $\rho + p = \dot{\phi}^2$, (5) follows. The product rule and $\dot{\mathcal{V}} = 3H\mathcal{V}$ give (6). Equivalently, the scalar stress tensor is covariantly conserved on its equation of motion. These are local stress conservation and comoving pressure work, not an assertion of a generally defined global gravitational energy. $\square$

Proposition 2 — an expanding and a contracting branch of the same source. For $\varepsilon_b \in \{+1, -1\}$, (3) admits

$$H_{\varepsilon_b} = \varepsilon_b h(1 - d^2), \dot{d} = \varepsilon_b \gamma h d, (7)$$

and therefore, with $\tau = h(t - t_i)$,

$$\begin{aligned} d_{\varepsilon_b}(\tau) &= d_i e^{\varepsilon_b \gamma \tau}, \\ \frac{a_{\varepsilon_b}(\tau)}{a_i} &= \exp \left[\varepsilon_b \tau - \frac{d_i^2}{2\gamma} (e^{2\varepsilon_b \gamma \tau} - 1) \right]. \end{aligned} (8)$$

These solutions exist on every interval on which d_{ε_b} remains in the chosen local window and the source-to-role and metric applications remain valid.

Proof. For a monotone scalar, the Hamilton–Jacobi relations are $\dot{\phi} = -2M^2 H_{,\phi}$ and $V = 3M^2 H^2 - 2M^4 H_{,\phi}^2$ [P1]. Substitution of H_{ε_b} and $\mu^2 = 4M^2/\gamma$ gives (3) and (7). Direct substitution also verifies all three equations (4), including the scalar equation; no fluid equation of state is imposed independently. Integrating $\dot{d}/d = \varepsilon_b \gamma h$ and $\dot{a}/a = H_{\varepsilon_b}$ gives (8). $\square$

On either branch,

$$\begin{aligned} \rho &= 3M^2 h^2 (1 - d^2)^2, \\ p &= M^2 h^2 [4\gamma d^2 - 3(1 - d^2)^2], \\ \rho + p &= 4\gamma M^2 h^2 d^2, \\ w \equiv p/\rho &= -1 + \frac{4\gamma d^2}{3(1 - d^2)^2}. \end{aligned} (9)$$

The same d has the same source density and pressure on the two branches, but opposite signs of H and $\dot{d}$. Equations (4) are invariant under the corresponding reversal of time and velocities. This mathematical relation constructs two admissible histories of one model; it does not establish that an actual black-hole history is the temporal inverse of cosmological inflation. Their preparations, clocks and larger receiving contexts may differ. The inward solution climbs toward the potential maximum with the selected initial velocity; contraction or high density alone does not prepare that motion.

At $d=0$, the formal model has de Sitter geometry, nonzero potential $3M^2 h^2$ and scalar fluctuation modes. Thus $d=0$ is an undifferentiated *effective-order* limit, not the bare persistence bearer or AMetric closure. An exact solution initially at $d=0$ stays there; the outward branch needs a nonzero seed. On the inward branch, $d=0$ is reached only as $t \rightarrow \infty$ if the metric model remains applicable that long.

3.2. What the response does to expansion and contraction

Define the usual Hubble acceleration parameter

$$\epsilon_H = -\frac{\dot{H}}{H^2} = \frac{2\gamma d^2}{(1 - d^2)^2}. (10)$$

Proposition 3 — differentiation exit and inward braking. On $0 < d < 1$, ϵ_H increases strictly with d , and

$$\frac{\ddot{a}}{a} = h^2 [(1 - d^2)^2 - 2\gamma d^2]. \quad (11)$$

Acceleration is positive precisely when $d < d_e$, where

$$d_e^2 = 1 + \gamma - \sqrt{\gamma(\gamma + 2)}. \quad (12)$$

The outward branch crosses from acceleration to deceleration as d increases through d_e . The inward branch enters or remains in positive acceleration as d decreases below d_e ; while $\dot{a} < 0$, this reduces $|\dot{a}|$, rather than reversing its sign.

Proof. Equation (11) is $H^2 + \dot{H}$. Differentiating (10) shows strict increase for $0 < d < 1$. Solving $(1 - d^2)^2 = 2\gamma d^2$ gives the unique root (12) in that interval. The sign of $\dot{d}$ in (7) determines the direction of crossing. Finally, while $\dot{a} < 0$, $d|\dot{a}|/d t = -\ddot{a}$. $\square$

The exit in Proposition 3 is the end of accelerated expansion in this effective source. It is not a derivation of thermalization, particle production, a stable late vacuum or the complete reheating bridge. Those require further degrees of freedom or a subsequent matched regime. The distinction also applies to the motivation: the model shows that an explicit differentiation-dependent potential *can* make the two effects correlate; the published structural inflation role alone does not select that potential.

Proposition 4 — no flat canonical bounce, and no inward Hubble-length analogue of inflationary shrinking. Every solution of (4) satisfies $\dot{H} \leq 0$. A contracting solution therefore cannot cross upward through $H = 0$ into expansion while this closure applies. On the exact inward branch in the accelerating interval, the comoving Hubble length $(a|H|)^{-1}$ grows; on the outward accelerating branch it shrinks.

Proof. The first assertion follows immediately from (4), independently of the sign of V . On a contracting branch, $a|H| = -\dot{a}$, so positive $\ddot{a}$ makes $a|H|$ decrease. On an expanding branch, $aH = \dot{a}$, which increases under the same acceleration sign. Taking reciprocals proves the second assertion. $\square$

An inward inflation-like *source response* consequently does not reproduce the usual expanding-inflation behavior of the comoving Hubble scale. A Hubble length is not by itself an event horizon, but the difference is enough to prevent importing the usual mode-exit story unchanged. Nor is any faster-than-light material motion involved. The canonical scalar's principal propagation speed is the local light speed; recession or separation rates between distant points are a different quantity. The negative-pressure response is specified by the stress in (9), not caused by a separation speed exceeding light speed.

For inward evolution, d decreases and ρ rises toward $3M^2 h^2$, while w approaches -1 . In the positive-acceleration interval $p < 0$; the decreasing physical volume then gives $\dot{E} < 0$ by (6). Density rises although the energy of a comoving cell falls. The kinetic contribution decreases while the potential contribution increases; the changes are governed together by the scalar equation and pressure work, rather than by equating the potential gain to the kinetic loss alone. No extra sink in "the metric" is required or introduced.

3.3. A worked finite inward regime

Set $\gamma = 1$, so $\mu = 2M$, and use the local window $0 < d < 0.6$. Here the potential is positive; the zero of (3) occurs at $d_V = \sqrt{(4 - \sqrt{7})/3} \approx 0.671875$, outside the window. The acceleration exit

$d_e = \sqrt{2 - \sqrt{3}} \approx 0.517638$ lies inside it. Thus the same local potential includes an outward exit and a substantial inward accelerating interval.

Choose the inward branch from $d_i = 1/2$ to $d_f = 1/4$, and normalize $a_i = 1$. Equations (8) give

$$\Delta t = \frac{\log 2}{h}, N_f \equiv -\log(a_f/a_i) = \log 2 - \frac{3}{32} \simeq 0.599397, a_f \simeq 0.549143. (13)$$

Here N_f counts contraction e-folds, not elapsed governance depth. The duration is a licensed proper-time duration; no identification of h with a GUT or closure scale is needed.

Table 2. Exact reference values for the finite halving experiment.

Quantity	Initial $d=1/2$	Final $d=1/4$
H/h	-0.75	-0.9375
$\rho/(M^2 h^2)$	$27/16=1.6875$	$675/256=2.63671875$
$p/(M^2 h^2)$	$-11/16=-0.6875$	$-611/256=-2.38671875$
w	$-11/27 \simeq -0.407407$	$-611/675 \simeq -0.905185$
ϵ_H	$8/9$	$32/225$
$\dot{\phi}^2/(2M^2 h^2)$	$1/2$	$1/8$
$V/(M^2 h^2)$	$19/16$	$643/256$

Every designated contrast with $p_\alpha=1$ halves. The density increases by a factor $25/16$, while the energy in the same comoving cell changes by

$$\frac{E_f}{E_i} = \frac{25}{16} e^{-3N_f} \simeq 0.258747. (14)$$

The magnitude of the contraction speed changes by $|\dot{a}_f|/|\dot{a}_i| = (5/4) e^{-N_f} \simeq 0.6864$. Thus contraction slows by about 31% even though $|H|$ grows. These are different statements: $|H|$ measures fractional contraction per unit proper time, whereas $|\dot{a}|$ measures the rate of change of the chosen scale factor. Their simultaneous behavior follows exactly from (8), without any exotic energy condition violation.

The worked regime is nonempty and finite. It is an exact accelerated solution, not a slow-roll approximation: $\eta_H = 2M^2 H_{,\phi\phi}/H = -\gamma/(1-d^2)$ has magnitude greater than one in this benchmark. It does not reach $d=0$, and it requires no assertion that its observed contrasts already exhaust the full bearer-return target. It demonstrates the specific transitional possibility at issue: correlated loss of designated differentiation can coexist with negative-pressure braking in a conserved source.

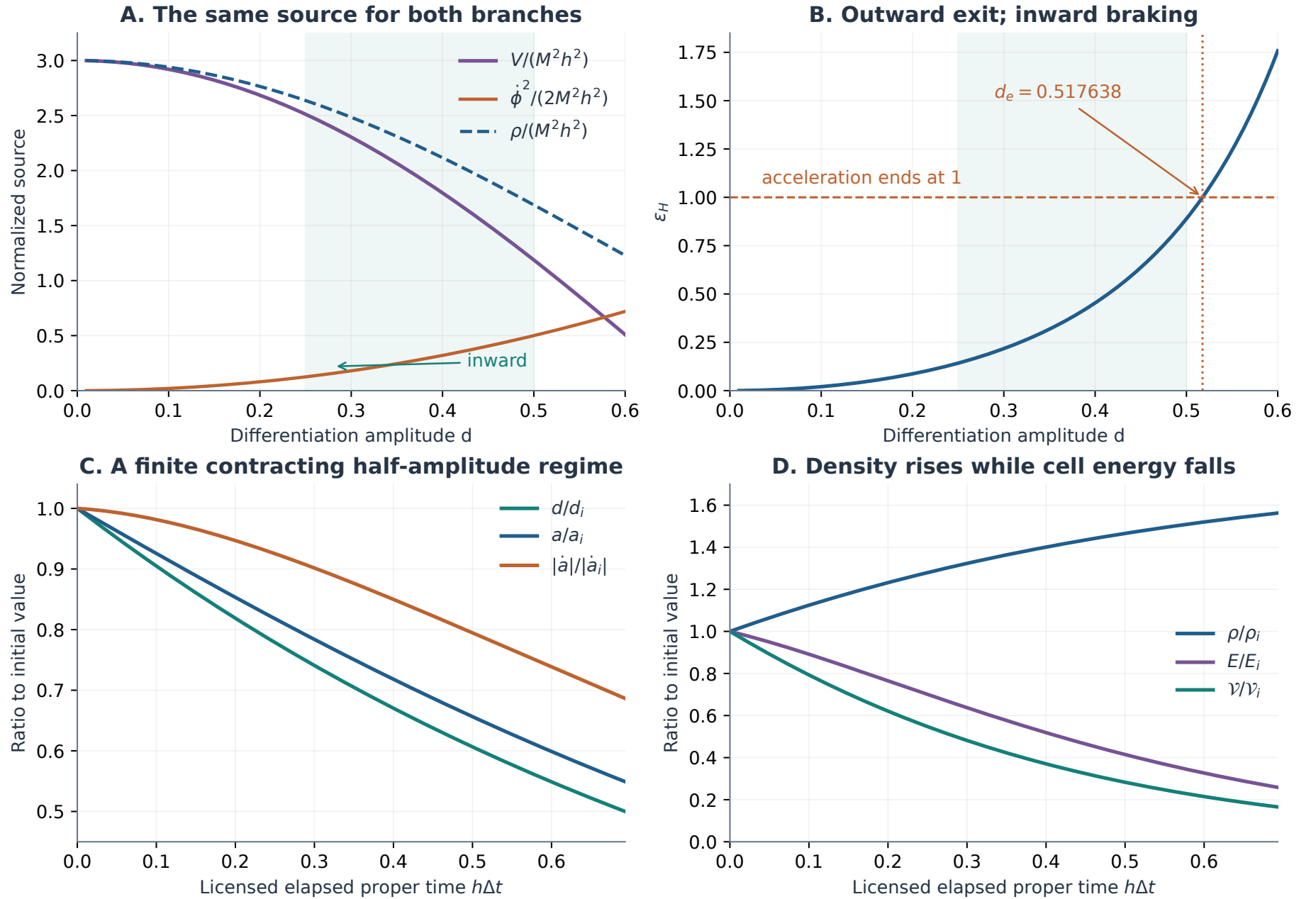


Figure 2. The $\gamma=1$ model. Panels A–B show the common source and acceleration threshold as functions of the differentiation amplitude; the shaded band is the inward benchmark. Panels C–D show its finite proper-time evolution. These are exact model curves; no source-to-support cutoff is inferred from them.

The accompanying computation integrates the full homogeneous scalar and Einstein equations independently of the exact solution. It checks the Friedmann constraint and pressure-work identity and compares successive step refinements. Both signs of 0.1% and 1% changes in initial inward field velocity also reach $d=1/4$ with positive acceleration on the sampled trajectories. These numerical checks are supporting evidence, not a replacement for the exact solution or the nonlinear open-potential and initial-data theorem below.

The remaining analysis separates three questions: which nearby source histories preserve this finite response, which original response channels actually decrease along them, and what limits a longer or more complicated inward phase. General profile inequalities first identify the mechanism; an explicit nonlinear corridor then supplies the main preparation and duration guarantee.

4. Robust reduction and its preparation cost

4.1. A wider family of inward source responses

The polynomial reference is one member of a much larger class. The following result isolates the shape conditions responsible for the response. It uses the established Hamilton–Jacobi construction [P1]; it does not claim that choosing a profile proves that arbitrary initial data follow it.

Theorem 5 — a finite inward response from profile inequalities. Let $\beta = 2 M^2 / \mu^2 > 0$, $h > 0$, and let $F \in C^3([d_f, d_i])$ obey

$$F > 0, F_{,d} < 0, F_{,dd} < 0, \beta F_{,d}^2 < F^2. \quad (15)$$

Define the effective potential and two prepared histories by

$$V = M^2 h^2 (3 F^2 - \beta F_{,d}^2), H_{\varepsilon_b} = \varepsilon_b h F, \dot{d} = -\varepsilon_b \beta h F_{,d}, \quad (16)$$

$$\varepsilon_b = \pm 1.$$

They solve the canonical field and Einstein equations. On the inward history $\varepsilon_b = -1$, the field coordinate and kinetic energy decrease, potential energy and density increase, the equation of state becomes more vacuum-like, and contraction slows without reversing. Both the proper-time duration and the accumulated contraction between the two field values are finite.

Proof. The kinetic energy is $T_\phi = \beta M^2 h^2 F_{,d}^2$, the density is $3 M^2 h^2 F^2$, and $\dot{H} = -\beta h^2 F_{,d}^2 = -\dot{\phi}^2 / (2 M^2)$. These identities verify the constraint and Raychaudhuri equation; their derivative gives the scalar equation because $\dot{\phi} \neq 0$. Acceleration is $h^2 (F^2 - \beta F_{,d}^2) > 0$. Along the inward branch,

$$\begin{aligned} \dot{T}_\phi &= 2 \beta^2 M^2 h^3 F_{,d}^2 F_{,dd} < 0, \\ V_{,d} &= 2 M^2 h^2 F_{,d} (3 F - \beta F_{,dd}) < 0. \end{aligned} \quad (17)$$

Thus $\dot{V} > 0$. Also $\epsilon_H = \beta (F_{,d}/F)^2$ has positive derivative with respect to d , since $F_{,d} < 0$ and $F F_{,dd} - F_{,d}^2 < 0$. It decreases in inward time, as does $w = -1 + 2\epsilon_H/3$. Density growth and pressure work follow from Proposition 1. Finally, put $m = \min(-F_{,d}) > 0$, $L = \max(-F_{,d})$, and $F_{\min} = \min F$, $F_{\max} = \max F$. Integration gives

$$\begin{aligned} \frac{d_i - d_f}{\beta L} \leq h \Delta t \leq \frac{d_i - d_f}{\beta m}, N = \frac{1}{\beta} \int_{d_f}^{d_i} \frac{F}{-F_{,d}} dd, \\ \frac{F_{\min}(d_i - d_f)}{\beta L} \leq N \leq \frac{F_{\max}(d_i - d_f)}{\beta m}. \end{aligned} \quad (18)$$

Here and below $N = \log(a_i/a)$ denotes accumulated contraction. All signs have strict compact-interval margins. $\square$

For example, $F = 1 - d^2 + \eta d^4$, $\beta = 1/2$, $d \in [1/4, 1/2]$, and $|\eta| \leq 1/10$ satisfy the theorem. One has $F \geq 0.74375$, $|F_{,d}| \leq 1.05$, $F_{,dd} \leq -1.7$, and the conservative acceleration margin is at least $0.74375^2 - (1/2)1.05^2 = 49/25600 > 0$. The generated potential is generally of degree eight. The strict conditions persist under sufficiently small C^2 profile changes on this interval.

This establishes a class of source shapes, not an attractor. Openness of profile inequalities alone does not establish a quantified neighborhood of arbitrary potentials and initial data. That stronger statement is the purpose of the next theorem. Neither result requires slow roll or selects a microscopic source-to-role match.

4.2. An open potential and initial-data class through a full halving

The main quantitative result replaces exact preparation by a finite-width class of admissible preparations and permits uncertain potential shape. Its proof follows the nonlinear constraint in field coordinates. It does not infer a finite neighborhood from a successful sample of integrations.

Fix $\mu = 2M$, $\tau = h(t - t_i)$, $Q = -H/h$, and $v = -d'$. From here onward, a prime denotes differentiation with respect to τ ; field and spatial derivatives have explicit subscripts. On the local band $J = [0.2475, 0.505]$, put

$$\begin{aligned} W(d) = \frac{V(d)}{M^2 h^2} = 3(1 - d^2)^2 - 2d^2 + \Delta W(d), \\ |\Delta W(d)| < 0.005 d^2, |(\Delta W)_{,d}(d)| < 0.01 d. \end{aligned} \quad (19)$$

The subscript $,d$ denotes a field derivative. The function ΔW is C^2 on a neighborhood of J . These strict conditions define an open local potential class in the C^2 topology; they do not constrain an unknown global potential or require a stationary point at the origin. The complete perturbed potential supplies the same conserved scalar stress. It is not an external force or an energy drain.

Theorem 6 — robust inward halving. For every potential (19), take initial data

$$\begin{aligned} 0.495 < d_i < 0.505, |Q_i - (1 - d_i^2)| < 0.0009, \\ v_i = \sqrt{\frac{3Q_i^2 - W(d_i)}{2}} > 0. \end{aligned} \quad (20)$$

The canonical solution reaches $d_f = d_i/2$ in finite time. Throughout that interval it is contracting, its field amplitude and dimensional kinetic energy strictly decrease, its potential fraction strictly increases, and its positive acceleration per unit scale factor, $\ddot{a}/a$, increases. With $m = \sqrt{159/400}$ and $z = \sqrt{641/400}$, it obeys

$$|Q - (1 - d^2)| < \frac{1}{5} d^2, m d < v < z d, v' < -0.19980 d, \quad (21)$$

$$\frac{\ddot{a}}{a h^2} > 0.040.$$

Its proper-time duration and contraction satisfy the uniform bounds

$$\frac{\log 2}{z} \leq h \Delta t \leq \frac{\log 2}{m},$$

$$\frac{\log 2 - \frac{3}{8}(1 + \frac{1}{5})(0.505)^2}{z} \leq N_f \leq \frac{\log 2 - \frac{3}{8}(1 - \frac{1}{5})(0.495)^2}{m}. \quad (22)$$

Thus $0.547553 < h \Delta t < 1.099403$ and $0.456897 < N_f < 0.982813$ are conservative outward-rounded summaries of the exact bounds. The initial-data set is open on the contracting constrained two-dimensional scalar section, with coordinates (d_i, Q_i) . No measure or probability of these preparations is asserted.

Proof outline. The exact equations are

$$3 Q^2 = 2 v^2 + W, d' = -v, Q' = 2 v^2, v' = 3 Q v - g(d), \quad (23)$$

$$g(d) = -W_{,d}/4 = d(4 - 3 d^2) - (\Delta W)_{,d}/4.$$

Write $f = 1 - d^2$, $\Delta = Q - f$, and $\ell_d = d_i - d$. Rationalizing the constraint gives

$$\Delta_{\ell_d} = \frac{12 f \Delta + 6 \Delta^2 - 2 \Delta W}{\sqrt{4 d^2 + 12 f \Delta + 6 \Delta^2 - 2 \Delta W + 2 d}}. \quad (24)$$

Appendix A bounds this nonlinear equation in the strip $|\Delta| \leq d^2/5$. The resulting strict estimate is $|\Delta| < 0.012 < 0.01225125$, the smallest allowed strip width before the half crossing. A first-exit argument therefore closes the strip for the complete interval. Both Q and v have strictly positive lower bounds there, excluding a field turn and guaranteeing actual finite crossing. The same strip gives $g(d) - 3 Q v > 0.19980 d$, so v decreases while Q increases. Hence kinetic energy decreases and the potential fraction grows. Finally $\mathcal{A} = Q^2 - 2 v^2$ starts above 0.040, and $\mathcal{A}' = 4 v (g(d) - 2 Q v) > 0$. Integrating $d' = -v$ and $N' = Q$ proves (22). The appendix gives the full estimates, including elementary rational bounds; numerical curves are not used to establish the theorem. $\square$

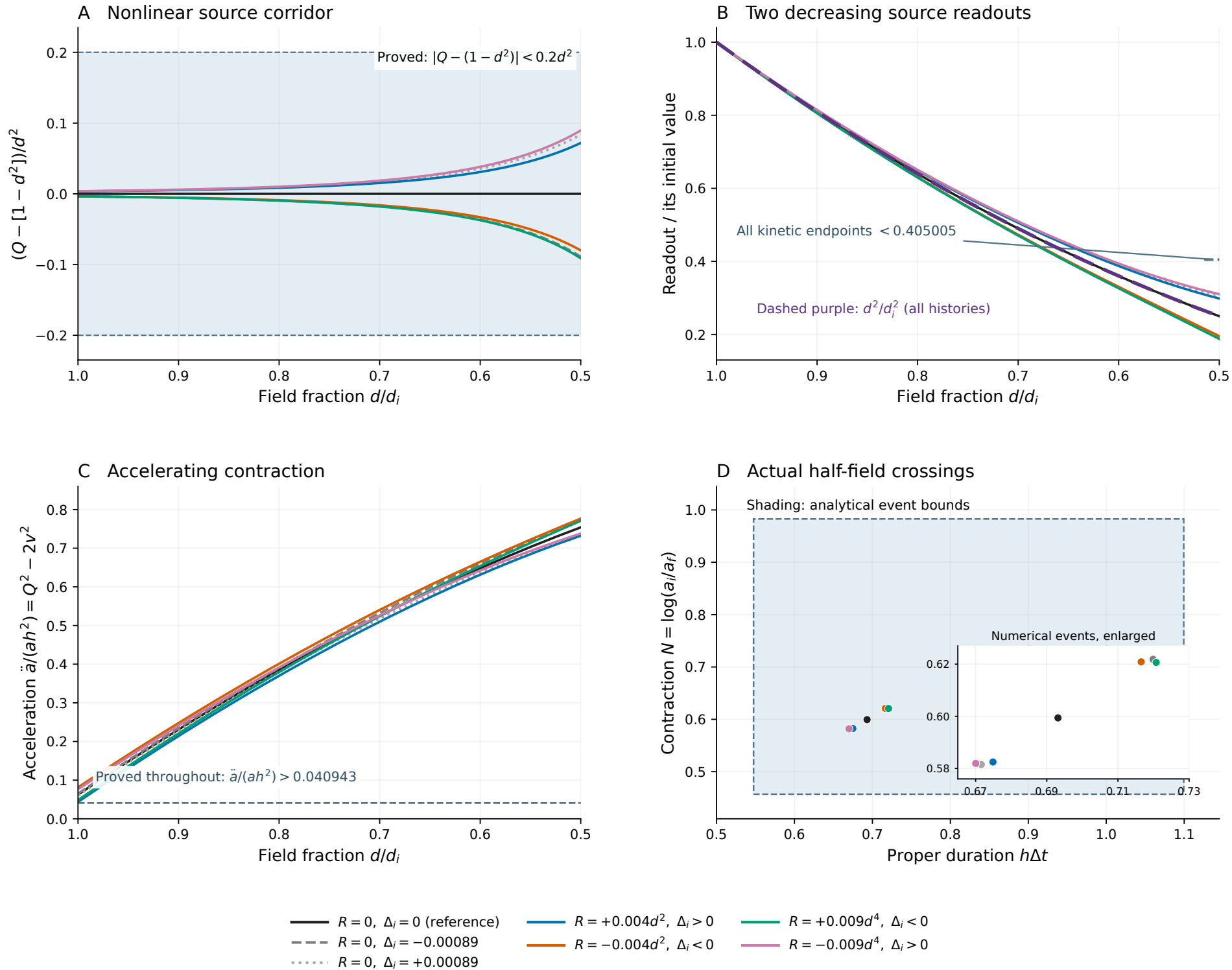
4.2.1. Preparation width and genuine potential uncertainty

At $\Delta W = 0$ and $d_i = 1/2$, the admitted set includes every relative initial inward-velocity perturbation $|r| \leq 0.004$, where $v_i = (1 + r)/2$. This is a rigorous $\pm 0.4\%$ preparation interval; a generic four-variable Gronwall bound gives the smaller sufficient interval $\pm 0.0018\%$. That comparison concerns two sufficient guarantees, not a measured basin size. The stronger theorem also allows the initial field section to vary and an infinite-dimensional class of potential corrections. It does not include arbitrary shear or additional fields; those require the separate analysis below.

The potential class is demonstrably nonempty beyond the reference polynomial. For example, $\Delta W = \pm 0.004 d^2$ and $\Delta W = 0.008 d^4$ satisfy (19) throughout J . Perturbed histories are solved using the full equations (23), not forced onto the reference $Q = f$. Their target times differ. The proof guarantees the signs and finite bounds for the entire class; computations illustrate its behavior and check implementation.

A finite inward regime for open potentials and initial data

Full canonical source: $W = 3(1 - d^2)^2 - 2d^2 + R(d)$; every event is $d_f = d_i/2$



Lines and points: full ODE illustrations. Shading and labelled bounds: analytical theorem.
The open-data guarantee is finite; it is not an attractor, an anisotropic stability theorem, or a metric-exhaustion result.

Figure 3. The analytic scalar corridor and seven full-equation examples: the reference history, lower and upper initial Q , quadratic corrections $\Delta W = \pm 0.004 d^2$, and quartic corrections $\Delta W = \pm 0.009 d^4$, with the declared constrained preparations. The admitted strip and rate bounds are analytical guarantees. The plotted histories are numerical examples inside that class; their actual halving events need not occur at a common time. In the preserved figure, $R(d)$ denotes the potential perturbation $\Delta W(d)$; the two normalized readouts are O_d and O_w , and panel C plots $\mathcal{A}$. This is finite persistence of the stated response, not attraction toward the reference curve.

4.2.2. Wider preparation ranges over shorter field bands

The full-halving certificate favors a substantial field reduction. A different comparison gives wider speed intervals when the required excursion is shorter, without constructing a Hamilton–Jacobi reference profile. Define $\xi = -\dot{\phi}/(\sqrt{6} M |H|)$, $\lambda_V = -M V_{,\phi}/(\sqrt{6} V)$, and inward field distance $\mathcal{Y}_\phi = (\phi_i - \phi)/(\sqrt{6} M)$. For positive decreasing potentials the exact phase plane is $\xi_N = 3(1 - \xi^2)(\xi - \lambda_V)$, $\mathcal{Y}_{\phi,N} = \xi$, and $\epsilon_H = 3\xi^2$.

Appendix B proves two useful sufficient criteria. On a field band of width $M/10$ with potential ratio $1 < V_f/V_i < 26/25$, every $\xi_i \in (0.25, 0.45)$ crosses inward with positive acceleration. On a band of width $M/20$ with $0.305 < \lambda_V < 0.36$, every $\xi_i \in (0.21, 0.30)$ crosses with $0.10 < \xi < 0.30$ and increasing potential fraction. Restrictions of the same reference potential supply both examples. These intervals contain a substantial range of kinetic fractions, but cover less field change. The second criterion does not by itself prove decreasing dimensional kinetic energy. The distinction matters for interpreting independent response channels.

4.3. Independent response channels with controlled matching error

The exact power laws in Postulate D are restrictive once their original preparations and comparisons are fixed. They nevertheless describe a one-coordinate family. A physical neighborhood can explore directions away from that curve, and small errors in response values need not be small errors in response rates. We therefore state the stronger condition required to infer coordinated decrease.

Keep the original source, question family, common-input domains, normalization, clock and shared incidences fixed before choosing a history. A withdrawal of a response's application is recorded as withdrawal; no derivative is evaluated through an absent domain. The following theorem concerns current admitted responses, while preserving the bearer's standing, identity, comparison, transport and historical records. A finite diagnostic family is not silently promoted to the complete AASC role family. This distinction follows the original-domain and source-direction obligations of [A2, Definition 2.1 and Theorems 3.5, 4.3, 5.4; A14, Theorem 4.2].

Theorem 7 — controlled multichannel decrease. On a compact received state tube, let the actual dynamics be $x' = F_0(x) + r(\tau, x)$, with $\tau = h(t - t_i)$, and let the matched response to an original input q in channel α be

$$C_\alpha(\tau, x; q) = O_\alpha(x; q) + e_\alpha(\tau, x; q). \quad (25)$$

Suppose, uniformly on the claimed domains, that $DO_\alpha[F_0] \leq -m_\alpha < 0$, $\|DO_\alpha\|_* \leq G_\alpha$, $\|F_0\| \leq B$, $\|r\| \leq \epsilon_F$, $|\partial_\tau e_\alpha| \leq \epsilon_{t,\alpha}$, and $\|D_x e_\alpha\|_* \leq \epsilon_{x,\alpha}$. If

$$b_\alpha = G_\alpha \epsilon_F + \epsilon_{t,\alpha} + \epsilon_{x,\alpha}(B + \epsilon_F) < m_\alpha, \text{ then } \quad (26)$$

$$C'_\alpha \leq -(m_\alpha - b_\alpha) < 0.$$

Proof. Differentiate (25) along the actual velocity. The nominal term is at most $-m_\alpha$; the three other terms are bounded by the displayed dual-norm estimates. Integration also bounds the response change by $-(m_\alpha - b_\alpha)\Delta\tau$. $\square$

The substantive premise is a strict common decrease direction for the original response differentials. The chain-rule proof is elementary; defining a cone without supplying a physically occupied trajectory would make it empty of application. Theorem 6 supplies a nonempty dynamical region. A receiving construction must still establish its response matching. For infinitely many original inputs, uniform conclusions require the displayed bounds uniformly over that whole family. Faithful smooth changes of state presentation preserve the

directional derivatives; the chosen norms only express sufficient error budgets in fixed calibrated units. Any new field responsible for r must have its full stress and evolution included in the dynamics establishing the tube.

4.3.1. Two independent ports in the effective source

A concrete example need not add an unaccounted spectator field. In the same scalar theory, with the already applicable comoving unit clock vector n^a , introduce independent compact source ports through

$$\Delta S[j_d, j_u] = \mu^2 h^2 \int \sqrt{-g} \left[j_d (\phi / \mu)^2 + j_u \left(\frac{n^a \nabla_a \phi}{\mu h} \right)^2 \right] d^4 x. \quad (27)$$

At zero ports the unprobed action and stress are unchanged. The normalized classical action insertions, obtained by differentiating with the fields and received clock prescription held fixed and then evaluating on the occupied homogeneous history, are

$$O_d = d^2, O_u = u^2 = v^2, D_{(d,u)}(O_d, O_u) = \begin{pmatrix} 2d & 0 \\ 0 & 2u \end{pmatrix}. \quad (28)$$

Here $u = d' = -v$. The determinant $4du$ is nonzero in the corridor: these are functionally independent phase-space responses to different ports, even though their values coincide on the exact curve $u = -d$. They are classical insertion readouts, not quantum expectations, susceptibilities or a detector construction. Positive normalized spatial smearing gives the same readouts on a homogeneous history, so this entire declared smearing class is received without inferring coverage from sampled tests. A genuine open probe neighborhood exists: $|j_u| < 1/4$ leaves the temporal kinetic coefficient $1/2 + j_u > 1/4$ and the local scalar equation hyperbolic. Nonzero varying ports would require their full metric, field and clock variations. The rank statement concerns current phase-space/source-direction independence; it does not assert irreducibility against every whole-history question.

Theorem 6 gives $d \geq 0.2475$, $v > 0.630476 d$, and $u' > 0.19980 d$. Therefore conservative strict margins are

$$O_d' < -0.07724, O_u' < -0.01543. \quad (29)$$

For $x = (d, u, Q)$ in that same corridor, $\|F_0\|_\infty < 1.64$. With no additional dynamical error, a concrete matching budget $|e_\alpha| \leq 0.001$, $|\partial_\tau e_\alpha| \leq 0.001$, $\|D_x e_\alpha\|_1 \leq 0.001$ contributes at most 0.00264 to either rate. Both matched responses remain positive and strictly decreasing, with $C_d' < -0.07460$ and $C_u' < -0.01279$. The nominal amplitude readout falls by a factor four at the halving event; the conservative kinetic endpoint ratio is below 0.406, or below 0.412 after this value-error budget. The corresponding matched amplitude ratio is below 0.256. These are finite, independently testable response predictions, not merely two names for d .

A nonzero example fixes two slightly cross-coupled source insertions before the evolution: $C_d = d^2 + \lambda d u$ and $C_u = u^2 + \lambda d^2$, with $\lambda = 0.0005$. These follow by replacing the two port insertions in (27) by those declared expressions; the zero-port dynamics is still the same source. Their value and first-derivative discrepancies satisfy the displayed budget, and their response Jacobian remains nonsingular (its determinant has magnitude greater than 0.15407 throughout the corridor). Thus the matching allowance contains an explicit distinct source protocol, not only the identity match. Its normalization and source directions remain fixed throughout the comparison.

These ports are new effective diagnostics. The kinetic port explicitly retains metric and clock support. Neither their decrease nor equality establishes a merger of atlas-level roles, disappearance of an original operation's application, or complete bearer return. The example strengthens the matching discipline without pretending to have completed that larger source interpretation.

4.3.2. Two failures that delimit the result

Inward direction and positive acceleration do not suffice. In the unperturbed model, choose $d = 1/4$, $u = -2/5$, and the negative constraint root $H/h = -\sqrt{0.94390625}$. Acceleration is $0.62390625 h^2 > 0$, yet $u' = 0.953125 - 1.2\sqrt{0.94390625} < 0$, so $(u^2)' > 0$. The strict source-rate condition used above is a real restriction.

Nor does a small value error imply monotonicity. On a finite interval, $C(\tau) = e^{-2\tau} + \varepsilon \sin(\omega \tau)$ can remain positive and uniformly close to its decreasing nominal value while acquiring positive derivative for large ω . A value-error bound can still certify an endpoint decrease when the nominal gap exceeds twice that error; it cannot replace derivative control between endpoints.

4.4. Nonlinear separation and the cost of a long inward phase

The finite theorem should not be read as the discovery of a contracting attractor. The reference potential has

$$V_{,\phi\phi}(0) = -\gamma(3+\gamma)h^2, \delta \ddot{\phi} - 3h\delta \dot{\phi} - \gamma(3+\gamma)h^2\delta\phi = 0. \quad (30)$$

The homogeneous modes at the contracting de Sitter limit are $e^{-\gamma h t}$ and $e^{(3+\gamma)h t}$. The exact inward branch selects the decaying one. At first order and fixed field, the familiar Hamilton–Jacobi separation relation gives $\delta H \propto a^{-3}$ [P1]. Its amplification over the reference halving interval is about 6.0387, while fractional velocity separation has a different amplification, about 30.194. Fixed-field comparisons are not arbitrary fixed-time perturbations.

There is also an exact nonlinear statement, valid beyond the specific polynomial. It makes the preparation cost explicit without silently reversing the stability of expanding inflation.

Theorem 8 — exact nonlinear separation at fixed field. Consider two inward monotone contracting canonical solutions of the same potential on a common field interval, with distinct Hubble values at the same initial field: $Q_1(d_i) \neq Q_0(d_i)$. Thus they define different $H(\phi)$ curves, not merely different time origins on one curve. Put $\beta = 2M^2/\mu^2$, $U = V/(M^2 h^2)$, $Q_j = -H_j/h > 0$, $v_j = -\dot{d}_j/h > 0$, and $\ell_d = d_i - d$. For $D = Q_1 - Q_0$,

$$\begin{aligned} U &= 3Q_j^2 - v_j^2\beta, \frac{dQ_j}{d\ell_d} = v_j\beta, \\ \frac{dD}{d\ell_d} &= \frac{3(Q_1 + Q_0)}{v_1 + v_0} D. \end{aligned} \quad (31)$$

Thus the sign of D is preserved and its absolute value strictly grows. In particular,

$$\begin{aligned} |D(\ell_d)| &= |D(0)| e^{3N_{\text{mix}}(\ell_d)}, \\ N_{\text{mix}}(\ell_d) &= \int_0^{\ell_d} \frac{Q_1 + Q_0}{v_1 + v_0} d\ell_d. \end{aligned} \quad (32)$$

If both histories have $\epsilon_j = v_j^2 / (\beta Q_j^2) \leq \epsilon_* < 1$, then

$$\frac{d \log |D|}{d \ell_d} \geq \frac{3}{\sqrt{\beta \epsilon_*}}, \frac{d \log (|D|/Q_j)}{d \ell_d} \geq \frac{3 - \epsilon_*}{\sqrt{\beta \epsilon_*}}. \quad (33)$$

Proof. Subtract the two constraints to obtain $v_1^2 - v_0^2 = 3\beta(Q_1^2 - Q_0^2)$, and rationalize the difference of $dQ_j/d\ell_d$. This gives the last identity in (31) exactly, not as a linearization. Integration proves (32). The acceleration bound implies $v_j \leq \sqrt{\beta \epsilon_*} Q_j$, while $d \log Q_j/d\ell_d \leq \sqrt{\epsilon_*/\beta}$. These two bounds prove (33). $\square$

The mixed integral is a velocity-weighted average of the local contraction rates of the two histories. It is not the physical clock or the contraction count of either one separately. In the coincident-history limit it becomes the ordinary contraction count and recovers the linear relation. Its finite nonlinear identity is the useful result: a prescribed final Hubble tolerance over a common mixed-contraction interval requires an exponentially smaller initial tolerance.

This excludes convergence of the stated monotone $H(\phi)$ histories at fixed field during contraction. It does not exclude every mechanism, norm or multifield theory. A turning field or a failed effective application ends its scope. Substantial finite sets of initial data can still share the same response signs and target crossing, as Theorem 6 proves. For contrast, the reference choice $\gamma=0.1$, $d_i=0.6$, $d_f=0.1$ yields about 16.1676 contraction e-folds and a linear Hubble-separation factor near 1.16×10^{21} . A slower ideal approach is not a free improvement in robustness.

The canonical kinetic term has positive signature and the usual luminal short-wavelength principal part. These properties do not remove growing long-wavelength modes or prove stability of a general inhomogeneous black-hole interior. The component analysis below also shows why matter and anisotropy that were initially small can impose an independent duration limit.

5. Attained scalar entry, turn, exit and return

5.1. A distinct pre-braking preparation

Specialize the canonical action (2) and reference potential (3) to $\gamma=1$, so $\phi=2Md$ and $W=W_0=3(1-d^2)^2-2d^2$. This section uses the same conserved flat homogeneous scalar source, but a different preparation from the near-reference halving theorem (Theorem 6). Its history begins before acceleration, enters a finite braking regime, turns at a positive field value, and leaves acceleration before returning to its initial field. The component extensions below show that acceleration can instead end while the radial responses are still decreasing.

The openly extended local window $0 < d < 0.62$ includes the entire scalar construction. It has positive potential; no global completion of this polynomial is required. Its source application and operational matching over the new preparation and return portions are declared receiving hypotheses; the narrower near-reference halving corridor alone does not license those additional physical domains. Put $\tau = h(t - t_i)$, $u = d'$, $Q = -H/h > 0$, $N = \log(a_i/a)$, $f = 1 - d^2$, and $g = d(4 - 3d^2) = -W_{,d}/4$. The equations are

$$d' = u, u' = 3Qu + g, Q' = 2u^2, N' = Q, 3Q^2 = 2u^2 + W. \quad (34)$$

The constraint is preserved. Define

$$\mathcal{A} = \frac{\ddot{a}}{a h^2} = Q^2 - 2u^2 = W - 2Q^2, \bar{\rho} = 3Q^2, \quad (35)$$

$$\bar{p} = 2u^2 - W = -Q^2 - 2\mathcal{A},$$

where bars divide physical density and pressure by $M^2 h^2$. Here *braking* means $H < 0$ and $\mathcal{A} > 0$: the scale factor continues contracting while $\dot{a}$ becomes less negative. It does not mean $|H|$ falls. Since $Q' \geq 0$, $|H|$ increases except at the isolated scalar turn. The quantity whose derivative is evaluated below is $\ddot{a}/a$, not $\ddot{a}$.

Keep the two original zero-port action insertions $O_d = d^2$, $O_u = u^2$ from (27)–(28).

The physical scalar kinetic density is $T_\phi = 2 M^2 h^2 O_u$; O_u here is a normalized insertion. Their original compact source directions, clock/frame prescription and common inputs remain fixed. They are action readouts, not automatically microscopic detectors or complete atlas roles. Calling their decrease reduced differentiation requires the already declared independent receiving match.

The open constrained preparation class is

$$0.59 < d_i < 0.61, 0.005 < \delta < 0.015, Q_i = f(d_i) - \delta, \quad (36)$$

$$u_i = -v_i = -\sqrt{d_i^2 - 3f(d_i)\delta + 3/2\delta^2}.$$

This is open in the two-dimensional contracting constraint section. Relative to the exact inward reference speed $v = d$, its initial speeds are approximately 1.27%–4.26% smaller. Those are sufficient preparation limits, not basin probabilities. The effective phase starts on a genuine pre-braking section, rather than inside the near-reference accelerating corridor. Nothing asserts that generic prior collapse naturally selects this section. In fact, (38) keeps $\Delta \leq -\delta < -0.005$ on the inward leg. At any later crossing near $d = 0.5$, these histories therefore remain outside the $|Q - (1 - d^2)| < 0.0009$ preparation strip of Theorem 6. The two constructions are alternative preparations, not consecutive stages of one proved history.

5.2. Theorem 9: actual entry, positive-field turn and outgoing exit

For every preparation (36), the same solution has a unique incoming acceleration entry τ_e , a finite transverse field turn τ_t , a unique outgoing acceleration exit τ_x , and a finite return τ_r to d_i , with

$$0 < \tau_e < \tau_t < \tau_x < \tau_r < 3.205, 0 < d_t < d_x < d_e < d_i. \quad (37)$$

Both exact insertions decrease strictly up to the turn and increase strictly afterward through return. Acceleration is strictly increasing inward and strictly decreasing outward. The strict response-and-braking interval for these insertions is (τ_e, τ_t) ; the longer acceleration interval is (τ_e, τ_x) .

Proof. The preparation has $Q_i > q^* = 0.6129$, $v_i^2 \geq 0.319102$, $\mathcal{A}_i < -0.23256239$, and $\bar{p}_i > 0.05948317$. Also $W \geq W(0.61) = 0.43857523 > 0$ and $g > 0$ on the return band. The displayed endpoint constants bound the closure of the preparation rectangle; the actual initial inequalities are strict.

On the maximal inward segment put $v = -u > 0$, $\Delta = Q - f$. The constraint and time equations give

$$v^2 - d^2 = 3/2 \Delta(f + Q), \Delta' = 2v(v - d). \quad (38)$$

Initially $\Delta = -\delta$. As long as the field is inward, the sign remains negative and Δ decreases: $Q < f$, $0 < v < d$, and

$$v' = 3Qv - g < 3fd - g = -d. \quad (39)$$

This is an invariant sign argument on a maximal interval, not an assumed future turn. It shows kinetic decrease and gives an obstruction to indefinite inward continuation. To make the obstruction finite and avoid the origin, set $y = f - Q = -\Delta$, $\ell_d = d_i - d$, and

$$p^* = 3/2(0.6279 + 0.6129) = 1.8612, d_{lb} = [p^*(0.005)(0.59)^{p^*}]^{1/(p^*+2)}. \quad (40)$$

Since $y_{,\ell_d} = 3y(f+Q)/(v+d) \geq p_* y/d$, one has $y \geq \delta(d_i/d)^{p_*}$. Independently the constraint gives $0 \leq v^2 \leq d^2 - p_* y$. Thus $d \geq d_{\text{lb}} > 0.2309$ throughout the maximal inward interval. Its states stay in a compact set. Smooth proper-time continuation is available, and $v' < -d_{\text{lb}}$ forces an actual finite zero of v . At that turn $u' = g(d_t) > 0$. The monotone-field description is used only before the turn; the regular time equations carry the solution through it.

Inward,

$$\mathcal{A}' = 4v(g - 2Qv) > 0. \quad (41)$$

Initial $\mathcal{A} < 0$ and turn value $\mathcal{A}_t = Q_t^2 > 0$ yield exactly one actual acceleration entry. After the transverse turn, $u > 0$, $u' = 3Qu + g > 0$, and $\mathcal{A}' = -4u(g + 2Qu) < 0$.

It remains to prove return rather than merely assume continuation long enough to find an exit. While $d_t < d < d_i$, positive W implies

$$u^2 < 3/2 Q^2, Q_{,d} = 2u < \sqrt{6} Q, Q(d) \leq Q_t e^{\sqrt{6}(d-d_t)}. \quad (42)$$

This bounds Q, u on the entire return band, so finite-state blow-up cannot obstruct return. Moreover $u' \geq g(d_{\text{lb}}) > 0$; hence $d - d_t \geq g(d_{\text{lb}})(\tau - \tau_t)^2/2$. Finite return is therefore attained. There $Q_r > Q_i$ and $\mathcal{A}_r = W(d_i) - 2Q_r^2 < \mathcal{A}_i < 0$. Strict outgoing monotonicity yields one actual acceleration exit before return. At the same field value the outgoing state has larger Q and smaller $\mathcal{A}$, proving $d_x < d_e$. Equations (39) and (34) give the signs of $O'_d = 2du$ and $O'_u = 2uu'$. The duration bound is supplied below. $\square$

The finite-time argument is the nontrivial bridge from the prepared pre-braking state to occupied braking and an attained endpoint. Defining an endpoint as the first exit does not supply that proof. The field returns toward increasing response before it reaches zero. This is a proved obstruction to indefinite inward return for this open family, not a universal obstruction to inverse reheating or to all possible effective sources.

5.3. Event bounds, pressure work and a representative history

Let $\omega_* = \sqrt{1.015}$, $r_* = \sqrt{1 - 0.009381/(0.61)^2}$, $g_* = g(d_{\text{lb}})$, and $d_{\text{max}} = 0.61$. The comparison $d'' > \omega_*^2 d$, with the same initial position and velocity, gives

$$\begin{aligned} \tau_t < T_* &:= \omega_*^{-1} \operatorname{atanh}(r_*/\omega_*) < 2.281, \\ \tau_r - \tau_t < T_o &:= \sqrt{2(d_{\text{max}} - d_{\text{lb}})/g_*} < 0.925. \end{aligned} \quad (43)$$

These are bounds on actual attained events. The exact constants imply $T_* + T_o < 3.205$. Acceleration entry is transverse with $\mathcal{A}'_e > 1.1875$; the turn has $u'_t > 0.8867$, and outgoing exit has $-\mathcal{A}'_x > 1.1745$. They remain separated under sufficiently small compatible source and initial-data perturbations on the compact trajectory. This gives an additional local robustness statement by continuous dependence and the implicit-function theorem. It does not quantify an arbitrary-potential basin or remove the preparation cost.

For self-contained stage bounds, let e_- be the root in this field range of

$$\begin{aligned} (1 - e_-^2)^2 - 2e_-^2 + 4(1 - e_-^2)(0.005) - 2(0.005)^2 &= 0, \\ g_{\text{max}} &= g(d_{\text{max}}), V_E = 1/2 \sqrt{W(e_-)}, Q_A = \sqrt{W(d_{\text{lb}})/2}, \\ U_A &= Q_A/\sqrt{2}, a_{\text{max}} = 3Q_A U_A + g_{\text{max}}. \end{aligned} \quad (44)$$

At entry, $d_e > e_-$, hence $v_e < V_E$. Inward $\omega_*^2 d_{lb} < -v' < g_{max}$; outgoing while $\mathcal{A} \geq 0$, one has $Q < Q_A$, $u < U_A$, and $u' < a_{max}$. At both acceleration crossings the speed is $Q/\sqrt{2} > q_*/\sqrt{2}$. Integrating these speed-change bounds gives

$$\frac{q_*}{\sqrt{2} g_{max}} < \tau_t - \tau_e < \frac{V_E}{\omega_*^2 d_{lb}}, \frac{q_*}{\sqrt{2} a_{max}} < \tau_x - \tau_t < T_o, \quad (45)$$

$$\begin{aligned} \frac{q_*^2}{\sqrt{2} g_{max}} &< N_t - N_e < (1 - d_{lb}^2) \frac{V_E}{\omega_*^2 d_{lb}}, \\ \frac{q_*^2}{\sqrt{2} a_{max}} &< N_x - N_t < Q_A T_o. \end{aligned} \quad (46)$$

Their ordinary decimal evaluations, rounded outward for this summary, are:

Table 3. Uniform event bounds for the prepared scalar loop.

Actual stage	Dimensionless proper duration $h \Delta t$	Contraction ΔN
Acceleration entry to field turn	$0.2463 < h \Delta t < 2.1807$	$0.1510 < \Delta N < 2.0644$
Field turn to acceleration exit	$0.0963 < h \Delta t < 0.9247$	$0.0590 < \Delta N < 1.0506$

They are sufficient uniform bounds, not estimates of typical duration. The preparation interval is moderate but the model remains repelling relative to the inward reference trajectory. The exact nonlinear separation theorem (Theorem 8) still applies on each monotone inward branch. Entry through this open section is not a natural contracting attractor or a probability statement about black-hole interiors.

Pressure has its own attained crossings. Inward $\bar{p}' = 4u(3Qu + 2g) < 0$, because $u < 0$ and $g > 3Q|u|$; outgoing it is positive. Initial pressure is positive, turn pressure is $-\bar{p}$, and at return $\bar{p}_r > \bar{p}_i > 0$. Thus there is one incoming pressure zero before acceleration entry and one outgoing pressure zero after acceleration exit but before return. At either acceleration crossing $p = -\rho/3$, not zero.

The same stress tensor satisfies

$$\begin{aligned} \bar{p}' - 3Q(\bar{p} + \bar{p}) &= 0, \bar{E}' = 3Q\bar{p}a^3 = -\bar{p}(a^3)', \\ \bar{E} &= \bar{p}a^3. \end{aligned} \quad (47)$$

Density rises along the contraction, with zero instantaneous derivative at the scalar turn. Comoving-cell energy falls while pressure is negative, including at the turn. It rises before the incoming pressure zero and after the outgoing one. This is pressure work in a conserved source; there is no separate metric-energy sink and no energy ledger is carried across a future applicability failure.

For $d_i=0.6$, $\delta=0.01$, ordinary refined numerical integration illustrates the already proved sequence:

Table 4. A representative scalar loop, computed by ordinary integration.

Event	τ	d	Q
Incoming pressure zero	0.031604	0.581868	0.650808
Acceleration entry	0.132952	0.527956	0.708232
Field turn	0.801928	0.339284	0.840406
Acceleration exit	1.120329	0.425445	0.908367
Outgoing pressure zero	1.175018	0.465742	0.968102
Return to initial field	1.294103	0.600000	1.278946

The returning field value is not the returning whole source state. The larger Q forces a larger kinetic insertion at the repeated d . A one-coordinate power law cannot represent both original insertions over this full loop; phase-space and history data remain necessary.

6. Conserved components, curvature and charge

An extra source cannot be added while retaining the old exact Hubble history. Its stress must enter the constraint and evolution. The following extension gives a necessary duration limit that is independent of exact scalar tracking. It complements the sufficient scalar corridor; it does not silently extend that corridor's guarantee to new matter.

6.1. Two consistent geometries

Let $T_\phi = \dot{\phi}^2/2$ and add comoving perfect fluids with $\rho_j \geq 0$, constant $p_j = w_j \rho_j$, and separate covariant conservation. There is no scalar-fluid exchange. In Bianchi I, use the volume-mean scale factor $a = (a_1 a_2 a_3)^{1/3}$, $\sigma^2 = 1/2 \sigma_j^i \sigma_j^i$, and $\rho_\sigma = M^2 \sigma^2$. With no anisotropic matter stress,

$$3 M^2 H^2 = T_\phi + V + \sum_j \rho_j + \rho_\sigma, \quad (48)$$

$$\dot{H} = -\frac{T_\phi}{M^2} - \sum_j \frac{(1+w_j)\rho_j}{2 M^2} - \sigma^2.$$

The scalar equation is unchanged, while

$$\dot{\rho}_j = -3 H (1+w_j) \rho_j, \quad \dot{\rho}_\sigma = -6 H \rho_\sigma. \quad (49)$$

The shear density is geometric bookkeeping, not an additional material energy reservoir. These standard source and shear equations are used only in their stated domains [P2, P5].

For curved FLRW, set shear to zero and instead use a constant spatial-curvature parameter ℓ :

$$3 M^2 (H^2 + \ell/a^2) = T_\phi + V + \sum_j \rho_j, \quad (50)$$

$$\dot{H} = \frac{\ell}{a^2} - \frac{T_\phi}{M^2} - \sum_j \frac{(1+w_j)\rho_j}{2 M^2}.$$

A free FLRW curvature term cannot simply be combined with the Bianchi-I shear law to claim a general anisotropic-curvature solution. In either of these two consistent geometries, the acceleration equation is

$$3 M^2 \frac{\ddot{a}}{a} = V - 2 T_\phi - \sum_j \frac{1+3 w_j}{2} \rho_j - 2 \rho_\sigma, \quad (51)$$

with the shear term omitted for FLRW. Curvature cancels from this equation but remains in the constraint and in the evolution of the source.

6.2. An upper bound on continuing braking

Theorem 10 — bounded-potential acceleration budget. On a connected contracting interval, suppose $V \leq V_{\max} < \infty$, every added fluid has $w_j > -1/3$, and at least one fluid or Bianchi-I shear seed is positive. Put

$$B_{\text{comp}}(N) = \sum_j \frac{1+3 w_j}{2} \rho_{j,i} e^{3(1+w_j)N} + 2 \rho_{\sigma,i} e^{6N}. \quad (52)$$

If the initial data accelerate, there is a unique positive N_* satisfying $B_{\text{comp}}(N_*) = V_{\max}$. Positive acceleration at any later point requires $N < N_*$. In Bianchi I, or FLRW with $\tilde{f} \leq 0$, the supremal duration of the continuing accelerating contraction is at most

$$\Delta t_{\text{accel}} \leq \frac{N_*}{|H_i|}. \quad (53)$$

Proof. Conservation gives exactly $\rho_j = \rho_{j,i} e^{3(1+w_j)N}$ and $\rho_\sigma = \rho_{\sigma,i} e^{6N}$. Every nonzero term in (52) increases without bound. Equation (51) and $T_\phi \geq 0$ require $B_{\text{comp}}(N) < V \leq V_{\max}$ wherever acceleration is positive. Initially this strict inequality holds, proving existence and uniqueness of the root and the contraction cap. In the stated nonpositive-curvature or Bianchi-I cases, $\dot{H} \leq 0$, so $dN/dt = |H| \geq |H_i|$. This proves (53). $\square$

The scalar kinetic energy can end acceleration earlier; neither attainment of N_* nor a particular exit state has been proved. The potential window or source/metric application may also end first. A single nonzero component gives the weaker cap $N_* \leq \log[V_{\max}/(c_j \rho_{j,i})]/[3(1+w_j)]$, with $c_j = (1+3w_j)/2$, or $N_* \leq (1/6) \log[V_{\max}/(2\rho_{\sigma,i})]$ for shear. The combined root is sharper. A varying equation of state bounded below by $w_{j,\min} > -1/3$ gives an analogous comparison cap. Negative-pressure additions or source exchanges require a different budget.

This result quantifies why a bounded positive plateau does not indefinitely overcome persistent ordinary matter or shear during contraction. It is a limit on the braking regime, not an endpoint theorem for matter, a singularity theorem or evidence of metric exhaustion. The established benefit of sufficiently stiff contraction for anisotropy control [P2] concerns a different source mechanism. Expanding no-hair results [P3] do not provide the missing inward stability.

6.3. Energy accounting and a worked extended source

For total material density $\rho_m = T_\phi + V + \sum_j \rho_j$ and pressure $p_m = T_\phi - V + \sum_j w_j \rho_j$,

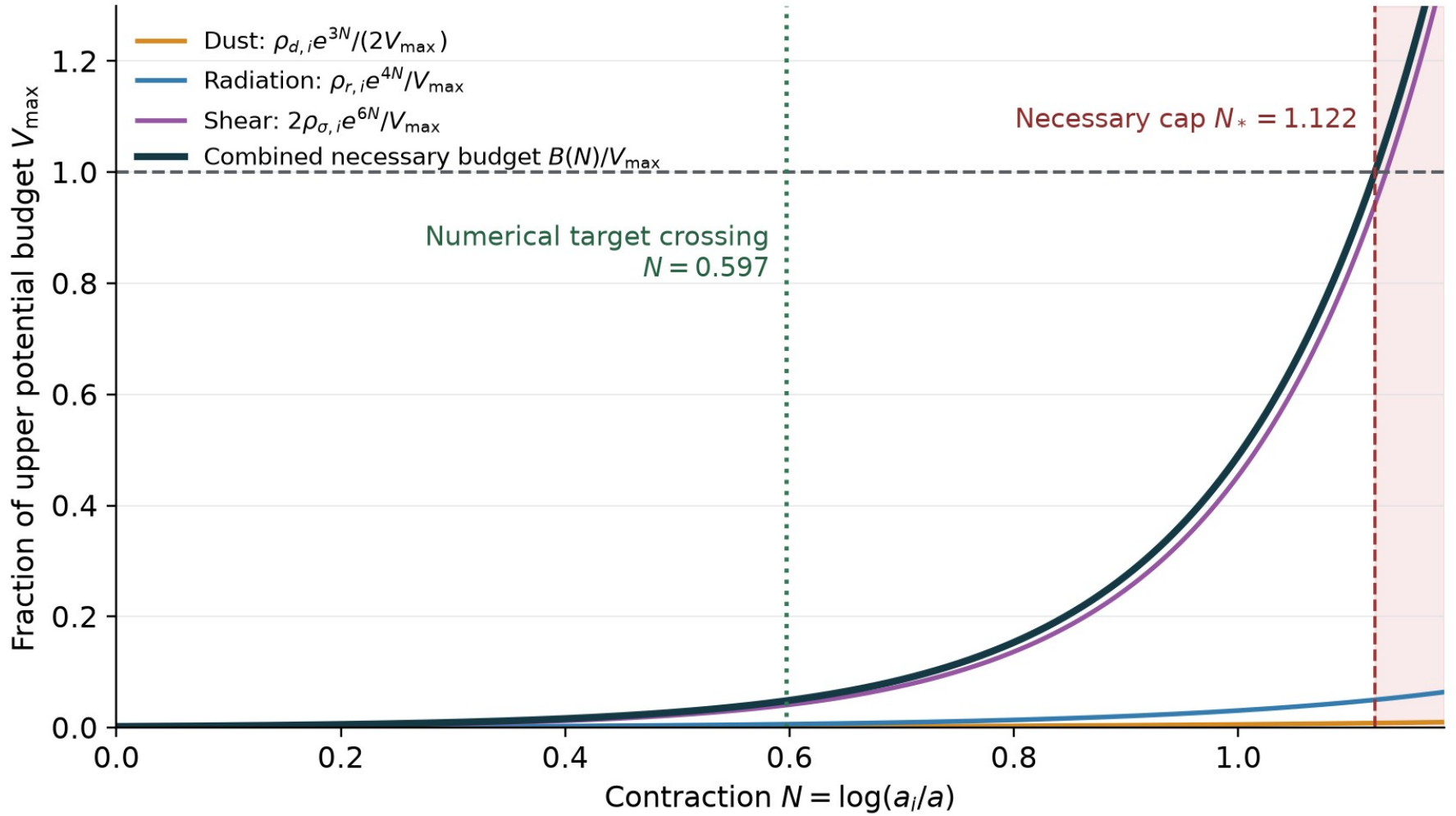
$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0, \quad \dot{E}_m = -p_m \dot{\mathcal{V}}, \quad E_m = \rho_m \mathcal{V}. \quad (54)$$

Shear bookkeeping is excluded from this material ledger. Negative total pressure makes cell energy fall in contraction; a negative scalar pressure alone does not determine the total sign. Dust cell energy stays constant

and radiation cell energy rises. When the total material density is nonnegative and mean acceleration is positive, (51) implies $\rho_m + 3 p_m < -4 \rho_\sigma \leq 0$, hence $p_m < 0$. The NEC makes density nondecreasing, strictly increasing when $\rho_m + p_m > 0$. Braking and growth of $(a | H |)^{-1}$ retain their kinematic meanings. In Bianchi I they concern the volume mean, not every directional scale factor.

For a reproducible example, keep the reference $\gamma = 1$ potential, choose $d_i = 1/2$, $u_i = -1/2$, and give dust, radiation and shear bookkeeping each an initial density $0.001 (27/16) M^2 h^2$. The full contracting constraint gives $H_i/h = -0.7511241575$. Since $V \leq 3 M^2 h^2$ on the stated local source window, the analytical budget gives $N_* = 1.1216850130$ and $h \Delta t_{\text{accel}} \leq 1.4933416824$. Full-equation integration reaches $d = 1/4$ at $h \Delta t = 0.686576795840$, $N = 0.597216951162$, with $u = -0.260748208464$, $H/h = -0.955153459417$, and $E_m/E_{m,i} = 0.263818859430$. Sampled acceleration remains positive. This crossing is numerical evidence; the necessary cap is the analytical result. Neither calculation inserts the added components into the old exact trajectory without backreaction.

Nonzero components limit accelerating contraction



Necessary bound only: scalar kinetic energy or source-window exit can end acceleration earlier.

Figure 4. The plotted budget B is B_{comp} here. Exact separately conserved dust, radiation and shear contributions to the acceleration budget. The combined root is a necessary upper limit for the continuing bounded-potential braking regime. The marked full-source halving event is an independently integrated example below that limit, not an assertion that the limit is attained.

6.4. Curvature and a conserved multifield charge

The flat canonical no-bounce result of Proposition 4 extends to Bianchi I and nonpositive-curvature FLRW under these NEC-respecting sources. It does not extend to closed FLRW [P6]. For example, the same potential's stationary top permits the closed de Sitter solution $a = h^{-1} \cosh(h t)$, $H = h \tanh(h t)$, which bounces with saturated NEC. It is not the differentiated inward history or a completed bearer return. A closed contracting interval also needs a separate lower bound on $|H|$ before using (53). During accelerated contraction, the relative curvature magnitude $|\ddot{r}|/(a^2 H^2) = |\ddot{r}|/\dot{a}^2$ increases, so initially small curvature cannot be dismissed for an arbitrary duration.

A second scalar can introduce a physical obstruction that a single order coordinate hides. Take two canonical real fields with a smooth $O(2)$ -symmetric potential $V(\varrho)$, where $\phi_1 = \varrho \cos \theta$, $\phi_2 = \varrho \sin \theta$. The exact homogeneous charge and angular kinetic energy are [P7]

$$J_\theta = (a/a_i)^3 \varrho^2 \dot{\theta} = \text{constant}, T_\theta = \frac{J_\theta^2 e^{6N}}{2 \varrho^2}, \quad (55)$$

$$\ddot{\varrho} + 3H \dot{\varrho} + V_{,\varrho} = \frac{J_\theta^2 e^{6N}}{\varrho^3}.$$

No rapid-spinning approximation is assumed.

Corollary 11 — charged radial obstruction. With $J_\theta \neq 0$, $V \leq V_{\max}$, nonnegative budget (52), and accelerating contraction, the Cartesian two-field theory requires

$$\varrho^2 > \frac{J_\theta^2 e^{6N}}{V_{\max} - B_{\text{comp}}(N)} \geq \frac{J_\theta^2 e^{6N}}{V_{\max}}, \quad (56)$$

$$N_f < \frac{1}{6} \log \frac{V_{\max} \varrho_f^2}{J_\theta^2}.$$

Proof. Replace the single-field kinetic term in (51) by the total two-field kinetic density $T_{\text{kin}} = \dot{\varrho}^2/2 + T_\theta$.

Positive acceleration requires $V > \dot{\varrho}^2 + J_\theta^2 e^{6N}/\varrho^2 + B_{\text{comp}}(N)$. Drop the nonnegative radial term and use the potential bound; rearrangement gives both inequalities. $\square$

The denominator must be positive. A nonpositive right side in the target bound excludes that target during the stated accelerating contraction; a positive one is only necessary. If ϱ decreases, the same bound with ϱ_i limits total contraction even without fluids or shear. Arbitrarily small nonzero charge need not spoil a finite window, but it cannot be ignored in an arbitrarily deep accelerating radial return. This is a charge/kinetic obstruction in a Cartesian field theory, not an artifact of singular polar coordinates. Symmetry breaking or charge exchange would require a different source. The result does not identify radius with the complete original role family or prohibit every multifield mechanism.

For general positive-definite multifield kinetic metrics, the total-kinetic energy, work and acceleration-budget arguments still apply. They do not establish transverse or entropy-mode stability. Those questions need the actual field-space geometry, couplings and source matching.

6.5. Component competition and attained exits

6.5.1. Theorem 12: retained shear and conserved angular charge

For a bounded extension, take Bianchi I mean expansion with no anisotropic material stress, and an $O(2)$ canonical field $\phi_1 + i \phi_2 = \varrho e^{i\theta}$, $\varrho = 2M d$. For the following attained-event results, fix the radial potential to $V = M^2 h^2 W_0(d)$, $W_0 = 3 - 8d^2 + 3d^4$, on $0 < d < .62$. Set

$$Z = \frac{T_\theta}{M^2 h^2} \geq 0, \Sigma = \frac{\sigma^2}{h^2} \geq 0, J_\theta = (a/a_i)^3 \varrho^2 \dot{\theta}, \quad (57)$$

$$\dot{J}_\theta = 0.$$

Σ is shear geometry, not a new material reservoir. The exact coupled system is

$$\begin{aligned} d' &= u, u' = 3Q u + g + \frac{Z}{2d}, \\ Q' &= 2u^2 + Z + \Sigma, Z' = (6Q - 2u/d)Z, \quad (58) \\ \Sigma' &= 6Q\Sigma, N' = Q, \\ 3Q^2 &= 2u^2 + W + Z + \Sigma, \mathcal{A} = W - 2Q^2. \end{aligned}$$

The normalized material ledger is $\bar{\rho}_m = 2u^2 + W + Z$, $\bar{p}_m = 2u^2 + Z - W$, and obeys $\bar{\rho}_m' - 3Q(\bar{\rho}_m + \bar{p}_m) = 0$. The mean acceleration also obeys

$$3\mathcal{A} = W - 4u^2 - 2Z - 2\Sigma, \mathcal{A}' = -4[g u + Q(2u^2 + Z + \Sigma)]. \quad (59)$$

(a) A nonempty relatively open small-component class preserves scalar event order. For each fixed interior preparation (36), sufficiently small nonnegative component seeds and nearby constrained initial data retain actual acceleration entry, a positive-radius transverse turn with $\mathcal{A}_t > 0$, and subsequent actual acceleration exit. Both exact radial insertions decrease to the turn. This follows from the compact scalar trajectory, strict margins in Theorem 9, and smooth continuous dependence for $d > 0$; $u' > 0$ has a uniform margin over the incoming leg. The statement is existential about the perturbation size. It is not the later explicit component box theorem and does not assert a full basin.

(b) Any reached positive-radius turn with $\mathcal{A}_t > 0$ has an attained outgoing exit before return to the initial field, provided the initial state has $\mathcal{A}_i < 0$ and the return band has $W, g > 0$. This is not merely a component ceiling. After the turn put $F = g + Z/(2d) > 0$. Then $u' > 0$, $\mathcal{A}' < 0$, and

$$\left(\frac{Q}{u}\right)' = -\frac{W}{u} - \frac{QF}{u^2} < 0 \quad (u > 0). \quad (60)$$

Take any short time after the regular turn. Thereafter Q/u is bounded above by its finite value there. Since $Q' = 3Q^2 - W$, $Q_d \leq 3(Q/u)Q$ bounds Q exponentially over the remaining finite field band. The constraint bounds u, Z, Σ . Smooth continuation and $u' \geq \min g > 0$ force finite return, with $\Delta\tau \leq \sqrt{2(d_i - d_t)/\min g}$. At return $Q_r > Q_i$, hence $\mathcal{A}_r < \mathcal{A}_i < 0$; strict monotonicity gives the unique outgoing exit. No monotone-field formula is evaluated at $u = 0$. $\square$

On the inward leg, nonzero angular energy increases: $Z' = (6Q - 2u/d)Z > 0$. The phrase decreasing differentiation therefore concerns the explicitly fixed **radial** response family. It does not cover every charge-related response or imply decreasing total kinetic energy. A charged radial turn is not $w = -1$. For general matched probes depending on Q, Z, Σ , their nonzero rates at that turn must be retained.

6.5.2. Theorem 13: open component sets with an earlier acceleration exit

There are nonempty explicit open boxes of contracting constrained initial data for which first acceleration entry and exit are both attained before the first radial turn. Within the documented outward-rounding arithmetic, interval Taylor propagation certifies this for all data in the following boxes. Uniqueness and exclusion of extra events refer to the complete recorded finite interval. The independent coordinates lie strictly within $\varepsilon_{\text{box}} = 10^{-5}$ of the listed centers; initial $N=0$, and u_i is the negative square root of the exact constraint.

Table 5. Independently varied component preparation boxes.

Source section	Center $(d_i, Q_i, Z_i, \Sigma_i)$	Independently varied coordinates
Scalar plus shear	$(0.6, 0.63, 0, 0.04)$	$d_i, Q_i, \Sigma_i; Z_i=0$
Charged scalar, isotropic	$(0.6, 0.63, 0.08, 0)$	$d_i, Q_i, Z_i; \Sigma_i=0$
Charged scalar plus shear	$(0.6, 0.63, 0.04, 0.04)$	d_i, Q_i, Z_i, Σ_i

The first two are open in their fixed-source constraint sections; the joint box is open in all four independent source coordinates. The conservative closed enclosures used by the computation include every member of these open boxes. They are witnesses of robustness, not claims about maximal widths or naturally selected preparations.

Table 6. Certified whole-box component event brackets and braking durations.

Source section	Entry τ_e	First acceleration exit τ_x	Radial turn τ_t	Braking duration $\tau_x - \tau_e$
Shear	$(\frac{17}{128}, \frac{20}{128})$	$(\frac{84}{128}, \frac{86}{128})$	$(\frac{90}{128}, \frac{92}{128})$	$(\frac{64}{128}, \frac{69}{128})$
Charge	$(\frac{20}{128}, \frac{25}{128})$	$(\frac{45}{128}, \frac{48}{128})$	$(\frac{54}{128}, \frac{56}{128})$	$(\frac{20}{128}, \frac{28}{128})$
Joint	$(\frac{19}{128}, \frac{24}{128})$	$(\frac{52}{128}, \frac{54}{128})$	$(\frac{61}{128}, \frac{63}{128})$	$(\frac{28}{128}, \frac{35}{128})$

All inequalities are strict: endpoint values exclude zero. The record encloses every continuous step, not just sampled endpoint values. It establishes $d, Q, u' > 0$, $u < 0$ through acceleration exit, opposite signs across each event bracket, monotone crossing within every event bracket, and strict acceleration signs outside those brackets. Thus no earlier missed or tangential acceleration crossing is admitted. The radial ports still decrease at the actual acceleration exit.

Proof mechanism and scope. The vector field (58) is smooth on each recorded positive-radius tube. At each exact step $1/128$, an interval box Y satisfies the strict Picard inclusion $x_0 + [0, 1/128]F(Y) \subset \text{int } Y$. This encloses the whole solution step and supplies continuation. Normalized Taylor jets give the endpoint polynomial through order seven and an interval eighth-order remainder evaluated on Y . Each elementary operation is rounded outward by `nextafter`, under the documented IEEE-754 nearest-rounding assumptions for basic operations and square root. Stepwise induction encloses every initial state in the box. Sign and

derivative inequalities on the tubes then prove the stated first events. The preserved evidence files `inputs/v3/validated_events.py`, `inputs/v3/certify_events.py`, `inputs/v3/VALIDATED_EVENT_ENCLOSURES.json`, `inputs/v3/CERTIFIED_EVENT_SUMMARY.json` and their independent reproduction accompany the calculation. This is a computer-assisted theorem about a declared ODE, not a finite audit promoted to a semantic support proof. $\square$

There is a precise but limited acceleration-ledger attribution. Along the same coupled solution,

$$\mathcal{A}' = \underbrace{\frac{W_{,d} u - 8 u u'}{3}}_{R_{\text{rad}}} + \underbrace{-\frac{2}{3}(Z' + \Sigma')}_{R_{\text{comp}}}. \quad (61)$$

Over each certified exit bracket, $R_{\text{rad}} > 0$, $R_{\text{comp}} < 0$, and the total $\mathcal{A}' < 0$. For the shear box the enclosed radial contribution is between 0.2184 and 0.3377 while the component contribution is between -4.0459 and -3.5728. For the charge box the corresponding intervals are (0.8492, 1.1995) and (-2.7469, -2.3422). Material pressure remains negative in both. This identifies which term reverses the acceleration rate in the chosen ledger; it is not a claim about deleting a component while keeping the other trajectory fixed. In particular u' itself depends on angular charge.

The conserved-component and charge caps in Theorem 10 and Corollary 11 remain useful necessary conditions: $\Sigma = \Sigma_i e^{6N}$, $Z = Z_i (d_i/d)^2 e^{6N}$, and $\mathcal{A} > 0$ requires $2(Z + \Sigma) < W \leq W_{\text{max}}$. A cap does not prove its own attainment, and generally is later than loss of acceleration when radial kinetic energy remains nonzero. Theorem 13 instead follows the coupled trajectory and proves actual exits.

The continuation argument in Theorem 12(b) requires only a reached regular positive-radius turn for its finite-return part. It therefore also applies after these certified turns, where $\mathcal{A}_t < 0$. Strict outgoing decrease of $\mathcal{A}$ rules out re-entry into acceleration before first return to d_i . No claim about arbitrary later continuation outside that field band follows.

The first acceleration exit can precede the radial turn

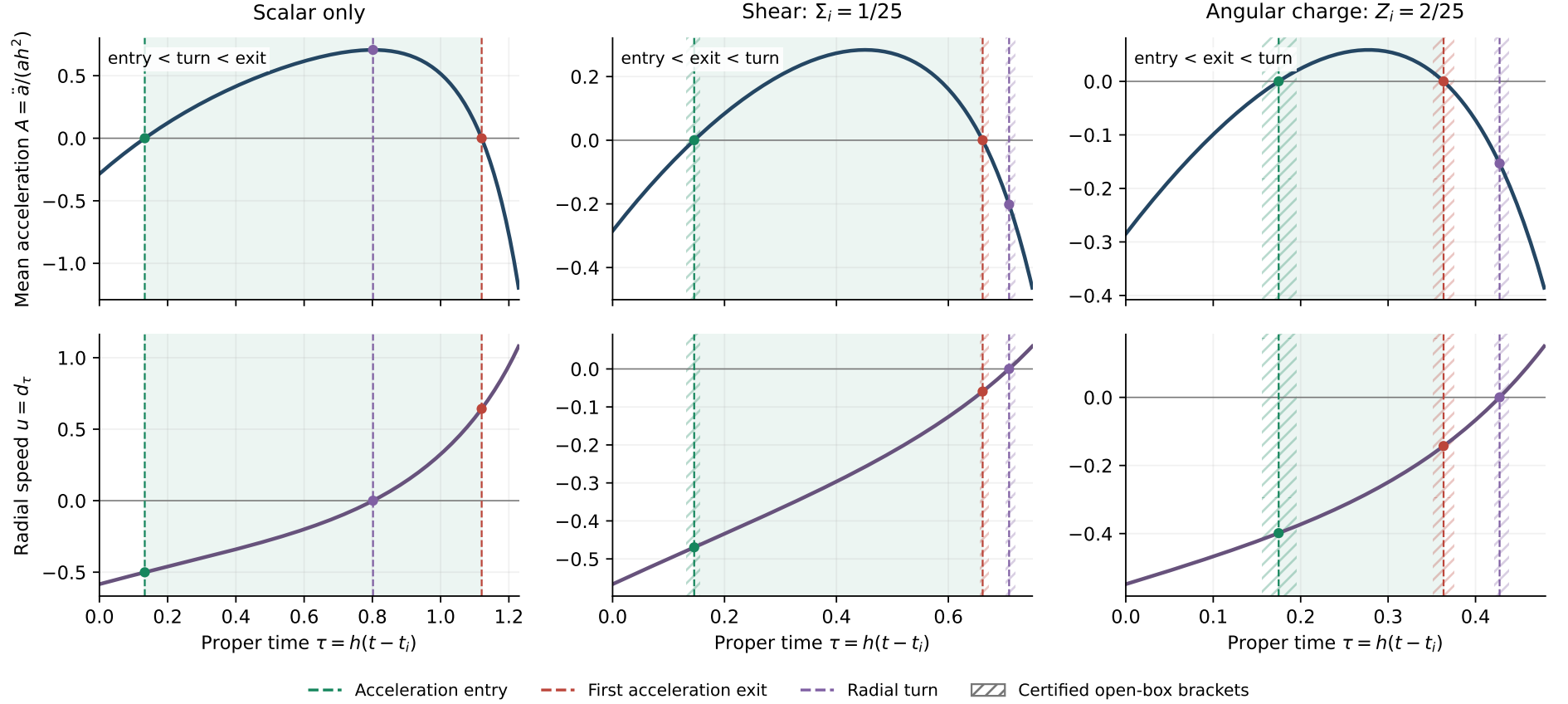


Figure 5. The first acceleration exit can precede the radial turn. The plot label A denotes $\mathcal{A}$ in the text. All panels use the same radial potential, exact initial $d_i = 3/5$, $Q_i = 63/100$, and the inward speed supplied by the complete constraint. The scalar panel has $\Sigma_i = Z_i = 0$; the shear panel has $\Sigma_i = 1/25$, $Z_i = 0$; the charge panel has $Z_i = 2/25$, $\Sigma_i = 0$. Solid curves and dashed event locations are ordinary refined numerical trajectories. Pale green spans mark their positive-acceleration intervals. Hatched bands enclose the events for the full certified initial-data boxes of half-width 10^{-5} ; they are not error bars inferred from the plotted curves. Axes use different time windows to resolve each ordering. Acceleration exits after the radial turn in the scalar example and before it in the component examples. In the latter, both radial insertions still decrease at exit. The figure does not depict a support boundary or the complete differentiated-role family.

7. Fixed response protocols through a turn

7.1. Theorem 14: original matched responses determine the first response exit

For any fixed actually matched family C_α , the physical claim concerns a connected component of

$$\{ \tau : \text{all claimed original applications survive}, H < 0, \mathcal{A} > 0, C'_\alpha < 0 \text{ for every claimed } \alpha \}. \quad (62)$$

Defining this set does not prove it is nonempty. Theorems 9, 12 and 13 establish this for the exact radial ports when their receiving match applies; parts (a)–(c) below give the stated conditions and examples for other fixed families. For the exact radial ports in Theorem 9, the first endpoint is the field turn. In Theorem 13 it is acceleration exit. For other fixed ports it can be a distinct earlier response event.

At the uncharged scalar turn of Theorem 9, put $\Delta\tau = \tau - \tau_t$, $d_T = d_t > 0$, $g_T = g(d_T) > 0$, $q = Q_t$. Then

$$\begin{aligned} d &= d_T + 1/2 g_T (\Delta\tau)^2 + 1/2 q g_T (\Delta\tau)^3 + O((\Delta\tau)^4), \\ u &= g_T \Delta\tau + 3/2 q g_T (\Delta\tau)^2 + O((\Delta\tau)^3), \\ O''_d(\tau_t) &= 2 d_T g_T > 0, O''_u(\tau_t) = 2 g_T^2 > 0. \end{aligned} \quad (63)$$

Both ports have regular defined minima. The determinant $\det D(O_d, O_u) = 4 d u$ vanishes at the turn, but their original operations remain available. This is not proof of role coalescence or standing loss.

(a) Buffered matching. On a closed incoming interval with $d \geq d_* > 0$, $-u \geq v_* > 0$, $u' \geq a_* > 0$, the exact rate margins are $-O'_d \geq 2 d_* v_*$, $-O'_u \geq 2 v_* a_*$. The chain-rule error theorem (Theorem 7) applies to errors smaller than those actual margins. They vanish as the interval approaches the turn. A fixed absolute first-derivative error budget cannot preserve the signs all the way through it. With along-history second-derivative control, positive convexity plus endpoint derivative signs instead gives nearby unique response minima, potentially at different times.

(b) The already fixed mixed protocol has an earlier response minimum. For $C_d = O_d + \lambda d u$, $C_u = O_u + \lambda O_d$, $\lambda > 0$,

$$C'_d(\tau_t) = \lambda d_t g(d_t) > 0, C'_u(\tau_t) = 0. \quad (64)$$

For the literal fixed value $\lambda = 0.0005$, every preparation in Theorem 9 has

$$C'_d(\tau_e) = -2 d_e v_e + \lambda (v_e^2 + d_e u'_e) < -\sqrt{2} q_* d_{lb} + 0.0005 [d_{max}^2 + d_{max} g(d_{max})] < -0.1993. \quad (65)$$

Together with its strictly positive turn derivative, this proves an attained first rate failure strictly between acceleration entry and the field turn. The other port has $C'_u = 2 u (u' + \lambda d) < 0$ throughout the inward leg. The source equations and ports remain defined at that earlier response endpoint. Separately, for sufficiently small fixed λ , the implicit-function theorem places the first port's local minimum at $\tau_t - \lambda/2 + O(\lambda^2)$; the fixed-value crossing argument does not require an unquantified asymptotic radius or claim uniqueness of all possible earlier stationary points. These original probes cannot be silently replaced to make all minima coincide.

(c) A distinct independently declared class can pin the common turn. If a fixed protocol has $C_i = F_i(O_d, O_u)$, with no explicit-time drift, then

$$C'_i = 2 u [d F_{i,O_d} + u' F_{i,O_u}]. \quad (66)$$

Uniform positivity of the bracket, checked against independently specified responses, gives decrease before and increase after the turn. For example $F_1 = O_d + \lambda O_u$, $F_2 = O_u + \lambda O_d$, $0 < \lambda < 1$, is an invertible, nonzero mixed family with this property wherever $u' > 0$. It is a new declared source protocol, not a rewrite of part (b). More generally for $F_1 = O_d + e_1$, $F_2 = O_u + e_2$, $d \in [d^*, d_{\max}]$, $u' \in [a^*, a_{\max}]$, sufficient budgets are

$$\begin{aligned} d_{\max} \|e_{1,O_d}\|_{\infty} + a_{\max} \|e_{1,O_u}\|_{\infty} &< d^*, \\ d_{\max} \|e_{2,O_d}\|_{\infty} + a_{\max} \|e_{2,O_u}\|_{\infty} &< a^*. \end{aligned} \quad (67)$$

The derivative identity is elementary. Its content here is a fixed, nonempty receiving class on a proved physical branch, with its scope and alternatives preserved. It is not a proof that all original material roles have disappeared. $\square$

8. Trapped spherical dynamics

The following construction examines a finite scalar-response experiment in a homogeneous trapped spherical interior. It retains the fixed gamma-one scalar source while including the curvature and signed shear required by that geometry. Its fixed insertions $O_d = d^2$ and $O_u = u^2$ retain their original source directions; their physical receiving match must apply to this geometry and observer prescription. They are not a replacement for the full intended role family.

8.1. Source and spherical geometry

Use the covariant canonical action (2), now on a Kantowski–Sachs metric rather than the flat or Bianchi I geometry. Retain the fixed gamma-one scalar source on its stated local window:

$$\mu = 2M, \phi = 2Md, V = M^2 h^2 W(d), W = W_0 = 3 - 8d^2 + 3d^4, \quad (68)$$

$$0 < d < 0.62.$$

The scales and units are those of the common action. The Kantowski–Sachs metric is

$$\begin{aligned} ds^2 &= -dt^2 + b(t)^2 dz^2 + R(t)^2 d\Omega^2, H = \frac{H_{\parallel} + 2H_{\perp}}{3} = -hQ, \\ s &= \frac{H_{\parallel} - H_{\perp}}{3h}, k_{\Omega} = \frac{1}{h^2 R^2} > 0. \end{aligned} \quad (69)$$

Thus R is the spheres' areal radius, b their longitudinal scale factor, and $\sigma^2 = 3h^2 s^2$ the shear scalar. Retain $\tau = h(t - t_i)$, $u = d'$ and $N = \log(a_i/a)$, now with mean scale factor $a = (bR^2)^{1/3}$; put $g = -W_{,d}/4 = d(4 - 3d^2)$. The full equations are

$$\begin{aligned} d' &= u, u' = 3Qu + g, Q' = 2u^2 + 3s^2 - k_{\Omega}/3, \\ s' &= 3Qs + k_{\Omega}/3, k'_{\Omega} = 2(Q + s)k_{\Omega}, N' = Q, \quad (70) \\ 3Q^2 &= 2u^2 + W + 3s^2 - k_{\Omega}. \end{aligned}$$

These are the canonical Einstein–scalar equations specialized to this geometry; the curvature term in the shear equation is essential. They agree with the conventions of Fadrakas–Leon–Saridakis after setting their $\sigma_+ = -hs$ [P8]. For the polynomial system, the derivative of $C = 3Q^2 - 2u^2 - W - 3s^2 + k_{\Omega}$ vanishes identically. Initial constraint satisfaction therefore propagates. Replacing the shear equation with the Bianchi I free-shear law would change the model.

Define the directional contraction rates

$$X = Q + s = -H_{\perp}/h, Y = Q - 2s = -H_{\parallel}/h, R'/R = -X, b'/b = -Y. \quad (71)$$

Every positive k_Ω corresponds to a finite sphere radius. The comparison $k_\Omega \rightarrow 0$ used below is a local dynamical limit; it does not identify the global topology with that of the flat scalar model.

8.2. Exact trapping, two accelerations and the source ledger

The future null normals $\partial_t \pm b^{-1} \partial_z$ give

$$\theta_+ = \theta_- = -2hX, X' = 3sX + 2u^2, Y' = 3QY - W. \quad (72)$$

Lemma 15 (trapping persists). If $X_i > 0$, all symmetry spheres remain strictly future trapped on every subsequent regular interval. This needs no smallness assumption on shear or curvature.

Proof. Multiplication of the first rate equation by $\exp(-\int 3s d\tau)$ gives a positive initial value plus a nonnegative integral. Equivalently,

$$\frac{d}{dt} \left(\frac{\dot{R}}{b} \right) = -\frac{R \dot{\phi}^2}{2M^2 b} \leq 0. \quad (73)$$

Initial $\dot{R} < 0$ therefore persists while $b, R > 0$. In particular, positive areal acceleration does not produce an areal bounce or escape. $\square$

Keep the two acceleration observables distinct:

$$\begin{aligned} \mathcal{A} &= \frac{\ddot{a}}{a h^2} = \frac{W - 4u^2 - 6s^2}{3} = W - 2Q^2 - \frac{2k_\Omega}{3}, \\ \mathcal{B} &= \frac{\ddot{R}}{R h^2} = XY - 2u^2 = Q^2 - 2u^2 - Qs - 2s^2. \end{aligned} \quad (74)$$

Mean braking means $Q > 0, \mathcal{A} > 0$. Areal braking means $X > 0, \mathcal{B} > 0$. In a trapped region the latter also forces $Y > 0$, hence mean contraction. Their derivatives are

$$\begin{aligned} \mathcal{A}' &= -4u(g + 2Qu) - 12Qs^2 - \frac{4}{3}sk_\Omega, \\ \mathcal{B}' &= -4ug - 2(4Q + s)u^2 - X(3sX + k_\Omega). \end{aligned} \quad (75)$$

On a trapped branch $X > 0$, for $s \geq 0$, any areal acceleration root satisfies

$$\mathcal{A} = \mathcal{B} + s(Q - s) + k_\Omega/3 > 0. \quad (76)$$

Indeed $XY = 2u^2$ gives $Y \geq 0$, hence $Q \geq 2s$. Areal braking is the stricter condition on that branch. At a turn with $Q > 0, s \geq 0$,

$$\mathcal{A}_T > 0 \Leftrightarrow W_T > 6s_T^2, \mathcal{B}_T > 0 \Leftrightarrow W_T > 9s_T^2 + k_{\Omega,T}. \quad (77)$$

The second equivalence has the stated nonnegative-shear hypothesis. Once $Y \leq 0$, (72) prevents its restoration while $W > 0$; on the trapped branch $X > 0$, this gives $\mathcal{B} = XY - 2u^2 \leq 0$. These are finite-size obstructions, not merely small-perturbation observations.

The material ledger, with bars denoting division by $M^2 h^2$, is

$$\begin{aligned} \bar{\rho} &= 2u^2 + W = 3Q^2 - 3s^2 + k_\Omega, \bar{\rho} = 2u^2 - W, \\ \bar{\rho}' &= 12Qu^2 = 3Q(\bar{\rho} + \bar{\rho}). \end{aligned} \quad (78)$$

For a fixed coordinate cell of volume $\mathcal{V} = 4\pi \Delta z b R^2$,

$$\mathcal{V}' = -3 Q \mathcal{V}, (\rho \mathcal{V})' = 3 Q p \mathcal{V} = -p \mathcal{V}'. \quad (79)$$

Density increases during mean contraction except at the isolated field turn; cell energy falls when pressure is negative. This is conserved pressure work. The matter density is not $3 M^2 h^2 Q^2$ here, and neither curvature nor shear is an extra material reservoir. Also Q' need not be positive: the flat one-field increase of $|H|$ cannot be imported into this geometry. At a mean acceleration root $\bar{p} = -\bar{\rho}/3 - 4 s^2$; at an areal root $\bar{p} = -(X^2 + k_\Omega)$. Both have negative pressure.

8.3. Analytic survival and a finite outgoing exit

Theorem 16 (an open trapped survival family). Fix a compact rectangle strictly inside the preparation family of Theorem 9, namely $d_i \in (.59, .61)$, $\delta \in (.005, .015)$. Set

$$Q_i = 1 - d_i^2 - \delta, u_i = -\sqrt{(3 Q_i^2 - W(d_i) - 3 s_i^2 + k_{\Omega,i})/2}. \quad (80)$$

Uniformly over that rectangle, sufficiently small $|s_i|$ and positive $k_{\Omega,i}$ give regular, mean-contracting, strictly trapped solutions through field return. Each acceleration has exactly one entry before the positive-field scalar turn and one exit after it. The exact insertions $O_d = d^2$, $O_u = u^2$ decrease to the turn and increase afterwards. For $s_i > 0$, the events obey

$$\tau_{\mathcal{A},E} < \tau_{\mathcal{B},E} < \tau_T < \tau_{\mathcal{B},X} < \tau_{\mathcal{A},X} < \tau_R. \quad (81)$$

Proof. At $s = k_\Omega = 0$, this is the flat scalar system in Theorem 9. Its compact prepared family has attained, transverse entry, turn, exit and return events, strict signs between them, a positive lower field bound and a strict $u' > 0$ margin. Its finite return proof supplies compact regular solution graphs. Local continuation from their bounded terminal states gives a small extension beyond each return, uniformly over the compact family.

The polynomial vector field and the initial negative constraint root depend smoothly on the data. Proper-time continuous dependence keeps small compatible perturbations in regular compact tubes. Choose disjoint reference neighborhoods for entry, turn and exit; each perturbed acceleration is treated separately even though they coincide at zero curvature. Their derivatives stay away from zero within the corresponding neighborhoods, and their values stay away from zero outside. This gives existence, uniqueness and exclusion of extra events. Treat the return root on the outgoing leg, excluding the initial equality $d = d_i$. Positivity of Q, d, W, u' survives, and Lemma 15 preserves trapping. Geometry reconstruction keeps $b, R > 0$. For positive initial shear, $s' = 3 Q s + k_\Omega/3$ preserves its sign; (76) gives strict nesting of the two braking intervals. Finally $O_d' = 2 d u$, $O_u' = 2 u u'$ give the response signs. $\square$

This theorem gives a genuine nonempty open positive-curvature family but no numerical perturbation radius. The explicit signed-shear witnesses below supply quantitative examples. It is finite-time event stability, not an attractor or generic-capture theorem.

Theorem 17 (an attained areal exit without small curvature). Suppose a regular inward history starts at d_i , reaches a positive-field scalar turn $d_T < d_i$, and has $Q > 0, s \geq 0, k_\Omega > 0, X > 0$ on that inward segment. Suppose $g, W > 0$ on $[d_T, d_i]$ and $\mathcal{B}_i < 0 < \mathcal{B}_T$. The outgoing solution then attains a unique areal acceleration exit in finite time before any return to d_i . At the exit, mean acceleration remains positive.

Proof. The turn has $u' = g(d_T) > 0$. As long as $\mathcal{B} \geq 0$, one has $Y \geq 0, Q > 0$, and $0 \leq s \leq X/3$. The constraint becomes

$$X^2 + 2 X Y + k_\Omega = 2 u^2 + W, X^2 + k_\Omega + 2 u^2 \leq W. \quad (82)$$

Thus X, Q, s, k_Ω, u remain bounded by constants depending only on $W_* := \max W$ on the field band. No finite-state continuation obstruction precedes exit or return. With $g_* := \min g > 0, u' \geq g_*$, so one of those events occurs within

$$\tau - \tau_T \leq \sqrt{2(d_i - d_T)/g_*}. \quad (83)$$

If return occurred first with no areal exit, the exact density law would give $u_R^2 > u_i^2$. Also X is nondecreasing and k_Ω increases. But $\mathcal{B} = (W - k_\Omega - X^2 - 2u^2)/2$ would then give $\mathcal{B}_R < \mathcal{B}_i < 0$, a contradiction. At any outgoing areal root, $Y = 2u^2/X > 0$, and (75) makes $\mathcal{B}' < 0$. Every possible crossing points downwards, excluding a second outgoing root while this positive-force band persists. Equation (76) gives $\mathcal{A} > 0$ at the exit. $\square$

Mean exit requires a further distinction. If also $\mathcal{A}_i < 0$, then after the areal exit, while $Q > 0, \mathcal{A} \geq 0$,

$$4u^2 + 6s^2 \leq W_*, 3Q^2 \leq 3W_*/2, k_\Omega \leq 3W_*/2. \quad (84)$$

The same continuation argument forces a mean exit, a first zero of Q , or field return. Return without either earlier event contradicts density growth and increasing shear. Hence mean exit or a first zero of Q occurs before return. A zero with $\mathcal{A} > 0$ is a mean-volume bounce, since $Q' = -\mathcal{A} < 0$, despite continuing areal trapping. This is an alternative in the general theorem, not a claimed bounce in the certified examples; those retain $Q > 0$.

8.4. Explicit signed-shear witnesses

Theorem 18 (explicit survival and competing exits). Independently vary each of $(d_i, Q_i, s_i, k_{\Omega,i})$ by strictly less than $\varepsilon_{KS} = 1/100000$ around each listed centre:

$$\begin{aligned} (d_i, Q_i, s_i, k_{\Omega,i}) &= \left(\frac{3}{5}, \frac{63}{100}, s_c, \frac{1}{100} \right), \\ s_c &\in \left\{ \frac{1}{20}, -\frac{1}{20}, \frac{1}{10}, -\frac{3}{20} \right\}, \\ u_i &= -\sqrt{\frac{3Q_i^2 - W(d_i) - 3s_i^2 + k_{\Omega,i}}{2}}. \end{aligned} \quad (85)$$

Each box is open in the four independent constrained KS preparation coordinates. For every datum in each box, the recorded solution is regular, mean contracting and strictly trapped, with $d > 0, W > 0, u' > 0$. Both accelerations have exactly one entry and one exit, and the scalar has exactly one turn, on that box's complete recorded interval. The following strict enclosures and first-event statements are certified by full-step interval propagation. **Every table entry denotes an open interval whose two endpoints are divided by 256.**

Table 7. Certified whole-box signed-shear event brackets.

Shear centre s_c	Mean entry	Areal entry	Scalar turn	Areal exit	Mean exit
1/20	(35,36)	(42,43)	(210,212)	(230,232)	(245,247)
-1/20	(35,36)	(29,30)	(210,212)	(294,304)	(251,253)
1/10	(37,38)	(51,54)	(195,197)	(159,163)	(182,184)
-3/20	(40,42)	(18,20)	(173,175)	(225,242)	(138,140)

The recording horizons are respectively $320/256, 320/256, 225/256, 242/256$. Uniqueness here concerns these complete finite intervals; it is not a claim about all later evolution. All boxes begin with negative mean and areal acceleration and positive pressure. The two original insertions decrease to their regular turn and increase afterwards over the record. In every case a positive-duration interval has both geometric braking and decreasing insertions. Uniform guaranteed subintervals are respectively $(43, 210)/256, (36, 210)/256, (54, 159)/256$, and $(42, 138)/256$.

Proof mechanism. Each exact step of length $1/256$ has an interval tube Z satisfying strict Picard inclusion $z_0 + [0, 1/256] F(Z) \subset \text{int } Z$. Normalized Taylor coefficients give the endpoint polynomial through degree seven with the eighth-order remainder evaluated over that entire tube. Every elementary operation and square root is rounded outward by `nextafter`, under the stated assumption of IEEE-754 binary64 nearest rounding. Induction encloses every exact constrained initial datum in each open box. Within the recorded event brackets the event derivative has one strict sign; endpoint values have opposite strict signs. Outside those brackets, full-tube signs exclude missed, tangential or repeated crossings. The same records prove $d, Q, k_\Omega, W, g, \rho, u' > 0$. The analytic constraint identity preserves the constraint of the true enclosed trajectories, even though interval boxes also contain off-constraint points.

Trapping has an additional integrating-factor certificate from (72); this matters in the strongest negative-shear case, where independent coordinate intervals for $Q+s$ alone are too wide to prove its sign. The original radial responses obey $O'_d = 2d u, O'_u = 2u u'$. Thus the enclosed turn and strict $u' > 0$ supply their decrease and increase. These are computer-assisted theorems about the declared ODE; the computations are not semantic proofs of bearer or metric-support claims. $\square$

For later geometric reception, put $\hat{b} = b/b_i$. Full-tube sums, with a range-reduced Taylor series and positive-tail bound for the exponential, also enclose

$$\begin{aligned} \hat{b}(\tau) &= \exp \left[\int_0^\tau (-Q + 2s) d\sigma \right], \\ J_\#(T) &= \int_0^T \hat{b}(\tau)^{-1} d\tau. \end{aligned} \quad (86)$$

Table 8. Certified trapping and optical bounds for the complete KS records.

Shear centre	Lower bound for X	Upper bound for $J_\#(T)$
1/20	0.6799	1.688
− 1/20	0.1571	3.026
1/10	0.7299	1.028
− 3/20	0.0641	2.512

These displayed bounds are deliberately rounded outward. They cover the complete corresponding records, not only their braking portions. `validated_ks_events.py`, `certify_ks_events.py` and the full enclosure record are retained in the computational supplement. Independent reproduction and certificate checking cover all 2,214 continuous steps in the four central and four open-box runs.

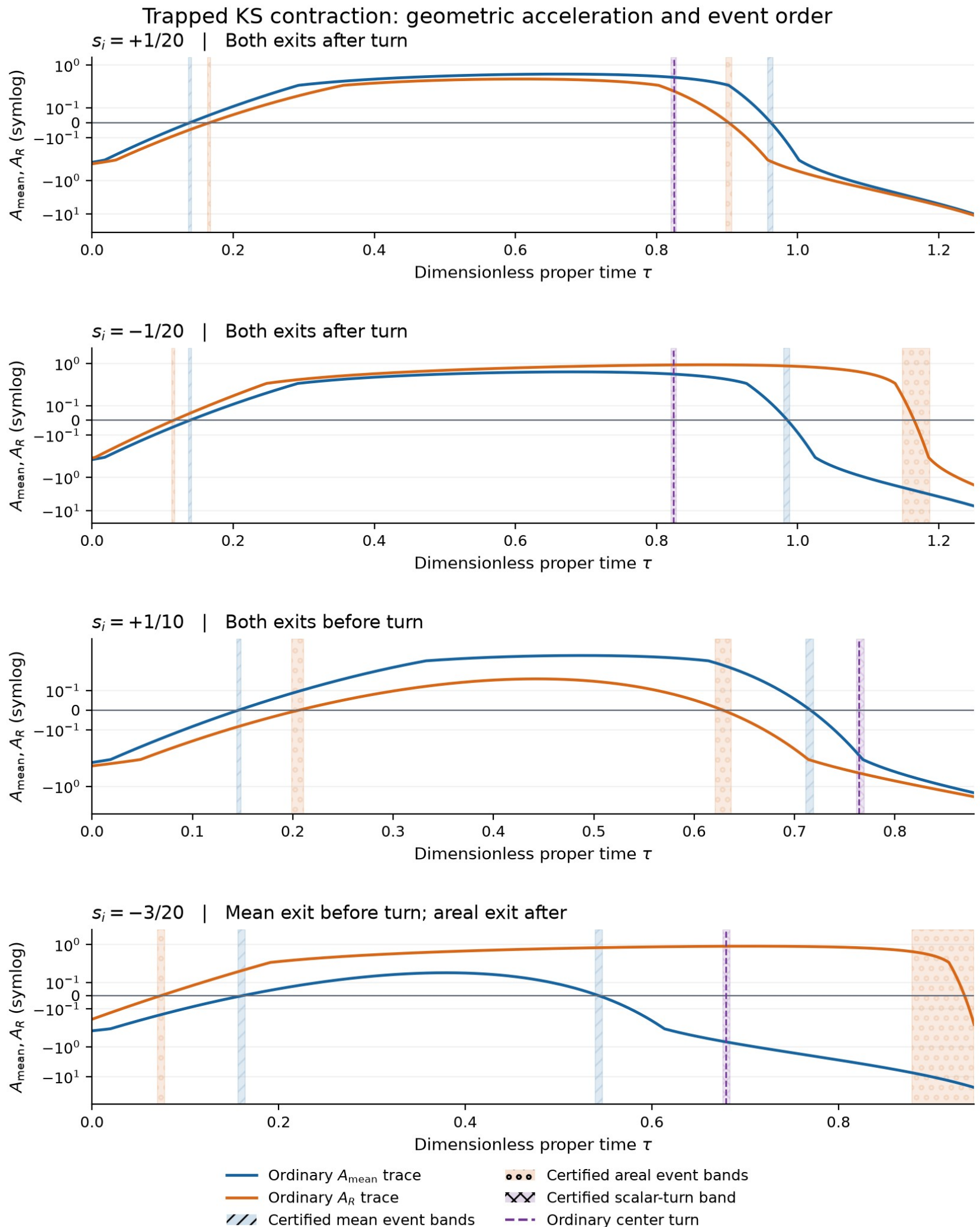


Figure 6. Signed shear changes the event order. All panels use curvature centre $k_{\Omega,i} = 1/100$; their shear centres are labelled. The plotted A_{mean} and A_R are respectively $\mathcal{A}$ and $\mathcal{B}$. Solid lines are ordinary centre-parameter trajectories at RK4 step 1/8192, and dashed purple lines mark their scalar turns. Hatched bands are the independently certified brackets for the complete open preparation boxes, not error bars inferred from those curves. Each panel uses its complete certificate time window. The symmetric-log ordinate is linear for $|A| \leq .25$, retaining both the positive regimes and large negative tails. Both geometric exits follow the turn in the first two panels, both precede it in the third, and they straddle it in the fourth. No band represents a semantic support boundary.

8.5. Physical meaning of the finite regimes

In the received KS experiments, the joint regime is the overlap of mean contraction, both geometric braking intervals and decreasing fixed source responses. For the exact insertions it is

$$\{ \tau : Q > 0, X > 0, \mathcal{A} > 0, \mathcal{B} > 0, (d^2)' < 0, (u^2)' < 0 \}. \quad (87)$$

This definition alone would be tautological evidence for its nonemptiness. The attained events, strict interval margins and actual geometric embedding supply the content. In a survival case the scalar turn ends the decreasing-response part while braking continues. In a competing case a geometric exit ends the joint regime sooner. Other independently fixed source responses can exit earlier still; the fixed mixed-port counterexample of Theorem 14 is not erased by the new geometry.

This supplies an intermediate physical consequence under the proposed receiving match: a finite decrease of the selected responses can coexist with braking of actual trapped spheres, while density rises and negative pressure reduces comoving matter energy through pressure work. It does not say energy is removed from a separate metric reservoir. The spheres remain trapped throughout, and no boundary crossing is described.

The broader flat results and the actual spherical realization support complementary conclusions. The open-potential theorem establishes a robust field reduction under quantified source uncertainty. The scalar loop and component boxes establish attained entry and competing exits for different preparations. The globally received KS boxes establish a finite coexistence of decreasing selected responses and braking of actual trapped spheres. None of these additions turns the contracting reference history into an attractor or identifies the selected response family with the entire differentiated source.

9. Global black-hole reception

A further question is global: a homogeneous trapped region could otherwise be part of a cosmological spacetime. The following construction supplies a regular centre, one asymptotically flat end and an actual black-hole region containing the experiment. Its price is an explicit outside-window completion of the effective potential and prepared initial data that already contain trapped spheres.

Theorem 19 (one-ended black-hole reception). For every preparation in the four boxes of Theorem 18, specified in (85), there are smooth, complete, one-ended asymptotically flat Einstein–scalar constraint data whose maximal development contains the corresponding complete recorded event interval inside its black-hole region. A nonzero central spatial subslab is received, not just one worldline. Future null infinity is complete in the sense of the imported spherical causal theorem. The claim uses the fixed potential completion below; it does not establish formation from wholly untrapped initial data or stability under arbitrary spatial perturbations.

Proof, stage 1: complete the source without changing the local experiment. Let $S_{\text{step}}(z)$ be zero for $z \leq 0$, one for $z \geq 1$, and $e^{-1/z} / (e^{-1/z} + e^{-1/(1-z)})$ between. Define

$$\begin{aligned} \chi(d) &= 1 - S_{\text{step}}\left(\frac{d^2 - .62^2}{.65^2 - .62^2}\right), \\ \widehat{W}(d) &= \chi(d) W(d), \quad V(\phi) = M^2 h^2 \widehat{W}(\phi / (2M)). \end{aligned} \quad (88)$$

This fixed smooth potential is nonnegative, equals W on $|d| \leq .62$, and vanishes identically on $|d| \geq .65$. The polynomial remains positive on the cutoff support: $W(.65) = .15551875$. Take exterior vacuum $d_v = .7$, where both potential and force vanish. The local interval is unchanged. This is an openly added global model hypothesis; the original polynomial has no stationary zero-energy vacuum.

Stage 2: solve both initial constraints. Use dimensionless proper radial distance $x = h \ell$, dimensionless sphere radius $r_g = h R$, and conventional extrinsic curvature $K = -1/2 \mathcal{L}_n g_S$. Put

$$\begin{aligned} g_S &= h^{-2} (d x^2 + r_g(x)^2 d \Omega^2), \\ K_j^i &= h \text{diag}(\kappa_{\parallel}, \kappa_{\perp}, \kappa_{\perp}), \Pi = \frac{n \phi}{M h}, \quad (89) \\ r_{g,0} &= k_{\Omega,i}^{-1/2}, \kappa_{\parallel,0} = Q_i - 2 s_i, \kappa_{\perp,0} = Q_i + s_i. \end{aligned}$$

Spatial subscripts in this proof denote spatial derivatives, not the temporal prime of the KS system. Each stage has zero scalar momentum current because either $d_x = 0$ or $\Pi = 0$. Its constraints are

$$\begin{aligned} -\frac{4 r_{g,xx}}{r_g} + \frac{2(1 - r_{g,x}^2)}{r_g^2} + 4 \kappa_{\parallel} \kappa_{\perp} + 2 \kappa_{\perp}^2 &= \Pi^2 + 4 d_x^2 + 2 \widehat{W}, \\ \kappa_{\perp,x} &= \frac{r_{g,x}}{r_g} (\kappa_{\parallel} - \kappa_{\perp}). \end{aligned} \quad (90)$$

If $\kappa_{\perp} = \kappa_{\perp}(r_g)$, the choice $\kappa_{\parallel} = \kappa_{\perp} + r_g \kappa_{\perp,r_g}$ solves the momentum constraint, including at a cylindrical plateau. Choose smooth $0 \leq \vartheta \leq 1$, equal to one near zero and zero for $|z| \geq \eta = .01$. Let

$$\psi(z) = \int_0^z \vartheta(v) dv, \text{ and set}$$

$$\begin{aligned} \kappa_{\perp}(r_g) &= \kappa_{\perp,0} - 3 s_i \psi(\log(r_g/r_{g,0})), \\ \kappa_{\parallel}(r_g) &= \kappa_{\perp}(r_g) - 3 s_i \vartheta(\log(r_g/r_{g,0})). \end{aligned} \quad (91)$$

Near the centre this profile is constant, so its extension is smooth. At $r_{g,0}$ it has exactly $\kappa_{\perp,0}, \kappa_{\parallel,0}$.

Throughout the slightly larger envelope

$|d_i - .6| \leq 10^{-5}, |Q_i - .63| \leq 10^{-5}, |s_i| \leq .151, |k_{\Omega,i} - .01| \leq 10^{-5}$, its variation from $\kappa_{\perp,0}$ is at most .00453 and $\kappa_{\perp} > .474$. With $W_{\max} = W(.59999)$,

$$4 \kappa_{\parallel} \kappa_{\perp} + 2 \kappa_{\perp}^2 - 2 W(d_i) \geq \begin{cases} 6(.62999 - .151 - .00453)^2 - 2 W_{\max} > .33, & s_i \leq 0, \\ 6(.62999 - .00453)^2 - 6(.151)^2 - 2 W_{\max} > 1.19, & s_i \geq 0. \end{cases} \quad (92)$$

For the second bound use $\kappa_{\parallel} \geq \kappa_{\perp} - 3 s_i$ and $\kappa_{\perp} = Q_i + s_i + e$, $|e| \leq .00453$. For the first use $\kappa_{\parallel} \geq \kappa_{\perp}$. These explicit positive margins keep the scalar momentum real through the cap and opening.

Stage 3: add the centre, cylinder and opening. Choose a smooth nonincreasing $v_{\text{cap}}: [0, 1] \rightarrow [0, 1]$, equal to one near zero and zero near one. If $I = \int_0^1 v_{\text{cap}}$, set $L_c = r_{g,0}/I$ and $r_g(x) = L_c \int_0^{x/L_c} v_{\text{cap}}(t) dt$. Then $r_g = x$ near the regular centre, $0 \leq r_{g,x} \leq 1, r_{g,xx} \leq 0$, and $r_g = r_{g,0}$ near its outer end. Set $d = d_i$, use (91), and choose

$$\Pi = -\sqrt{-4 r_{g,xx}/r_g + 2(1 - r_{g,x}^2)/r_g^2 + 4 \kappa_{\parallel} \kappa_{\perp} + 2 \kappa_{\perp}^2 - 2 W(d_i)}. \quad (93)$$

The radicand exceeds .33. At the centre its apparent inverse-radius terms vanish exactly, and the remaining data are constant and isotropic. At the far end all data are cylindrical and $\Pi = 2 u_i$. Insert an arbitrarily long exact cylinder with those same values; all endpoint jets agree.

Open the far end by smoothly increasing $r_{g,x}$ from zero to one, with flat endpoint jets and $0 \leq r_{g,xx} \leq .01$. Dilation of the ramp's proper length achieves this bound. Continue (91) and (93). Since $r_g \geq r_{g,0} > 9$, the

radicand is greater than $.33 - .04/9 > .325$. Continue with $r_{g,x}=1$ until $\kappa_{\perp}=\kappa_{\perp,\text{out}}$ is constant. The result is a smooth flat spatial collar with constant negative $\Pi_{\text{out}} = -\sqrt{6\kappa_{\perp,\text{out}}^2 - 2W(d_i)}$.

Stage 4: taper the matter and finish an ordinary vacuum end. Define the dimensionless Hawking mass

$$\begin{aligned} m &= \frac{r_g}{2} (1 - r_{g,x}^2 + \kappa_{\perp}^2 r_g^2), \\ m_{,r_g} &= \frac{r_g}{4} (\Pi^2 + 4d_x^2 + 2\widehat{W})(r_{g,x} > 0). \end{aligned} \quad (94)$$

The second identity follows from (90) and the zero-current momentum equation. In the flat collar $r_{g,x}=1$, first keep $d=d_i$ and prescribe a smooth taper of Π from its constant value to zero. Then keep $\Pi=0$ and smoothly change d from d_i to $.7$. Constant neighborhoods at the endpoints make every jet match. Solve the elementary mass integrals and reconstruct

$$m_{,r_g} = \begin{cases} r_g^2 (\Pi^2 + 2W(d_i))/4, & \text{momentum taper,} \\ r_g^2 (d_{,r_g}^2 + \widehat{W}(d)/2), & \text{field transition,} \end{cases} \quad \kappa_{\perp} = \sqrt{2m/r_g^3}, \kappa_{\parallel} = \kappa_{\perp} + r_g \kappa_{\perp,r_g}. \quad (95)$$

The mass stays positive and becomes a finite constant m_{∞} once the field reaches vacuum. Both constraints hold throughout; the two transitions never require simultaneous nonzero Π and d_x .

Continue in vacuum until $r_{g,a} > 2m_{\infty}$. Choose a smooth cutoff $0 \leq z(r_g) \leq 1$, equal to one near $r_{g,a}$ and zero beyond some finite $r_{g,b}$. Set

$$\begin{aligned} \kappa_{\perp} &= z(r_g) \sqrt{2m_{\infty}/r_g^3}, F = 1 + \kappa_{\perp}^2 r_g^2 - 2m_{\infty}/r_g > 0, \\ r_{g,x} &= \sqrt{F}, \kappa_{\parallel} = \kappa_{\perp} + r_g \kappa_{\perp,r_g}, \Pi = 0, d = .7. \end{aligned} \quad (96)$$

These solve the vacuum constraints with constant mass. They match the preceding flat collar where $z=1$. Beyond $r_{g,b}$, $\kappa_{\parallel}=\kappa_{\perp}=0$ and $F=1-2m_{\infty}/r_g$; the end is exactly a time-symmetric Schwarzschild end with standard asymptotic falloff. It is complete, has one infinity, and is joined through smooth constraint data rather than a shell. No optimized mass or compact outer radius is claimed.

Stage 5: receive the event interval and establish black-hole membership. Every noncentral initial sphere has strictly negative ingoing expansion $\theta_{-} = 2h(-\kappa_{\perp} - r_{g,x}/r_g) < 0$. The cylinder also has negative outgoing expansion because $r_{g,x}=0, \kappa_{\perp}=\kappa_{\perp,0} > 0$. The initial mass is bounded, and the vacuum end supplies nonempty future null infinity. The globally nonnegative smooth scalar potential satisfies the scalar extension principle [P10]. Thus the hypotheses of the spherical trapped-surface theorem [P9] apply: its causal result gives an event horizon, a black-hole region and future-complete null infinity. In particular its trapped region is disjoint from the causal past of future null infinity. No generic-censorship premise is added.

Normalize the initial longitudinal scale factor to one, centre its dimensionless coordinate $\mathcal{Y}_{\parallel}$ on a cylinder of half-length L , and use the proved optical width (86). Radial causal speed is at most $1/\hat{b}$; angular motion cannot increase that speed. Therefore

$$|\mathcal{Y}_{\parallel}| + J_{\parallel}(T) < L \quad (97)$$

keeps every past-directed causal curve from a recorded point in the KS patch until it meets the initial cylinder. Local uniqueness and domain of dependence identify this region of the global development with the exact KS solution. The certified common bound $J_{\parallel}(T) < 3.026$ permits $L=4$, a physical initial cylinder length $8/h$, and the full central subslab $|\mathcal{Y}_{\parallel}| < .9$. Its initial spheres are trapped; the complement of the causal past of

infinity is future invariant. The entire received event interval therefore lies in the black-hole region. Lemma 15 independently ensures that its spheres stay strictly trapped. $\square$

This closes the global reception gate for the declared completed scalar model. The four open boxes describe finite-dimensional constrained preparation parameters. An exact homogeneous cylinder is not an open condition in the full space of arbitrary initial-data functions. The construction is globally spatially varying, but it does not yet prove stability to spatial variation inside the experimental core.

10. Fixed horizon calibration

The preceding construction places the complete recorded KS experiments inside a one-ended black hole. It does not, by itself, select a numerical horizon size. This section supplies that missing calibration: first a bound valid across the admitted completions, then a fixed completion whose initial event horizon is identified exactly. The attained braking and scalar-turn events can consequently be compared in one fixed geometric unit. The subsequent endpoint analysis will use this same completion; the calibration itself does not locate a loss of metric support.

10.1. The reference cut and the contracting spheres

Let Σ_0 be the initial Cauchy slice of Theorem 19. In its classical completed development set

$$\begin{aligned} \mathcal{H} &= \partial J^-(\mathcal{I}^+), \mathcal{C}_H = \mathcal{H} \cap \Sigma_0, A_H = 4\pi R_H^2, \\ \eta(\tau) &= \frac{A(\tau)}{A_H}, A(\tau) = 4\pi R(\tau)^2. \end{aligned} \quad (98)$$

The cut faces the single asymptotically flat end. The initially trapped cylinder and escaping exterior lie on one connected spacelike quotient slice. The single outgoing horizon ray supplied by [P9] separates them and intersects that slice once, at positive radius. Thus $A_H > 0$ is well-defined for each completed dataset. Freeze it thereafter. Different datasets need not have the same numerical denominator.

The sphere at $\mathcal{C}_H$ is not an unrecorded first event on the central observer's trajectory. The local records begin strictly inside the hole. Nor is an initial marginally outer trapped sphere substituted for an event horizon. We continue to use the original normal observer, $\tau = h(t - t_i)$, $k_\Omega = (hR)^{-2}$, and $X = Q + s = -R'/R$.

Proposition 20 (strict area decrease in fixed horizon units). On every regular received portion of any of the four KS histories,

$$\begin{aligned} \frac{A(\tau)}{A_i} &= \frac{k_{\Omega,i}}{k_\Omega(\tau)} = \exp \left[-2 \int_0^\tau X(v) dv \right], \\ \eta(\tau) &= \eta_i \frac{k_{\Omega,i}}{k_\Omega(\tau)}, \eta_i = \frac{A_i}{A_H}. \end{aligned} \quad (99)$$

Both A and η decrease strictly, including through either braking interval.

Proof. The unchanged equations give $k'_\Omega = 2Xk_\Omega$ and $X' = 3sX + 2u^2$. All four initial boxes have $X_i \geq .47998 > 0$. Hence

$$X(\tau) = e^{\int_0^\tau 3s(v)dv} \left[X_i + \int_0^\tau 2u(v)^2 e^{-\int_0^v 3s(w)dw} dv \right] > 0. \quad (100)$$

Integrating $k'_{\Omega}/k_{\Omega}=2X$ proves (99), and $A'=-2XA<0$. No sign assumption on the shear or monotonicity of Q is needed. $\square$

Mean acceleration $\mathcal{A}$, areal-radius acceleration $\mathcal{B}$, and scalar turn $u=0$ remain distinct. In particular, because the denominator is fixed,

$$\frac{(R/R_H)''}{R/R_H} = \mathcal{B}, \frac{\eta''}{\eta} = 2(\mathcal{B} + X^2). \quad (101)$$

An inflection of an area curve therefore does not mark the entry or exit of areal-radius braking. Positive $\mathcal{B}$ slows the already contracting radius; it need not reverse that contraction.

10.2. A bound before choosing the exterior profiles

Write $r_g = hR$ and $x = h\ell$ on the initial slice. Let m be the dimensionless Hawking mass, m_{∞} the constant mass of the asymptotic Schwarzschild end, and m_f the dimensionless final Bondi mass. In units $c=1$, the ADM mass is $m_{\infty}/(Gh)$. This physical mass is distinct from the scalar normalization M in the action. The zero-current constraints of (90)-(95) give

$$\begin{aligned} m &= \frac{r_g}{2} (1 - r_{g,x}^2 + \kappa_{\perp}^2 r_g^2), \\ m_{,r_g} &= \frac{r_g^2}{4} (\Pi^2 + 4d_x^2 + 2\widehat{W}(d)) \geq 0, \quad (102) \\ 2m_0 &= r_0 \left(1 + \frac{X_i^2}{K_{\Omega,i}} \right), \quad r_0 = k_{\Omega,i}^{-1/2}. \end{aligned}$$

The second line applies where radius increases; mass is constant on the intervening cylinder. It follows directly by differentiating the first line and substituting the Hamiltonian constraint and $\kappa_{\parallel} = \kappa_{\perp} + r_g \kappa_{\perp,r_g}$. The two matter tapers never have simultaneous nonzero d_x and Π , so the zero-current identity is retained throughout.

Proposition 21 (same-slice horizon bound). Every completion admitted by the preceding global construction obeys

$$2m_0 \leq r_{A,\text{out}} \leq hR_H \leq 2m_f \leq 2m_{\infty}, \quad \eta_i \leq \left(1 + \frac{X_i^2}{K_{\Omega,i}} \right)^{-2}. \quad (103)$$

Here $r_{A,\text{out}}$ is the dimensionless radius of the outermost initial marginally outer trapped sphere.

Proof. Mass is nondecreasing outward by (102). Every realized initial radius $r_0 \leq r_g < 2m_0$ therefore has $2m/r_g > 1$. The ingoing expansion is negative everywhere on this slice. The Hawking identity then forces the outgoing expansion to be negative on that interval as well. These trapped spheres cannot belong to the causal past of future null infinity. The horizon cut lies outside them, giving $hR_H \geq 2m_0$ by continuity. The outgoing expansion is negative on the cylinder and positive sufficiently far along the asymptotic end. Its outer zero set is nonempty, closed and bounded, so an outermost initial marginal sphere exists. It cannot occur below $2m_0$, and marginal spheres also cannot escape to future null infinity under [P9, equation (6)]; hence $2m_0 \leq r_{A,\text{out}} \leq hR_H$. This argument neither assumes uniqueness of marginal spheres nor treats the initial horizon as marginal.

The upper horizon bound is [P9, Lemma 3]; its hypotheses were received in Theorem 19 using the scalar extension principle [P10]. The exact Schwarzschild end supplies an early constant-mass portion of null

infinity, and Bondi mass is nonincreasing, giving $m_f \leq m_\infty$. Dividing r_0^2 by the squared lower horizon bound gives the area inequality. $\square$

Here [P9] is used with its one-ended quotient, energy signs, initial negative ingoing expansion and extension hypothesis. [P10] proves that extension property for potentials bounded below; the nonnegative completed potential additionally supplies the signs needed for [P9]. Neither result imports regularity of the black-hole interior. The primary results are [P9, equation \(6\), Theorem 1 and Lemma 3](#) and [P10, Theorem 3.1 and Sections 5-6](#).

Evaluating (103) with the minimum X_i and maximum $k_{\Omega,i}$ in each independent 10^{-5} preparation box gives:

Table 9. General whole-box bounds on the initial interior-to-horizon area ratio.

Initial shear centre	Centre $2 m_0$	Rigorous whole-box upper bound on η_i
+.05	472.4	.000449035
-.05	346.4	.000835114
+.10	542.9	.000339985
-.15	240.4	.001733934

These outward-rounded bounds do not measure the actual horizon. Its area-mass $R_H/(2G)$, its Hawking mass, ADM mass and final Bondi mass are not generally interchangeable.

10.3. An explicit completion and its exact horizon

We now fix, rather than vary, the freedoms in the global construction. Retain the same $\widehat{W}$ from (88), the exterior vacuum $d_v=.7$, and the initial cylinder half-length 4. Use the smooth step S , equal to zero for $z \leq 0$, one for $z \geq 1$, and $e^{-1/z}/(e^{-1/z} + e^{-1/(1-z)})$ between. Its symmetry gives $S(1-z)=1-S(z)$. It also obeys $0 \leq S' \leq 2$: with $v=2z-1$ and $w=2v/(1-v^2)$,

$$S'(z) = \frac{2(1+v^2)}{(1-v^2)^2} \operatorname{sech}^2 w \leq \frac{2}{1+v^2} \leq 2, \quad (104)$$

where $\cosh^2 w \geq 1+w^2$.

For the regular cap take $v_{\text{cap}}(z)=1-S(2z-1/2)$ on $[0,1]$. Its integral is $1/2$, so the cap length is $2r_0$. The cap formula in the preceding proof gives $r_g=x$ near the centre and $r_g=r_0$ near its outer end. In the extrinsic-curvature profile set

$$\begin{aligned} \vartheta(z) &= \begin{cases} 1, & |z| \leq .005, \\ 1 - S((|z| - .005)/.005), & .005 < |z| < .01, \\ 0, & |z| \geq .01, \end{cases} \quad \psi(z) = \int_0^z \vartheta(v) dv, \\ \kappa_\perp(r_g) &= Q_i + s_i - 3s_i \psi(\log(r_g/r_0)), \quad \kappa_\parallel(r_g) = \kappa_\perp(r_g) - 3s_i \vartheta(\log(r_g/r_0)), \\ a_c &= \frac{391}{400}, \quad B_c = Q_i + a_c s_i. \end{aligned} \quad (105)$$

The positive-side integral of ϑ is exactly .0075. Thus $\kappa_\parallel = \kappa_\perp = B_c$ on the opening and constant-profile stage wherever $r_0 e^{.01} \leq r_g \leq r_c$. The later tapers reconstruct their varying extrinsic curvature from the mass constraint. The coefficient a_c is a consequence of this fixed profile.

Beyond the cylinder, use opening coordinate $0 \leq \xi \leq 200$ and set

$$r_{g,x} = S(\xi/200), 0 \leq r_{g,x} \leq .01, r_c = r_0 + 100, \quad \Pi_c = -\sqrt{6B_c^2 - 2W(d_i)}. \quad (106)$$

The radial increase is 100 by the step symmetry. The radicand on the cap exceeds .33, by (92); the opening subtracts at most $.04/r_0 < .04/9$, leaving a margin greater than .325. Hence the negative Hamiltonian root remains real and smooth. At the end of the opening, the radius derivative is one with flat matching jets, the extrinsic profile is constant, and the data have exactly the displayed Π_c .

Keep $r_{g,x} = 1$ through the next two radial units and prescribe

$$\begin{aligned} r_c \leq r_g \leq r_c + 1: & \quad d = d_i, & \quad \Pi = \Pi_c [1 - S(r_g - r_c)], \\ r_c + 1 \leq r_g \leq r_c + 2: & \quad d = d_i + (.7 - d_i) S(r_g - r_c - 1), & \quad \Pi = 0. \end{aligned} \quad (107)$$

Define m by (102) and reconstruct $\kappa_\perp = \sqrt{2m/r_g^3}$, $\kappa_\parallel = \kappa_\perp + r_g \kappa_{\perp, r_g}$. Both constraints hold, all endpoint jets match, and the scalar current stays zero. Matter ends at $r_v = r_c + 2$.

Theorem 22 (exact fixed-completion horizon). The profiles (105)-(107), with the vacuum continuation specified below, produce an admitted completion for every preparation in the four boxes. Its final mass is

$$\begin{aligned} m_\infty = & \frac{B_c^2 r_c^3}{2} + \int_0^1 \frac{(r_c + z)^2}{4} \{ \Pi_c^2 [1 - S(z)]^2 + 2W(d_i) \} dz \\ & + \int_0^1 (r_c + 1 + z)^2 \{ (.7 - d_i)^2 S'(z)^2 + 1/2 \widehat{W}(d_i + (.7 - d_i) S(z)) \} dz. \end{aligned} \quad (108)$$

Its initial horizon and all regular received interior sphere areas satisfy

$$h R_H = 2 m_\infty, A_H = \frac{16 \pi m_\infty^2}{h^2}, m_f = m_\infty, \eta(\tau) = \frac{1}{4 m_\infty^2 k_\Omega(\tau)}. \quad (109)$$

Proof. At the start of the tapers, $r_{g,x} = 1$ and $\kappa_\perp = B_c$, so $m(r_c) = B_c^2 r_c^3 / 2$. Integrating the two nonnegative source densities in (102) with (107) gives exactly (108). Continue the resulting vacuum Painlevé-Gullstrand collar to $r_a = 4 m_\infty$. In (96) take the vacuum slicing cutoff $\zeta(r_g) = 1 - S(r_g - r_a)$. There $F = 1 + \kappa_\perp^2 r_g^2 - 2 m_\infty / r_g \geq 1/2$. It matches the flat vacuum collar to all orders and becomes the time-symmetric Schwarzschild end beyond $r_a + 1$. The centre, cylinder, opening and transitions therefore remain smooth complete constraint data with one asymptotically flat end.

Uniformly on all four boxes, $B_c > .474$, $r_c > 109$, and $r_v < 113$. The nonnegative mass increments imply $2 m_\infty \geq B_c^2 r_c^3 > r_v$. Thus an exact vacuum neighborhood of the initial Schwarzschild radius is present on both sides of $r_g = 2 m_\infty$. Positive $\kappa_\perp$ in the flat vacuum collar selects the future black-hole branch: both expansions below that radius are negative.

In the reference Schwarzschild future of this initial vacuum segment, no future causal curve from $r_g < 2 m_\infty$ reaches its exterior or horizon. Every past causal foot of a future exterior or horizon point therefore lies on the initial vacuum segment. Cauchy uniqueness and domain of dependence identify that complete future exterior and horizon with a region of the scalar development. Initial spheres above $2 m_\infty$ escape through the received exterior; those just below are trapped. Their boundary is precisely $\mathcal{C}_H$, proving $h R_H = 2 m_\infty$. The exterior stays exactly Schwarzschild, so $m_f = m_\infty$. The remaining identities follow from (98) and $k_\Omega = (h R)^{-2}$. $\square$

The causal step is essential: equality is not inferred from Birkhoff's theorem or an assumed final settling. The theorem concerns the future of this fixed initial slice; it is not a construction of untrapped formation.

10.4. Bounds and completion dependence of the numerical scale

There is an analytic enclosure independent of numerical quadrature:

$$m_c := \frac{B_c^2 r_c^3}{2} \leq m_\infty \leq m_c + \frac{3}{2} B_c^2 (r_c + 1)^2 + \left[4(.7 - d_i)^2 + 1/2 W(d_i) \right] (r_c + 2)^2. \quad (110)$$

Indeed $0 \leq 1 - S \leq 1$ makes the first integrand in (108) at most $(r_c + 1)^2 (\Pi_c^2 + 2 W(d_i)) / 4 = 3/2 B_c^2 (r_c + 1)^2$. Along the field transition, $\widehat{W} \leq W(d_i)$, while (104) bounds the squared field gradient by $4(.7 - d_i)^2$. The integrals have unit length, giving (110). At the centres, $r_c = 110$, and all coefficients in (110) are rational.

Table 10. Fixed-completion horizon normalization: numerical centres and analytic centre enclosures.

S_i	Numerical $h R_H$	Numerical η_i	Rigorous analytic centre enclosure for η_i
+.05	626692.9485	$2.5461875 \times 10^{-10}$	$[2.4579, 2.6576] \times 10^{-10}$
-.05	460939.0365	$4.7066621 \times 10^{-10}$	$[4.5393, 4.9496] \times 10^{-10}$
+.10	719214.2169	$1.9332298 \times 10^{-10}$	$[1.8668, 2.0125] \times 10^{-10}$
-.15	320903.2910	$9.7107251 \times 10^{-10}$	$[9.3512, 10.3400] \times 10^{-10}$

The numerical columns are ordinary centre diagnostics from independent Simpson and Gauss-Legendre quadratures. Agreement at the printed precision is not a quadrature error certificate. The last column follows instead from (110) by exact rational arithmetic and outward rounding. These are centre bounds, not those same narrow bounds over the whole initial box.

Proposition 23 (nonuniqueness of the horizon calibration). The same local KS preparation and its complete recorded experiment admit completions with arbitrarily small η_i .

Proof. Before the two unit tapers, insert an arbitrarily long allowed flat constant- B_c collar. The cylinder and its optical domain of dependence are unchanged, so its recorded metric/source experiment remains isometric. Let the new terminal collar radius be $\tilde{r}_c$. Its initial mass and the two taper estimates give

$$m_\infty = \frac{B_c^2 \tilde{r}_c^3}{2} + O(\tilde{r}_c^2), \quad \frac{A_i}{A_H} = \frac{r_0^2}{4 m_\infty^2} \rightarrow 0 \quad (111)$$

$$(\tilde{r}_c \rightarrow \infty).$$

For sufficiently large $\tilde{r}_c$, the matter endpoint $\tilde{r}_c + 2$ is below $2 m_\infty$, and the same vacuum causal proof applies. These are different actual event horizons of the admitted completed models, not merely different unspecified ADM masses. $\square$

The small ratios in the preceding table reflect this deliberately extended, massive completion. They are neither universal physical thresholds nor an astrophysical calibration. Even the chosen opening has a construction-specific scale cost: integrating $0 \leq r_{g,x} \leq .01$ while $r_{g,x}$ rises from zero to one gives

$1 = 2 \int r_{g,xx} dr_g \leq .02(r_c - r_0)$. It imposes $r_c \geq r_0 + 50$; the selected symmetric ramp uses $r_c = r_0 + 100$.

10.5. Attained events in the fixed units

Let M_+ and M_- denote mean-braking entry and exit, R_+ and R_- areal-radius-braking entry and exit, and T the scalar turn. The disjoint certified event brackets retain their different orders in all four boxes. The table lists those orders; by Proposition 20, their sphere areas have the reverse strict order.

Table 11. Twenty attained events: numerical centres, certified relative areas and fixed-unit calibration.

S_i	Event	Numerical centre τ	Numerical centre A/A_i	Certified whole-box A/A_i	Numerical centre $10^{10} \eta$
+.05	M_+	.138479	.816719	[.811400, .821606]	2.079519
	R_+	.166341	.781631	[.777107, .787027]	1.990180
	T	.824267	.207202	[.204286, .210220]	.527574
	R_-	.902290	.167585	[.164960, .170206]	.426702
	M_-	.962115	.140071	[.138221, .143007]	.356648
−.05	R_+	.115352	.868589	[.863934, .873551]	4.088155
	M_+	.138313	.843346	[.838270, .847686]	3.969347
	T	.823450	.341221	[.338290, .343589]	1.606014
	M_-	.985670	.288904	[.287180, .291336]	1.359773
	R_-	1.165116	.248760	[.243182, .253162]	1.170828
+.10	M_+	.145942	.793627	[.787678, .798177]	1.534263
	R_+	.206144	.714154	[.705646, .725534]	1.380623
	R_-	.628626	.280828	[.273487, .287875]	.542904
	M_-	.715845	.217218	[.214300, .221580]	.419932
	T	.764164	.185202	[.180953, .187653]	.358037
−.15	R_+	.073682	.929908	[.923081, .935778]	9.030083
	M_+	.161305	.849966	[.845073, .856938]	8.253787

S_i	Event	Numerical centre τ	Numerical centre A/A_i	Certified whole-box A/A_i	Numerical centre $10^{10} \eta$
	M_-	.543053	.585666	[.582108, .589338]	5.687242
	T	.679328	.529952	[.526948, .532738]	5.146221
	R_-	.935016	.473101	[.468805, .483202]	4.594152

The whole-box column is a new exact postprocessing of the original validated event enclosures, not a new interval ODE solve. For each certified event bracket, gather every intersecting full continuous-step tube, including both end rows, and verify coverage without gaps. If K_- and K_+ are the least lower and greatest upper curvature endpoints among those tubes, respectively, then

$$\frac{A(\tau_e)}{A_i} \in \left[\frac{999/100000}{K_+}, \frac{1001/100000}{K_-} \right]. \quad (112)$$

This follows from (99), the original interval for $k_{\Omega,i}$, positivity of curvature and coverage of the actual event by the tubes. All 61 tubes covering the twenty brackets are included. Their binary64 endpoints are converted to exact dyadic rationals before division; decimal endpoints are rounded outward by integer arithmetic. Correlation is discarded conservatively. The underlying enclosure qualification remains: Python binary64 basic operations and *math.sqrt* use nearest rounding, and one outward *nextafter* encloses each elementary result. Endpoint-only samples are not used to establish (112).

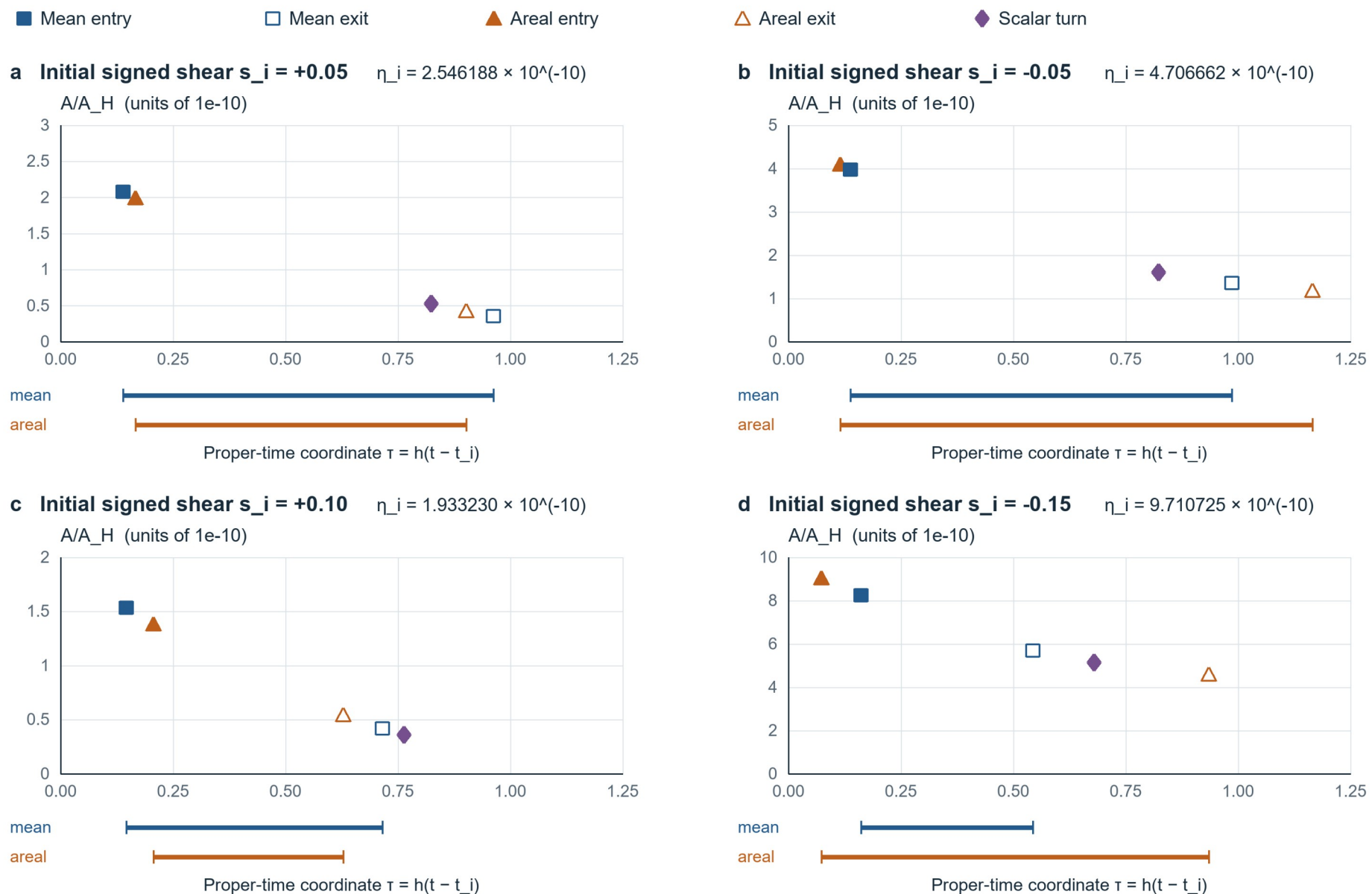
The independent centre integrations use RK4 at steps $1/8192, 1/16384, 1/32768$, with within-step event bisection and a separate integration of X to check both forms of (99). At the two finest steps, event times differ by at most 4.72×10^{-14} and relative areas by 3.29×10^{-14} . These consistency checks do not replace the continuous-tube certificate. In particular, the last column is not a narrow certified absolute-area interval.

If a valid event enclosure is $A/A_i \in [a, b]$ and a valid calibration enclosure is $\eta_i \in [c, d]$, then $\eta \in [ca, db]$; independence is unnecessary. Combining the centre calibration bounds above with whole-box event bounds certifies the centres only. Whole-box calibration can instead use (103), or a separate enclosure of (108) across the box.

Sphere areas at entry, scalar turn and exit

Chosen fixed completion; initial EH reference $A/A_H = 1$ outside plotted range; no support endpoint located.

Central numerical events only. Fixed initial horizon area A_H ; each panel has its own vertical range.



Bars: mean acceleration > 0 ; radius acceleration > 0 . The latter is not area acceleration; radius and area both decrease.

Horizon normalization comes from the selected global-completion quadrature. Marker positions are numerical, without certified error bars.

Figure 7. Numerical central events for the explicitly chosen completion (105)-(109), with separate vertical ranges. Every panel uses its own fixed initial event-horizon area; its numerical ordinate is $10^{10} A/A_H$. The two bars mark positive mean acceleration and positive areal-radius acceleration, respectively. The latter concerns R'' , not A'' , as (101) shows. The reference $A/A_H = 1$ lies far outside the panels. All twenty points are attained classical events; no support-loss marker is placed on this figure. Certified whole-box relative-area enclosures are in the table and have not been replaced by numerical error bars.

10.6. Geometric units and the next continuation question

The square root $\sqrt{\eta} = R/R_H$ is an areal-radius ratio, not proper distance to the horizon. Different longitudinal spheres on one homogeneous cylinder slice have the same area. To specify a local proper length or volume, fix a comoving strip in the received central region, $|y_{||}| < .9$, of width $\Delta y_{||} \leq 1.8$, use $b_i = 1$, $y_{||} = h z_{||}$, and let

$N = \int_0^{\tau} Q(v) dv$. Direct integration of the induced spatial metric gives

$$\begin{aligned} \frac{\ell}{R_H} &= \frac{b \Delta y_{||}}{h R_H}, \\ \frac{V_{\text{strip}}}{R_H^3} &= \frac{4 \pi b \Delta y_{||}}{k_{\Omega} (h R_H)^3}, \\ \frac{V_{\text{strip}}}{V_{\text{strip},i}} &= b \frac{k_{\Omega,i}}{k_{\Omega}} = e^{-3N}, \\ \frac{\Delta t}{R_H} &= \frac{\Delta \tau}{h R_H}. \end{aligned} \quad (113)$$

The volume identity uses $b'/b = -Q + 2s$ and $k'_{\Omega}/k_{\Omega} = 2(Q + s)$. A proper distance to the horizon or a volume of the entire black-hole interior would require a specified global slice and the metric throughout that integration region; (113) makes neither claim.

More generally, along any regular received approach $0 \leq \tau < \tau_*$, monotonicity gives the one-sided geometric limit

$$\eta_* = \eta_i \exp \left[-2 \int_0^{\tau_*} X(v) dv \right] \in [0, \eta_i]. \quad (114)$$

It is positive if the integral is finite and zero if it diverges. This is a limit of original metric-side answers, not an area assigned beyond their domain. A later identification of τ_* with a licensing or AMetric boundary would require the original support and operation comparison, including admission of the limiting question itself. The event horizon here likewise belongs to the complete classical model; if an independently established loss invalidates its assumed future, that horizon remains a comparison-model reference unless the changed future is received.

Imported support-to-radius criterion. Fix independently the source/physical records Z , the proposed retained support information $p: Z \rightarrow P$, and the admitted areal-radius answer $r: Z \rightarrow \mathbb{R}_{>0}$, with its original domain and independently warranted output transport to common units. Existence of a support-only radius decoder is characterized by

$$\begin{aligned} \exists \bar{r}: p(Z) \rightarrow \mathbb{R}_{>0}: r = \bar{r} \circ p &\Leftrightarrow \forall z, z' \in Z, \\ [p(z) = p(z') \Rightarrow r(z) = r(z')]. \end{aligned} \quad (115)$$

If the factorization exists, equal retained inputs have equal radii. Conversely, the fibre condition defines $\bar{r}(x)$ as the common radius on the nonempty fibre over each $x \in p(Z)$; equality on that fibre makes the value well defined and forces uniqueness. A claim on all of P additionally requires coverage $p(Z) = P$ or an independently warranted extension. This is the criterion of AMetric Theorem 7.2, p.36, and L-metric Lemma 4.2, p.14 [A12; A10], not a proof that this source's proposed support records determine radius. Original-incidence warrant remains required; any geometry already present in the complete original problem must be retained.

A one-way radius bound must hold over the complete relevant support fibre; failure of exact factorization alone does not refute such a bound. Monotonicity additionally requires the claimed inequality along an actual history ordered by the original support relation. Adding radius to p by definition makes its decoding tautological and does not establish a support-to-geometry law. Conversely, a radius-to-support determination or implication has its own fibres and conditions. These separate comparisons are needed before interpreting (114) as the geometric approach to an independently established loss of the specified application.

The next section asks what the same completed source actually does beyond the recorded events. It must prove both continuation of the homogeneous solution and its continued presence in the fixed global domain of dependence. A small value of η , a scalar turn, or either braking exit supplies neither proof.

11. Finite-time invariant endpoint

The finite braking experiments have a further, source-determined future in the completed scalar model. All four homogeneous preparation boxes reach an exact massless-scalar tail with divergent curvature in finite proper time. For the two positive-shear boxes, the entire approach remains inside the domain of dependence of the original cylinder of half-length 4. This establishes an invariant obstruction in the actual globally embedded histories, without replacing their scalar interiors by Schwarzschild vacuum. The corresponding future receipt remains unresolved for the negative-shear boxes.

11.1. The fixed source beyond the recorded experiment

Retain the canonical scalar, the KS variables in (69)–(71), and the smooth nonnegative potential completion in (88). Thus $\phi = 2 M d$, $\tau = h(t - t_i)$, $u = d'$, $k_\Omega = (h R)^{-2}$, $X = Q + s = -R'/R$, and $Y = Q - 2s = -b'/b$. Here $M, h > 0$, $b_i = 1$, and a prime denotes $d/d\tau$. The dimensionless longitudinal coordinate is $y = h z$. The already fixed completion has

$$\widehat{W}(d) = \left[1 - S_{\text{step}} \left(\frac{d^2 - .62^2}{.65^2 - .62^2} \right) \right] (3 - 8d^2 + 3d^4), \quad (116)$$

$$V = M^2 h^2 \widehat{W}(d).$$

The smooth step increases from zero to one on $(0, 1)$. Since the polynomial is decreasing and remains positive up to $d = .65$, $0 \leq \widehat{W} \leq 3$, $\widehat{g} = -\widehat{W}_{,d}/4 \geq 0$ for $d > 0$, and both $\widehat{W}$ and $\widehat{g}$ vanish for $d \geq .65$. The force is smooth and bounded. These properties concern the declared global completion; extrapolation of the uncompleted quartic would give a different source.

The constraint and evolution retain every term of the KS system:

$$\begin{aligned} d' &= u, u' = 3Q u + \widehat{g}, Q' = 2u^2 + 3s^2 - k_\Omega/3, \\ s' &= 3Q s + k_\Omega/3, k'_\Omega = 2X k_\Omega, 3Q^2 = 2u^2 + \widehat{W} + 3s^2 - k_\Omega, \\ Q' &= 3Q^2 - \widehat{W} + 2/3 k_\Omega, X' = 3s X + 2u^2, Y' = 3Q Y - \widehat{W}. \end{aligned} \quad (117)$$

Constraint satisfaction propagates. The integrating factor for X preserves $X > 0$ throughout subsequent regular evolution. In particular, all symmetry spheres remain future trapped. The scalar source obeys

$$\begin{aligned} \rho &= M^2 h^2 (2u^2 + \widehat{W}), p = M^2 h^2 (2u^2 - \widehat{W}), \\ \dot{\rho} + 3H(\rho + p) &= 0, H = -hQ. \end{aligned} \quad (118)$$

No extra material reservoir or change to the source ledger is introduced.

11.2. Attainment of the cutoff and the exact tail

Theorem 24 (finite homogeneous scalar-curvature end). Suppose a regular constrained outgoing state at τ_e satisfies $0 < d_e < .65$, $u_e > 0$, $Q_e > 1$, $k_{\Omega,e} > 0$, and $X_e > 0$. Its homogeneous evolution under (116) reaches $d_c = .65$ at a finite regular time τ_c . Its subsequent maximal future proper-time parameter τ_* is finite, and $d, u, k_{\Omega} \rightarrow +\infty$, $R \rightarrow 0$. The scalar curvature tends to $-\infty$, and $R_{ab}R^{ab} \rightarrow +\infty$.

Proof. From (117), $Q' \geq 3(Q^2 - 1) > 0$. Therefore $Q > 1$ persists, $u' = 3Qu + \hat{g} > 0$, and $u \geq u_e$. Integration of the Riccati comparison bounds the remaining regular lifespan by

$$\tau_* - \tau_e \leq F(Q_e), F(q) = \int_q^\infty \frac{dv}{3(v^2 - 1)} = \frac{1}{6} \log \frac{q+1}{q-1}. \quad (119)$$

This inequality alone does not identify an invariant singularity or prove arrival at the cutoff. To exclude termination at a finite field value, use d as the independent variable and set $x = X/u > 0$, $w = Y/u$. Direct differentiation gives

$$\begin{aligned} w_{,d} &= -\frac{\widehat{W} + w\hat{g}}{u^2}, x_{,d} = 2 - x^2 - 2xw - \frac{x\hat{g}}{u^2}, \\ (\log u)_{,d} &= 2x + w + \frac{\hat{g}}{u^2}. \end{aligned} \quad (120)$$

On a finite field interval $[d_e, D_0]$, the upper Dini derivative of $|w|$ is at most $3/u_e^2$. The term involving $\hat{g}$ damps its absolute value. Consequently, with $M_{D_0} = |w_e| + 3(D_0 - d_e)/u_e^2$,

$$|w| \leq M_{D_0}, x \leq \max\{x_e, M_{D_0} + \sqrt{M_{D_0}^2 + 2}\}. \quad (121)$$

The second estimate follows from $x_{,d} \leq 2 - x^2 + 2M_{D_0}x$. The last equation in (120) bounds u above on this interval, while its original evolution bounds it below by u_e . Thus $Q = (2x + w)u/3$ and $s = (x - w)u/3$ stay finite. The equations $(\log k_{\Omega})_{,d} = 2x$, $(\log b)_{,d} = -w$, and $(\log R)_{,d} = -x$ also keep k_{Ω}, b, R finite and strictly positive. Smooth ODE continuation excludes termination before any such finite field value. Since $d' \geq u_e$, the cutoff is actually reached, with $0 < \tau_c - \tau_e \leq (.65 - d_e)/u_e$.

At this regular crossing, put $D = d - d_c \geq 0$ and define

$$\begin{aligned} c &= Y_c/u_c, \alpha = \sqrt{c^2 + 2}, \zeta_c = X_c/u_c + c, \\ Z_c &= \operatorname{atanh}(\zeta_c/\alpha), G(D) = \frac{\cosh(Z_c + \alpha D)}{\cosh Z_c}. \end{aligned} \quad (122)$$

On the tail $Y/u = c$ is constant. The constraint gives $(x + c)^2 = \alpha^2 - k_{\Omega}/u^2 < \alpha^2$, so Z_c is real. Moreover $x_{,d} = \alpha^2 - (x + c)^2 = k_{\Omega}/u^2 > 0$. Integration therefore yields

$$\begin{aligned} x(D) &= \alpha \tanh(Z_c + \alpha D) - c, \\ u(D) &= u_c e^{-cD} G(D)^2, k_{\Omega}(D) = k_{\Omega,c} e^{-2cD} G(D)^2, \\ b(D) &= b_c e^{-cD}, R(D) = R_c e^{cD} / G(D), \\ Q(D) &= (2x(D) + c)u(D)/3, s(D) = (x(D) - c)u(D)/3, \\ \tau(D) - \tau_c &= \int_0^D \frac{dv}{u(v)}. \end{aligned} \quad (123)$$

These formulae solve the full constrained system for every finite D . Since $\alpha > |c|$, the final integral converges as $D \rightarrow \infty$, while $u, k_{\mathcal{D}}$ diverge and $R \rightarrow 0$. This gives the finite maximal time and identifies the variables that become singular. Writing $\beta = 2\alpha - c > 0$, the asymptotics include

$$u \sim \frac{1}{\beta(\tau_* - \tau)}, Q \sim \frac{1}{3(\tau_* - \tau)}, \quad (124)$$

$$R \sim C_R (\tau_* - \tau)^{(\alpha - c)/(2\alpha - c)}, 0 < \frac{\alpha - c}{2\alpha - c} < \frac{2}{3}.$$

The longitudinal scale may shrink, stay constant, or grow according to the sign of c ; the volume factor $b R^2$, proportional to $1/u$, tends to zero in every case. The kinetic scalar remains present: on this tail $\rho = p = 2 M^2 h^2 u^2 \rightarrow \infty$.

Finally, the canonical Einstein equation is $R_{ab} = M^{-2}(\partial_a \phi \partial_b \phi + V g_{ab})$. In an orthonormal frame it gives $R_{00} = h^2(4u^2 - \widehat{W})$, $R_{ii} = h^2 \widehat{W}$. Hence

$$\mathcal{R} = 4h^2(\widehat{W} - u^2), R_{ab}R^{ab} = h^4[(4u^2 - \widehat{W})^2 + 3\widehat{W}^2]. \quad (125)$$

They reduce to $\mathcal{R} = -4h^2 u^2 \rightarrow -\infty$ and $R_{ab}R^{ab} = 16h^4 u^4 \rightarrow +\infty$ after the cutoff. This proves invariant divergence, not only a divergent chart component. $\square$

11.3. Whole-box bounds and the unchanged global receipt

The four preparations remain the independent open boxes in (85), each with half-width 10^{-5} . Their final continuous-enclosure records satisfy every hypothesis of Theorem 24. The following simplified endpoint lower bounds and future upper bounds apply to every datum in the respective box. The quantities q_L and u_L are rounded downward; the time bounds are rounded upward.

Table 12. Homogeneous endpoint bounds for the complete preparation boxes.

Initial shear centre	Record end τ_e	$q_L \leq Q_e$	$u_L \leq u_e$	Upper $\tau_* - \tau_e$	Upper τ_*
1/20	320/256	2.31367	1.51797	.154205264922	1.404205264922
-1/20	320/256	2.01577	1.41771	.181368040286	1.431368040286
1/10	225/256	1.28989	.18183	.344459586631	1.223365836631
-3/20	242/256	3.48221	.93291	.098494489330	1.043806989330

Each endpoint enclosure also has $0 < d_e < .65$, $k_{\mathcal{D},e} > 0$, and $X_e > 0$. These are inherited whole-box enclosures, not sampled centre trajectories. Physical time is obtained by division by h .

Theorem 25 (global receipt through the positive-box singular approach). Keep the same explicit global completion and its initial cylinder half-length 4. For every datum in either positive-shear box, every regular point of the original central strip $|y| < .9$ is received in the global Cauchy development up to τ_* . Its spheres remain in the black-hole region. No enlargement of the cylinder is required.

Proof. The domain-of-dependence condition (97) is $|Y| + J(\tau) < 4$, where $J(\tau) = \int_0^\tau b(v)^{-1} dv$. At the positive-box record endpoints, the enclosures give respectively $Y_e < -1.45748$ and $Y_e < -.71786$. The equation $Y' = 3QY - \widehat{W}$, with $\widehat{W} \geq 0$, preserves strict negativity. Thus b increases after τ_e . For any inherited lower bound $b_L \leq b_e$, (119) gives

$$J_* \leq J_e + \frac{F(q_L)}{b_L}. \quad (126)$$

It suffices to use the weaker archived minimum of b over the entire recorded interval. Exact rational postprocessing gives the following bounds.

Table 13. Optical receipt for the entire original positive-box strip.

Initial shear centre	Archived b_L	Archived upper J_e	New upper J_*
1/20	.6602290193876296	1.6874434081403096	1.921006702684
1/10	.8046455490226936	1.0276567060996622	1.455745306166

Here the archived decimal inputs identify their stored binary64 numbers, interpreted as exact dyadic rationals in the postprocessing. The final column is rounded upward. Both bounds satisfy $J_* < 4$. Thus every regular point in the claimed strip remains in the initial cylinder's domain of dependence. Local uniqueness identifies the homogeneous solution there with the same global Einstein-scalar development for every $\tau < \tau_*$. The earlier causal construction, using the nonnegative-potential extension principle [P10] and trapped-region result [P9], places that approach in the black-hole region. No new censorship premise or regular endpoint is added. $\square$

For completeness, the arithmetic certificate for every time bound uses $z = 1/q_L$ and 64 terms of the positive series:

$$\frac{1}{3} \sum_{j=0}^{63} \frac{z^{2j+1}}{2j+1} \leq F(q_L) \leq \frac{1}{3} \sum_{j=0}^{63} \frac{z^{2j+1}}{2j+1} + \frac{z^{129}}{3 \cdot 129(1-z^2)}. \quad (127)$$

The remainder bound follows by replacing every subsequent denominator by 129 and summing the geometric series. All arithmetic in (126)–(127), including upward decimal rounding, is exact rational arithmetic. It inherits the enclosure solver's stated assumption that Python binary64 elementary operations and square root round to nearest and that the prescribed outward `nextafter` encloses each result. No new interval ODE solve across the cutoff is required: (120)–(123) provide that analytic bridge. The supplement preserves the complete archived endpoint intervals, exact fractions and independently reproduced certificate.

The negative-shear homogeneous solutions also satisfy Theorem 24, but the positive-box optical argument does not apply to them. Ordinary centre calculations give formal homogeneous optical reaches about 11.58847 and 54.33036, larger than the initial half-length 4. These calculations include the analytic tail remainder evaluated numerically; they are not whole-box certificates. The already received negative-box finite experiments remain valid. Their later homogeneous endpoints have not been shown to be endpoints of the actual globally embedded solution. Failure of this sufficient receipt does not prove nonsingularity of that global future.

The exact tail itself always has finite optical reach. Indeed $X_{,d} > 0$ gives

$$J_* - J_c = \frac{1}{u_c b_c} \int_0^\infty e^{2cD} G(D)^{-2} dD \leq \frac{1}{2b_c X_c}, \quad (128)$$

$$\tau_* - \tau_c \leq \frac{1}{3Q_c}.$$

To see the inequalities, use $(\log(bu))_{,d} = 2X \geq 2X_c/u_c$ and $(\log u)_{,d} = 2X + c \geq 3Q_c/u_c$. Finiteness alone is weaker than fitting inside a fixed cylinder; it does not repair the missing negative-box global receipt.

11.4. Incomplete geodesics and the precise extension classes

Theorem 26 (through-endpoint continuation obstruction). On either globally received positive-box history, the normal curves at fixed (y, θ, φ) , $|y| < .9$, are future-incomplete timelike geodesics of finite proper length. No isometric extension into a nondegenerate $\mathcal{C}^2$ Lorentzian metric can give this approached end a regular endpoint. Separately, no joint extension can give it an endpoint while retaining the original scalar as a finite real-valued continuous field, even if metric regularity is weakened to $\mathcal{C}^0$.

Proof. The synchronous metric has unit lapse, zero shift and $\Gamma_{tt}^i = 0$. The indicated curves are therefore unit timelike geodesics. Their remaining proper length is $(\tau_* - \tau)/h < \infty$. If any one extended as a geodesic in the original smooth development, continuity at its finite parameter endpoint would contradict (125). They are actual incomplete geodesics because Theorem 25 receives the entire approach, not merely a terminating coordinate patch.

For a nondegenerate $\mathcal{C}^2$ metric, the inverse metric and curvature tensor are continuous; their scalar contractions are finite and locally bounded at a regular point. Isometry would preserve the divergent values in (125) along a curve approaching such a point. This contradiction excludes every extension of the stated class, including nonsymmetric extensions and faithful smooth coordinate presentations. The same argument applies to any approach sequence in the received strip with $\tau \nearrow \tau_*$.

The separate source statement follows from $\phi = 2M d \rightarrow +\infty$. A finite real-valued continuous extension of that same scalar could not have this limit at a newly added point. This argument requires no curvature regularity of the proposed metric. A bounded field coordinate such as $\arctan \phi$ does not supply the original finite continuous scalar through its singular inverse. $\square$

The first conclusion is a through-this-end obstruction. It does not establish that every boundary component of the entire maximal development is inextendible. The curvature argument alone leaves metric-only $\mathcal{C}^0$ and $\mathcal{C}^1$ extendibility undecided; the next theorem settles the stated local metric-only question by a separate causal and intrinsic-distance argument. Nor is a weak or distributional successor supplied. Each weaker proposal must be tested against its own independently fixed original source and continuation duties. Every finite pre-endpoint segment constructed above remains regular.

11.5. A metric-only continuous-extension obstruction

The positive-box tails also obstruct a continuous nondegenerate Lorentzian endpoint, without requiring an extension of the scalar field. The additional argument uses longitudinal spatial distances and causal localization. Curvature divergence is used only to exclude passage through a smooth point of the original development; no curvature is assigned to a merely continuous proposed extension.

By Theorem 25, the actual smooth globally hyperbolic Einstein–scalar development contains

$$\mathcal{U} = (\tau_0, \tau_*) \times (-.9, .9) \times S^2, h^2 g = -d\tau^2 + b(\tau)^2 dy^2 + r(\tau)^2 d\Omega^2, r = hR. \quad (129)$$

Choose τ_0 on the already attained massless tail. The negative values of Y_e in the positive-box enclosures remain negative by (117), so $c = Y_c / u_c < 0$. Put $\beta = 2\alpha - c$, with α as in (122), and $\sigma = \tau_* - \tau$. Eliminating D from (123) gives

$$b \sim B \sigma^{p_1}, r \sim C \sigma^{p_2}, B, C > 0, \\ p_1 = \frac{c}{2\alpha - c} < 0, p_2 = \frac{\alpha - c}{2\alpha - c} \in (0, 2/3), p_1 + 2p_2 = 1. \quad (130)$$

Consequently both remaining causal-coordinate travel bounds vanish, while the longitudinal scale diverges:

$$b(\tau) \rightarrow \infty, L_Y(a) := \int_a^{\tau_*} \frac{d\tau}{b(\tau)} \rightarrow 0, L_\Omega(a) := \int_a^{\tau_*} \frac{d\tau}{r(\tau)} \rightarrow 0 (a \nearrow \tau_*). \quad (131)$$

The integrability follows from $p_1 < 1$ and $p_2 < 1$. Divergence of b alone would not prove continuous inextendibility: a shrinking causal tip need not have divergent spatial diameter. The proof must place a fixed-width spatial family inside a chart of the hypothetical extension.

Theorem 27 (local metric-only C^0 endpoint obstruction). For every datum in either globally received positive-shear box, let $(\mathcal{M}, g)$ be its original smooth globally hyperbolic development. There is no smooth isometric embedding of $\mathcal{M}$ into a same-dimensional manifold with a continuous, nondegenerate Lorentzian metric for which a sequence in (129), satisfying $\tau_n \rightarrow \tau_*$ and $y_n \rightarrow y_* \in (-.9, .9)$, has a limit point. No symmetry, preferred coordinates, field extension or field equation is required of the proposed extension. The conclusion concerns this received end, not every boundary component of the global spacetime.

Proof. We adapt the local continuous-extension method of Sbierski [P11] (Proposition 2.2 and proof of Theorem 1.1, pp.3-5, 8-16). The imported boundary-chart proposition applies to the actual globally hyperbolic development: a future boundary point reached by a piecewise smooth timelike curve admits a chart with metric uniformly close to Minkowski, original spacetime below an achronal Lipschitz boundary graph, and that curve as the vertical axis. The following argument supplies that accessibility from the sequence, confines the chart to the received scalar region, and proves the intrinsic-distance contradiction there. It does not apply a Schwarzschild or vacuum Kasner inextendibility theorem to a different source. The angular homotopy below is the local homogeneous-fibre construction in Proposition 4 of [P12] (pp.12-13), with its causal bounds supplied by (131).

Causal confinement and boundary accessibility. Every future causal curve in the received region can be parameterized by increasing τ , with

$$\left| \frac{dy}{d\tau} \right| \leq b^{-1}, \left| \frac{d\omega}{d\tau} \right|_{S^2} \leq r^{-1}. \quad (132)$$

Choose nested longitudinal intervals about y_* , with closures strictly inside $(-.9, .9)$. Choose τ_a late enough that $L_Y(\tau_a)$ is smaller than their separating buffer and $L_\Omega(\tau_a) < \pi/4$. A future curve beginning in the smaller interval after τ_a cannot first leave the received region through its longitudinal sides or past face. It cannot pass through τ_* at a smooth point of $\mathcal{M}$, because (125) diverges there. Its entire original-spacetime future therefore remains in the received region with a strict longitudinal buffer. This is a first-exit argument; the timelike-sided strip is not being declared globally hyperbolic.

Suppose that the sequence in the theorem has an extension limit point p . It cannot be an original interior point, where the original curvature invariant is continuous and finite. In a sufficiently small continuous-metric chart at p , pass to a subsequence whose original future orientation agrees with one chosen local cone orientation. Choose a fixed point p_+ strictly to the timelike future of p . For all sufficiently large indices, a straight timelike segment joins the sequence point to p_+ . Some such segment must leave the original image. If p_+ is

outside that image, this is immediate. If it is inside, a segment wholly in the original spacetime would, by confinement, force p_+ into the received region with $\tau(p_+) > \tau_n$, impossible for sufficiently large n .

Take the end of the connected initial parameter interval of that segment lying in the original image. This is a future boundary point, regardless of possible later re-entry. The time orientation cannot change on the connected initial segment. Its original part stays in the buffered region and has τ -limit τ^* . A smaller limit, together with (132), compactness of the sphere and the strict longitudinal buffer, would give a regular original limit, contradicting first exit. Thus a future-boundary curve is obtained without requiring the original normal tangent to extend. Arbitrary mixing of original coordinates in the extension is allowed.

The safe lower chart and spherical homotopy. Apply the boundary-chart proposition to this curve and the actual global development. Write the extension chart as $\tilde{\varphi}$, its vertical axis as $\tilde{\gamma}(v) = (v, 0)$, and its endpoint as zero. Its lower side is

$$\mathcal{B}_- = \{(v, z) : v < f(z)\} \subset \tilde{\varphi}[\iota(\mathcal{M}) \cap \tilde{U}], \quad \text{graph } f \subset \tilde{\varphi}[\partial^+ \iota(\mathcal{M}) \cap \tilde{U}]. \quad (133)$$

where $\tilde{U}$ is the chart domain and all displayed coordinate sets are restricted to its image. The function f is Lipschitz and its graph is achronal. Shorten the temporal chart along the axis until its bottom is a late physical point in the safe buffer. Then shrink its spatial width so that a compact bottom slab pulls back entirely into that buffer after τ_a . Inverse continuity on the compact regular axis segment and continuity of f justify this order of choices. Choose a height $v_b < 0$ in that slab and a closed spatial box strictly inside the chart, with $v_b < \min f$. Every vertical line above this bottom section and below f remains in the received region by confinement and tends to τ^* by the preceding compactness argument. Only the safe portion $v_b < v < f(z)$ is used below. All cone vertices, separators and boundary paths are chosen above v_b ; nothing is inferred about a lower, possibly unsafe part of the original chart.

The relevant timelike curves are homotopic with fixed endpoints through timelike curves in this same region. To prove this without flat angular interpolation, reverse the parameter for the homotopy construction and write each curve on a compact interval $[a, b]$ with common angular basepoint $\omega(a) = o$. Let

$\mathcal{L}(\xi) = \int_a^\xi |\omega'(v)| dv$. Its total length is below the sphere's injectivity radius by (131). For $a \leq \xi \leq \eta \leq b$, set

$$\omega_\eta(\xi) = \exp_o \left[\frac{\mathcal{L}(\xi)}{\mathcal{L}(\eta)} \exp_o^{-1} \omega(\eta) \right], \quad \omega_\eta(\xi) = \omega(\xi) (\xi \geq \eta). \quad (134)$$

If $\mathcal{L}(\eta) = 0$, the replaced part is the constant point o . Otherwise its angular speed is $|\omega'(\xi)| \text{dist}(o, \omega(\eta)) / \mathcal{L}(\eta) \leq |\omega'(\xi)|$. Leaving time and longitudinal coordinate unchanged therefore preserves timelikeness. At $\eta = b$, the two shortened paths lie on the same unique angular geodesic. Interpolate their true angular arclength coordinates and their longitudinal coordinates linearly at fixed τ . Convexity of the positive spatial quadratic form preserves the strict timelike inequality. If the angular endpoints coincide, both shortened paths are constant and only the longitudinal coordinate is interpolated. Compact piecewise- $\mathcal{C}^1$ junctions can be smoothed with fixed endpoints within the strict timelike margin. Confinement keeps all these curves inside the received strip. This proves the needed local one-connectedness, without asserting it for the whole inhomogeneous development.

Compact separation and chart trapping. Put $\gamma = \iota^{-1} \tilde{\gamma}$ on the axis below zero. For two late axis points $a_- \ll a_+$, define

$$\mathcal{K} = \left[\left(\bigcup_{v < 0} I^-(\gamma(v), \mathcal{M}) \right) \cap I^+(a_-, \mathcal{M}) \right] \setminus I^+(a_+, \mathcal{M}). \quad (135)$$

Its closure is compact in the original spacetime. Indeed, a compact regular timelike segment from a_+ to a later axis point has a strict timelike margin. Small smoothly introduced longitudinal translations and sphere rotations preserve that margin; continuing them as fixed local isometries produces a fixed coordinate-width tube about the subsequent axis lying in $I^+(a_+)$. By (132), every sufficiently late relevant point in the past of any later axis point lies in this tube. Hence $\mathcal{K}$ is bounded away from τ^* . Its regular past bound, longitudinal buffer and compact sphere then give compact closure. Small translations remain in the reserved buffer; no isometry of the entire global development is needed.

The set $\mathcal{K}$ timelike-separates the late axis from $I^-(a_-)$. A past timelike curve from the former to the latter leaves $I^+(a_+)$ before leaving $I^+(a_-)$. These exit parameters are distinct because global hyperbolicity gives causal simplicity and $\overline{I^+(a_+)} = J^+(a_+) \subset I^+(a_-)$. A point between the exit parameters lies in (135).

Here is the cone construction used to place a whole future in the chart. For a chart vector Z , let $C_a^\pm = \{ Z : \pm Z^0 / |Z|_E > a \}$, using the Euclidean norm. Choose the metric closeness small enough that the $C_{5/6}^\pm$ cones are timelike and all chart causal vectors lie in the wider $C_{5/8}^\pm$ cones. Select axis vertices x_- , x_+ and the late axis point a_- , all above v_b , such that

$$\overline{(x_+ + C_{5/6}^-) \cap (x_- + C_{5/6}^+)} \text{ is contained in the chart,} \quad (136)$$

$$C_{5/8}^- \cap (a_- + C_{5/8}^+) \subset \mathcal{D} := (x_+ + C_{6/7}^-) \cap (x_- + C_{6/7}^+).$$

Here points such as a_- denote their chart images in cone expressions; choose a_+ between a_- and zero. These choices fit inside an arbitrarily narrow safe box: take $x_\pm = (\pm a, 0)$ with $0 < a < -v_b$ and $a\sqrt{11}/5$ smaller than its spatial half-width, then choose a_- sufficiently near zero for the second strict inclusion. The chart chronological diamond between a_- and any later axis point is precompact. Every original timelike curve between those endpoints stays in the chart: the local fixed-endpoint homotopy joins it to the axis, and the set of homotopy parameters whose curves stay in the chart is nonempty, open and closed. Closedness follows from confinement in the compact chart diamond. Conversely, a chart past timelike curve starting below the achronal boundary graph remains below it and therefore in the original spacetime. The original and chart diamonds thus agree.

The diamond $\mathcal{D}$ can extend above the boundary graph. Use its original-spacetime preimage $\mathcal{W} = (\tilde{\varphi} \circ \iota)^{-1}[\mathcal{D} \cap \{v_b < v < f(z)\}]$. Diamond agreement and (136) imply $\overline{\mathcal{K}} \subset \mathcal{W}$. Compactness gives a positive original-coordinate distance from $\overline{\mathcal{K}}$ to the complement of $\mathcal{W}$. Choose a displacement δ whose neighborhood of $\mathcal{K}$ remains in $\mathcal{W}$, and so small that moving x_- by that displacement leaves it in $I^-(a_-)$. The small travel bounds allow a later axis point p such that every point of $I^+(p)$ differs by less than this joint displacement, in longitude and angle, from the equal- τ axis point.

Every past timelike curve from $I^+(p)$ to x_- must meet $\mathcal{W}$. Translate its longitudinal component and rotate its sphere component to place its starting point on that equal-time axis point. The moved endpoint remains in $I^-(a_-)$, so the moved curve crosses $\mathcal{K}$. The original curve crosses its δ -neighborhood, contained in $\mathcal{W}$.

If a future timelike curve from p first left $\mathcal{D}$, its first exit could be joined to x_- by a past timelike curve on the cone diamond's boundary avoiding $\mathcal{D}$. Its sides are strictly timelike by the choice $6/7 > 5/6$. Achronicity keeps the curve below f , and the vertices keep it above v_b . Its inverse would be an original curve avoiding $\mathcal{W}$, contrary to separation. An original curve cannot cross the boundary graph itself, and the compact cone diamond is met before a chart-side escape. We conclude

$$I^+(p, \mathcal{M}) \subset \mathcal{W}, (\tilde{\varphi} \circ \iota)[I^+(p, \mathcal{M})] \subset \mathcal{D} \cap \{v_b < v < f(z)\}. (137)$$

Physical slice graphs and the intrinsic contradiction. Choose $t_n \uparrow \tau_*$, later than the maximum physical τ on the compact bottom section. Every vertical line above that section crosses $\{\tau = t_n\}$ exactly once: its physical τ increases, starts below t_n , and tends to τ_* . Write the crossing as $v = h_n(z)$. Spacelikeness and the implicit function theorem give a smooth graph. Its tangent vectors obey

$$0 \leq \tilde{g}_{00}(\partial_i h_n)^2 + 2\tilde{g}_{0i}\partial_i h_n + \tilde{g}_{ii}. (138)$$

The chart has $\tilde{g}_{00} \leq -c_0 < 0$ and uniformly bounded coefficients. Thus $|\partial_i h_n|$ is bounded independently of n, z , as are the coefficients of the induced graph metric. Straight segments in the fixed spatial chart box lift to paths on the actual physical t_n -slice with one uniform finite length bound. This establishes the needed slice property directly; these local slices have not been called global Cauchy surfaces.

Openness of $I^+(p)$ gives a later point (t_1, y_1, ω_1) and $\ell > 0$ such that the interval $[y_1 - \ell, y_1 + \ell]$ at that time and angle is in $I^+(p)$. Normal timelike evolution keeps

$$P_n = (t_n, y_1 - \ell, \omega_1), Q_n = (t_n, y_1 + \ell, \omega_1) (139)$$

in $I^+(p)$ for every sufficiently large n . By (137) both are in the slice graph. Its bounded-length connecting path remains in the received physical slice. Every such path, however, satisfies

$$\text{Length}_g \geq h^{-1} b(t_n) \int |d y| \geq 2 h^{-1} \ell b(t_n) \rightarrow \infty. (140)$$

This contradicts the uniform chart bound and proves the theorem. The contradiction concerns intrinsic path lengths that the assumed continuous extension forces to be finite; it is not an inference from a divergent coordinate component. $\square$

The metric-only obstruction includes $\mathcal{C}^1$ and smoother nondegenerate Lorentzian isometric endpoints in this same local class. The separate scalar statement in Theorem 26 remains useful for proposals that retain a continuous original scalar but relax other metric duties. Neither result receives the negative-shear homogeneous tails into the fixed global completion or proves inextendibility at every other global boundary. A degenerate metric, conformal change, lower-dimensional quotient or normalized record is not an extension of the class just excluded; whether any such object meets a different original duty is a separate question. All original finite regular-side observations and the exact representation constructed below remain valid. Complete residual-support exhaustion still requires an independently supplied necessity and coverage result connecting every eligible realization of the fixed unresolved burden to an obstructed continuation duty.

11.6. Fixed-horizon limits and the ordering of exits

Corollary 28 (zero fixed-horizon limits after the braking exits). For the same explicit completion, fix its initial event-horizon cross-section and its already established radius and area, $R_H = 2 m_\infty / h$ and $A_H = 16 \pi m_\infty^2 / h^2 > 0$. For every datum in the two globally received positive boxes,

$$\lim_{\tau \uparrow \tau_*} \frac{R}{R_H} = 0, \lim_{\tau \uparrow \tau_*} \frac{A}{A_H} = 0, \frac{A}{A_H} = \frac{1}{4 m_\infty^2 k_\Omega(\tau)}. (141)$$

Both recorded braking exits occur before this approach, and neither mean nor areal braking re-enters later.

Proof. The fixed denominator is finite and positive for each completed dataset. Equation (123) gives $R \rightarrow 0$ and $k_\Omega \rightarrow \infty$, proving (141). The existing event enclosures give the strict order in Table 14. The record ends before cutoff arrival because its upper field bound is below .65; Theorem 24 gives $\tau_e < \tau_c < \tau_*$.

Table 14. Certified braking exits before the invariant end in the received boxes.

Initial shear centre	Areal braking exit	Mean braking exit	Record end τ_e	Later order
1/20	(230, 232)/256	(245, 247)/256	320/256	$\tau_e < \tau_c < \tau_*$
1/10	(159, 163)/256	(182, 184)/256	225/256	$\tau_e < \tau_c < \tau_*$

The complete recorded tubes already exclude additional crossings between exit and τ_e . After τ_e , both positive boxes have $Q > 32/25 > \sqrt{3/2}$, and (117) makes Q increase. With the completed potential,

$$\mathcal{A} = \widehat{W} - 2Q^2 - \frac{2k_\Omega}{3} \leq 3 - 2Q^2 < 0, \mathcal{B} = XY - 2u^2 < 0, (142)$$

where the second inequality uses $Y < 0 < X$. These signs cover both the transition region and the tail. On the tail, independently of the sign of Y , the constraint also gives $\mathcal{A} = -(4u^2 + 6s^2)/3 < 0$ and $\mathcal{B} = -(2u^2 + k_\Omega + X^2)/2 < 0$. $\square$

The numerical centre estimates $\tau_* \simeq 1.393883$ and 1.175552 illustrate the two positive-box histories; they are distinct from the outward-rounded whole-box bounds in Table 12. No proper time in seconds is selected without specifying h . The zero limits in (141) are limits of received metric-side answers relative to a fixed initial horizon. They assign no area or clock to an AMetric boundary. The stiff scalar stress grows without bound rather than annihilating, and no energy ledger is transported across the limiting end. What has been proved is failure of the stated classical endpoint-continuation classes, not exhaustion of every original residual use.

12. Exterior reconstruction and the original classical profile

The fixed completion makes it possible to ask a stronger question than whether an exterior metric survives: how much of the original interior does that complete exterior determine? The answer depends on the independently specified input. A continuous family within each received positive-shear box has identical entire future exterior fields but different original interior source answers. A different, disclosed preparation contract admits an exact exterior mass inverse. Both results concern the same completed scalar model.

Retain the action, scales h, M , potential $\widehat{W}$, marked initial normal congruence and negative constraint root. Use the two positive-shear boxes in (85), centred at

$$(d_i, Q_i, s_i, k_{\Omega,i}) = \left(\frac{3}{5}, \frac{63}{100}, s_c, \frac{1}{100} \right), (143)$$

$$s_c \in \left\{ \frac{1}{20}, \frac{1}{10} \right\},$$

with independent open half-widths 10^{-5} . Freeze every completion profile of Section 10, including the cylinder half-length 4, the two matter tapers and final vacuum slicing. These choices are inputs, not quantities adjusted separately after an exterior observation. The geometrical marking and signed rates remain those of (69)-(71). The singular approach proved in Section 11 applies to every member of these boxes; the reconstruction arguments below do not assume a regular endpoint.

12.1. What determines the exterior data

Write $r_g = h R$, $r_0 = k_{\Omega,i}^{-1/2}$, and retain $x = h \ell$, $\Pi = n \phi / (M h)$ from (89). Thus the x -derivatives below are spatial initial-data derivatives. Put

$$a_c = \frac{391}{400}, B_c = Q_i + a_c s_i, r_c = r_0 + 100, r_v = r_c + 2. \quad (144)$$

Here B_c is the constant scaled extrinsic-curvature value reached after the initial opening; it is not the time-dependent areal acceleration $\mathcal{B}$. The prescribed profile in Section 10 has $\psi(z) = \int_0^z \vartheta(v) dv$, with $\vartheta = 0$ for $z \geq .01$ and positive-side integral .0075. Thus (91) gives

$$\begin{aligned} \kappa_{\perp}(r_g) &= Q_i + s_i - 3 s_i \psi(\log(r_g/r_0)), \\ \kappa_{\parallel}(r_g) &= \kappa_{\perp}(r_g) - 3 s_i \vartheta(\log(r_g/r_0)), \\ \kappa_{\parallel} &= \kappa_{\perp} = Q_i + (1 - 3 \cdot .0075) s_i = B_c \text{ for } r_0 e^{.01} \leq r_g \leq r_c. \end{aligned} \quad (145)$$

The proper-radius profile depends on $k_{\Omega,i}$ and the fixed cap, cylinder and opening functions. On the remaining opening up to r_c , the scalar is still d_i , and the negative Hamiltonian root is

$$\Pi = -\sqrt{-\frac{4 r_{g,xx}}{r_g} + \frac{2(1 - r_{g,x}^2)}{r_g^2} + 6 B_c^2 - 2 W_0(d_i)}. \quad (146)$$

Consequently all outward initial metric, extrinsic-curvature, scalar and scalar-momentum data depend on the preparation only through $(d_i, k_{\Omega,i}, B_c)$. At the flat end of the opening, $\Pi_c^2 = 6 B_c^2 - 2 W_0(d_i)$. The fixed momentum taper, field taper, mass integral and vacuum slicing retain precisely that dependence. This identifies the complete outer Cauchy data, not merely one asymptotic coefficient.

12.2. Identical complete exteriors and different interior answers

Theorem 29 (exterior-only nonreconstruction). In each box (143), there is a continuous family with the same entire future exterior metric and scalar history, including the future horizon from the marked initial slice onward, but strictly different original initial kinetic readouts $U = u_i^2$. No single-valued reader of that complete exterior history recovers U , or the full marked classical history, on the box.

Proof. Hold $d_i = 3/5$, $k_{\Omega,i} = 1/100$, all scales and profiles fixed, and set

$$s_i = s_c + \epsilon, Q_i = \frac{63}{100} - a_c \epsilon, |\epsilon| < 10^{-5}. \quad (147)$$

Since $0 < a_c < 1$, every member is strictly inside its original box. Its original negative scalar-velocity root is

$$u_i = -\sqrt{\frac{3 Q_i^2 - 3 s_i^2 + k_{\Omega,i} - W_0(d_i)}{2}}. \quad (148)$$

The admitted full-box construction supplies its smooth constrained preparation. Equation (144) gives the constant $B_c = 63/100 + a_c s_c$. Equation (145), the fixed radius profile and (146) therefore make the initial data identical from $r_g = 10 e^{.01}$ outward. The full tapers, mass function, vacuum collar and final asymptotically flat slice coincide. Their outward proper-distance identification also coincides because the cap and opening depend on the fixed $k_{\Omega,i}$.

The exact-completion theorem of Section 10 gives $r_v = 112 < 2 m_\infty$. Thus this common initial vacuum region contains a neighbourhood on both sides of the Schwarzschild radius. Its positive extrinsic sign selects the future black-hole branch. The complete future exterior and its horizon belong to the domain of dependence of that initial vacuum region: a past causal curve from either cannot obtain an initial foot inside $r_g < 2 m_\infty$, since no future causal curve of this Schwarzschild branch exits that region into the exterior. Cauchy uniqueness therefore receives the same whole future exterior and horizon, with $d=.7$, for every member of (147). This uses the full causal receipt already proved in Section 10. Equality of mass alone would not establish it in a general scalar spacetime.

The original kinetic answer varies strictly:

$$U(\epsilon) = \frac{3 Q_i(\epsilon)^2 - 3 s_i(\epsilon)^2 + k_{\Omega,i} - W_0(d_i)}{2}, \quad (149)$$

$$\frac{dU}{d\epsilon} = -3(a_c Q_i + s_i) < 0.$$

The strict sign holds throughout both intervals in (147), since $a_c Q_i + s_i > a_c(.62999) + .04999 > 0$. Explicitly, $U(0) = 1711/5000$ for $s_c = 1/20$ and $U(0) = 6619/20000$ for $s_c = 1/10$, with

$$U(\epsilon) - U(0) = \begin{cases} -\frac{79899}{40000}\epsilon - \frac{21357}{320000}\epsilon^2, & s_c = \frac{1}{20}, \\ -\frac{85899}{40000}\epsilon - \frac{21357}{320000}\epsilon^2, & s_c = \frac{1}{10}. \end{cases}$$

For example, writing differences between $+\epsilon$ and $-\epsilon$, $\epsilon > 0$, gives

$$\Delta U = -6\left(a_c \frac{63}{100} + s_c\right)\epsilon,$$

$$\Delta(\mathcal{R}_i/h^2) = 24\left(a_c \frac{63}{100} + s_c\right)\epsilon, \quad (150)$$

$$\Delta(\sigma_i^2/h^2) = 12 s_c \epsilon.$$

Here $\mathcal{R}_i/h^2 = 4[W_0(d_i) - U]$ and $\sigma_i^2/h^2 = 3 s_i^2$. Thus a spacetime scalar invariant already separates the marked initial cylinders; the difference is not an arbitrary coordinate label.

Let E_{ext} be the complete future exterior metric/scalar history with that initial-slice identification and the fixed source conventions. Then (147)-(150) prove

$$E_{\text{ext}}(\epsilon_1) = E_{\text{ext}}(\epsilon_2), U(\epsilon_1) \neq U(\epsilon_2) (\epsilon_1 \neq \epsilon_2), \quad (151)$$

$$\ker E_{\text{ext}} \not\subseteq \ker U.$$

A reader of equal inputs cannot give unequal required outputs. Hence there is no r with $U = r \circ E_{\text{ext}}$ on the box. A reader recovering the marked history would recover its original U , so it is excluded too. A faithful encoding of E_{ext} preserves the same collision. $\square$

The conclusion covers the entire exterior-only observation algebra, including every defined functional of the complete exterior history, rather than a finite set of probes. For queries carrying a common independently supplied protocol, that protocol and its joint-domain conditions remain fixed on both sides. Added active fields, changed boundary experiments or separately retained preparation records are not inferred from the unperturbed exterior. The theorem asserts nothing about an unprovided pre-initial history. Its elementary equal-fibre proof is exactly the all-readers obstruction formalized by ATS and fixed-domain exhaustion [A14,

Theorem 3.5; A13, Theorem 3.3; A12, Theorem 7.2]. Those results do not require a finite audit of possible decoders.

12.3. The complete marked classical profile

There is a positive result on the full boxes, not only their one-dimensional subfamilies. Put

$$I = (d_i, Q_i, s_i, k_{\Omega, i}). \quad (152)$$

The fixed completion maps this tuple to a full Cauchy datum. Consider the full original classical-history task on that family, with the action, units, normal orientation, initial marking and completion fixed. Histories are identified by the admitted initial-surface-fixing isometry preserving the scalar and the supplied normal/spherical structures. Each original partial query retains its independently supplied context and protocol. The task is not reduced to a list of selected numerical observables.

Theorem 30 (exact classical profile). The attained coordinate I and the full original marked classical-history profile induce the same distinction relation on the two preparation boxes. On either independently fixed family (147), the attained U value also gives that complete profile.

Proof. The tuple I , negative root (148) and fixed construction determine all initial fields. The admitted Cauchy theorem then determines the marked regular development up to its original geometric identification. Equal I therefore gives equal original classical structures, and every originally specified query has the same output, definedness and joint conditions on them. For an additional supplied source protocol, its external input stays attached; this is not recovery of an absent experimental record from an unperturbed history.

Conversely, the original marked source and geometric readers recover

$$\begin{aligned} d_i &= \frac{\phi_i}{2M}, k_{\Omega, i} = (h R_i)^{-2}, Q_i = -\frac{H_i}{h}, \\ s_i &= \frac{H_{\parallel i} - H_{\perp i}}{3h}. \end{aligned} \quad (153)$$

The normal congruence, time orientation and sphere directions are original parts of the preparation, not newly selected observers. Equal complete profiles therefore imply equal I , proving both directions. This is minimality of the distinction profile: it does not assert a lower bound on the number of coordinates in an arbitrary encoding.

On (147), strict monotonicity in (149) gives a unique inverse from the attained U interval to ϵ . That determines Q_i, s_i ; the other preparation coordinates and all completion choices are fixed. Hence it recovers I , the entire marked regular history and every original operation with its original domain. The complete original profile contains U , so the converse follows as well. $\square$

Completeness here is proved by reconstructing the full structure before interpreting its questions. It is not inferred from successful finite measurements or from the word “complete.” It applies to the independently specified classical source family. It does not turn U into a complete coordinate of the four-dimensional box, identify this profile with all microscopic atlas roles, or certify the original BH residual task. In particular, preservation of the finite original U at its marked incidence is different from demanding finite ϕ at the later singular end.

12.4. A positive exterior inverse with disclosed preparation inputs

Proposition 31 (preparation-conditional mass reconstruction). Fix and disclose $h, M, d_i, k_{\Omega, i}, Q_i$ and the completion rule. Allow S_i to vary in an open interval within one positive box. Then the exterior mass reconstructs S_i , and thus the marked regular history, uniquely.

Proof. In the exact mass integral (108) of Section 10, collect the terms proportional to B_c^2 . With S the already fixed smooth step and $r_c = k_{\Omega, i}^{-1/2} + 100$, one obtains

$$m_\infty = K_m B_c^2 + C_m, \\ K_m = \frac{r_c^3}{2} + \frac{3}{2} \int_0^1 (r_c + z)^2 (1 - S(z))^2 dz > 0, \quad (154)$$

where

$$C_m = \frac{W_0(d_i)}{2} \int_0^1 (r_c + z)^2 \{1 - (1 - S(z))^2\} dz \\ + \int_0^1 (r_c + 1 + z)^2 \left\{ (.7 - d_i)^2 S'(z)^2 + \frac{1}{2} \widehat{W}(d_i + (.7 - d_i) S(z)) \right\} dz. \quad (155)$$

Both coefficients are fixed by the disclosed inputs. Since $B_c > 0$ on the admitted boxes, the unique inverse is

$$s_i = \frac{\sqrt{(m_\infty - C_m)/K_m} - Q_i}{a_c}. \quad (156)$$

The mass is an actual exterior geometric readout, not an inserted interior density. On an exterior symmetry sphere and its exterior neighbourhood,

$$m_\infty = \frac{hR}{2} (1 - g^{ab} \nabla_a R \nabla_b R). \quad (157)$$

All quantities in (157) are determined by the supplied exterior metric and public scale h . Formula (156), the negative root and the fixed construction recover I and the full Cauchy datum. The preceding theorem then reconstructs all originally defined regular-side classical questions. $\square$

This is an ideal exact inverse, not a finite-precision error estimate. The known Q_i is essential. It cannot be concealed as information obtained from mass: (147) keeps B_c , and hence the whole exterior, fixed while changing Q_i and S_i . Nor does regular-history reconstruction supply a finite endpoint value excluded in Section 11 or extend a local constitutive matching outside its stated window.

12.5. Witness contracts and the residual-duty question

The three reconstruction statements have different, explicit inputs.

Table 15. Reconstruction conclusions under three explicitly different witness contracts.

Original reconstruction input	Established result
The complete future exterior fields alone, on the original preparation boxes	The original U and full marked history do not factor through that record.

Original reconstruction input	Established result
Exterior fields plus disclosed fixed $d_i, k_{\Omega,i}, Q_i, h, M$ and the completion rule	The mass inverse (156) reconstructs the unknown-shear subfamily and its regular history.
Genuinely retained full original preparation data	The admitted Cauchy theorem reconstructs the regular history; actual retention is a separate premise.

A mathematical preparation specification is not automatically a physical memory outside the horizon. No stressed recorder is added to the closed scalar source by these proofs. Equally, if an original licensed certificate or prior record is genuinely retained, the horizon does not authorize deleting it from witness subtraction. The actual source/record incidence decides whether it is available. No particular detector, pre-initial history or formation from wholly untrapped data is imposed by this comparison.

The exterior gravitational field can warrant inference of its mass parameter under the admitted description. That is not a direct witness of interior material constitution. The positive inverse shows precisely when additional preparation knowledge makes the inferred parameter sufficient for a restricted interior problem.

The received future exterior also retains current sphere-area, curvature and proper-time comparisons along exterior timelike curves. They are actual continuing exterior applications, not merely entries in a historical description. Their survival does not establish that they perform a required duty of the unresolved interior target.

The exterior-only theorem therefore constructs a nonempty exterior-relative reconstruction ambiguity using an original source question. It does not by itself identify the BH corpus's complete witness-subtracted burden. The latter also fixes an old standing-positive target, all genuine exterior and prior witnesses, the standing-break incidence and the post-break continuation duties. Exact recovery of a regular history may remove a reconstruction burden where that recovery is an admitted witness; it is not automatically a surviving realization of a different unresolved duty. Section 13 gives the explicit regular-tail inverse, and Section 14 tests these stage and duty distinctions against the full original receiving contract. No exterior-only ambiguity, faithful encoding or source singularity is promoted here to complete residual-support exhaustion.

13. Exact reconstruction on the regular side

The singular scalar tail admits an exact compact presentation. Its normalized variables have a regular analytic limit, while explicit inverse maps recover the original source, metric, clocks and observations at every regular event. This supplies a constructive representation result for the same received history. It does not supply a physical endpoint or establish a successor for an independently fixed unresolved burden.

Retain the completed canonical source, normal observers, original occurrence labels, mathematical preparation specification and fixed global completion. The preceding endpoint construction receives the whole regular tail of both positive-shear boxes in the original central strip. Theorem 24 establishes the attained regular cutoff used below. In this section only that homogeneous tail is recoded. All original data outside it, including preparation and occurrence records, remain attached. Keeping those data in a mathematical source description does not establish that a physical exterior copy exists or is available as a witness. The local cutoff state alone is not asserted to reconstruct the entire inhomogeneous spacetime.

13.1. The compact coordinate and exact inverse

Use the original KS definitions (69)-(72): $\phi = 2 M d$, $\tau = h(t - t_i)$, $u = d'$, $X = Q + s = -R'/R$, $Y = Q - 2s = -b'/b$, and $k_\Omega = (hR)^{-2}$, with $h, M > 0$. The already attained regular cutoff is $d_c = .65$; a subscript c denotes its original data. On the subsequent tail the fixed completion (88) has $\widehat{W} = \widehat{g} = 0$. Set

$$\begin{aligned} c &= \frac{Y_c}{u_c}, \alpha = \sqrt{c^2 + 2}, \zeta_c = \frac{X_c}{u_c} + c, \lambda = \frac{\alpha - \zeta_c}{\alpha + \zeta_c}, \nu = \frac{c}{2\alpha}, \\ \frac{k_{\Omega,c}}{u_c^2} &= \alpha^2 - \zeta_c^2 = \frac{4\alpha^2\lambda}{(1+\lambda)^2} > 0, \lambda > 0, -1/2 < \nu < 1/2. \end{aligned} \quad (158)$$

Here c is a dimensionless ratio, not a speed of light. The strict inequality follows from the original positive-curvature scalar constraint. Define

$$\begin{aligned} q &= e^{-2\alpha(d-d_c)} \in (0, 1], L(q) = \frac{1+\lambda q}{1+\lambda}, \\ \zeta(q) &= \alpha \frac{1-\lambda q}{1+\lambda q}. \end{aligned} \quad (159)$$

The inverse source and geometric fields are

$$\begin{aligned} d(q) &= d_c - \frac{\log q}{2\alpha}, u(q) = u_c q^{\nu-1} L(q)^2, \\ b(q) &= b_c q^\nu, R(q) = R_c \frac{q^{1/2-\nu}}{L(q)}, \\ k_\Omega(q) &= k_{\Omega,c} q^{2\nu-1} L(q)^2, Q(q) = \frac{2\zeta(q) - c}{3} u(q), \\ s(q) &= \frac{\zeta(q) - 2c}{3} u(q), X(q) = (\zeta(q) - c) u(q), Y(q) = c u(q). \end{aligned} \quad (160)$$

They also recover the original scalar, stress, mean scale and contraction coordinate:

$$\begin{aligned} \phi(q) &= 2 M d_c - \frac{M}{\alpha} \log q, \dot{\phi}(q) = 2 M h u(q), \rho(q) = p(q) = 2 M^2 h^2 u(q)^2, H(q) = -h Q(q), \\ \frac{a(q)}{a(\tau_c)} &= q^{(1-\nu)/3} L(q)^{-2/3}, N(q) = N_c - \frac{1-\nu}{3} \log q + \frac{2}{3} \log L(q). \end{aligned} \quad (161)$$

The denominator $a(\tau_c)$ is the original mean scale at the cutoff. The cutoff calibration N_c , like the time origins, is retained from the original history. Put $J(\tau) = \int_0^\tau b(\sigma)^{-1} d\sigma$. The remaining proper-time parameter and longitudinal optical interval are

$$\begin{aligned} \tau_* - \tau(q) &= \frac{(1+\lambda)^2}{2\alpha u_c} \int_0^q \frac{x^{-\nu}}{(1+\lambda x)^2} dx, \\ J_* - J(q) &= \frac{(1+\lambda)^2}{2\alpha u_c b_c} \int_0^q \frac{x^{-2\nu}}{(1+\lambda x)^2} dx, t(q) = t_i + \tau(q)/h. \end{aligned} \quad (162)$$

The original τ_c, J_c fix τ_*, J_* by setting $q=1$; the representation introduces no new time or optical normalization. Both integrals converge at zero. With the original dimensional longitudinal coordinate denoted by $Z_\parallel$, so the dimensionless coordinate is $y = h Z_\parallel$, the original metric becomes

$$d s^2 = -\frac{q^{-2\nu}}{4 \alpha^2 h^2 u_c^2 L(q)^4} d q^2 + b_c^2 q^{2\nu} d z_{\parallel}^2 + \frac{R_c^2 q^{1-2\nu}}{L(q)^2} d \Omega^2. (163)$$

Proposition 32 (Exact source, metric and clock reconstruction). Equations (158)-(163), with the original source constants and marked coordinates retained, give a bijective change of presentation of the entire constrained outgoing tail on $0 < q \leq 1$. They recover the original metric and its derivatives, scalar and stress, proper time, optical time and mean contraction variable.

Proof. On the zero-potential tail, $u' = 3 Q u$ and $Y' = 3 Q Y$, so $Y/u = c$. Put $x = X/u$. The original constraint and the equation for X yield

$$(x+c)^2 = \alpha^2 - k_{\Omega}/u^2 < \alpha^2, x_{,d} = 2 - x^2 - 2 c x = \alpha^2 - (x+c)^2. (164)$$

Consequently $\zeta = x+c = \alpha \tanh(Z_c + \alpha(d-d_c))$, where $Z_c = \operatorname{atanh}(\zeta_c/\alpha)$. The definition of λ gives $e^{-2Z_c} = \lambda$. Factoring the exact hyperbolic functions therefore gives

$$\frac{\cosh(Z_c + \alpha(d-d_c))}{\cosh Z_c} = q^{-1/2} L(q). (165)$$

Equivalently, direct differentiation of (159)-(160) gives

$$\begin{aligned} q_{,d} &= -2 \alpha q, (\log u)_{,d} = 2 \zeta - c, (\log b)_{,d} = -c, \\ (\log R)_{,d} &= c - \zeta, (\log k_{\Omega})_{,d} = 2(\zeta - c), k_{\Omega}/u^2 = \alpha^2 - \zeta^2. \end{aligned} (166)$$

These are the original zero-potential scalar and directional Einstein equations with their constraint, under the displayed definitions of Q, s . At $q=1$ every field equals its original cutoff value. Since $d' = u > 0$, each finite $d \geq d_c$ labels exactly one regular event, and (159) is inverted uniquely by (160). Ordinary ODE uniqueness therefore identifies the resulting fields with the original tail at every finite d .

Integrating $d \tau = d d/u$ and $d J = d \tau/b$ gives (162); integrability uses $\nu < 1$ and $2\nu < 1$, both consequences of (158). The definition $a = (b R^2)^{1/3}$ gives (161), including $N' = Q$. Finally,

$$\frac{d}{d t} = -2 \alpha h q u(q) \frac{d}{d q} (167)$$

recovers every original normal-time derivative. Substituting $d t = d \tau/h$ and $d \tau/d q = -1/(2 \alpha q u)$ into the original KS line element gives (163). Its coordinate map is a diffeomorphism on $q > 0$; no metric or source coefficient has been altered. $\square$

13.2. An analytic normalized asymptotic record

The exact inverse exhibits finite normalized fields:

$$\begin{aligned} U(q) &= q^{1-\nu} u(q) = u_c L(q)^2, B(q) = q^{-\nu} b(q) = b_c, \\ R(q) &= q^{\nu-1/2} R(q) = R_c/L(q), K(q) = q^{1-2\nu} k_{\Omega}(q) = k_{\Omega,c} L(q)^2. \end{aligned} (168)$$

Their values at zero are respectively $u_c/(1+\lambda)^2$, b_c , $R_c(1+\lambda)$ and $k_{\Omega,c}/(1+\lambda)^2$, all positive; $\zeta(0) = \alpha$. Normalize the remaining intervals as

$$\begin{aligned}\tau(q) &= \frac{\tau_* - \tau(q)}{q^{1-\nu}} = \frac{(1+\lambda)^2}{2\alpha u_c} \int_0^1 \frac{r^{-\nu}}{(1+\lambda q r)^2} dr, \\ J(q) &= \frac{J_* - J(q)}{q^{1-2\nu}} = \frac{(1+\lambda)^2}{2\alpha u_c b_c} \int_0^1 \frac{r^{-2\nu}}{(1+\lambda q r)^2} dr.\end{aligned}\quad (169)$$

Proposition 33 (Analytic normalized limit). The functions U, B, R, K, ζ, T, J extend real-analytically to a neighborhood of $q=0$. Their cutoff parameters determine the full exact asymptotic germ, rather than a finite set of sampled asymptotic values.

Proof. The field functions are polynomial or rational in q , with nonzero denominators at zero. In (169), substitute $x = qr$ in (162). For either $\gamma = \nu$ or $\gamma = 2\nu$, $\gamma < 1$, and for $|\lambda q| < 1$,

$$\int_0^1 \frac{r^{-\gamma}}{(1+\lambda q r)^2} dr = \sum_{n=0}^{\infty} \frac{(-1)^n (n+1) (\lambda q)^n}{n+1-\gamma}. \quad (170)$$

On every smaller disk $|\lambda q| \leq r_0 < 1$, the absolute series is dominated by an integrable multiple of $r^{-\gamma}$, so termwise integration is justified. The convergent power series proves analyticity and gives

$$\begin{aligned}\tau(0) &= \frac{(1+\lambda)^2}{2\alpha u_c (1-\nu)}, \\ J(0) &= \frac{(1+\lambda)^2}{2\alpha u_c b_c (1-2\nu)}.\end{aligned}\quad (171)$$

Together with (168), this proves the claim. $\square$

No minimal-parameter claim is needed. The constrained cutoff data, original units and calibrations determine this homogeneous germ; the original global preparation and occurrence information remain attached for questions that require them. In particular ν is not erased: it distinguishes longitudinal contraction from expansion, fixes the areal exponent, and determines the relation between field time and proper time. Analytic normalization does not identify distinct source histories or merge their physical responses.

13.3. Every original regular-side operation retains its domain

The relevant conservation is preservation of the original work. Kernel necessity applies to the actual source incidence; it requires its original identity, qualification, joint conditions and historical attribution, independently of any observer [A11, Proposition 2.4, pp.14-17; Theorem 3.1, pp.24-26]. For a representation, the exactness criterion concerns every original observation and its definedness, rather than a selected qualification bit [A11, Theorem 2.8, pp.20-22; A15, Theorem 5.2, pp.15-16, and Lemma 8.4, p.27].

Let E_A denote the source encoding on each original sort A : use the explicit q -presentation for the tail, and retain the identity on attached external inputs or output sorts that are not recoded. Let $D_A = E_A^{-1}$ on its attained image. The encoding retains original occurrence labels and records; equality of displayed numbers does not replace their independent identity.

Proposition 34 (Complete regular-side operation receipt). Every independently specified original partial operation, including a joint constructor with its original relational domain, has an exact conjugate through these encodings. The conjugate preserves and reflects its full application domain, ordinary result values, original records, and all original observations. This holds jointly for the entire original operation and query family, without a finite-list or computability assumption. It asserts a semantic realization, not that every specified operation has been physically executed or algorithmically computed.

Proof. For an original operation $O: A_1 \times \cdots \times A_n \rightarrow B$, let $E_{\text{in}} = \prod_i E_{A_i}$ and $D_{\text{in}} = \prod_i D_{A_i}$. Define

$$\begin{aligned} \text{Dom}(O_{\text{enc}}) &= E_{\text{in}}[\text{Dom}(O)], O_{\text{enc}} = E_B \circ O \circ D_{\text{in}} \text{ on this domain,} \\ O_{\text{enc}} \circ E_{\text{in}} &= E_B \circ O, E_{\text{in}}^{-1}[\text{Dom}(O_{\text{enc}})] = \text{Dom}(O). \end{aligned} \quad (172)$$

Both identities follow by substitution of the actual inverses proved in Proposition 32, with identity on the attached data. The same product-inverse proof applies to any independently specified indexed family of original inputs. For a joint constructor, $\text{Dom}(O)$ is its actual relational applicability set; it is not replaced by the Cartesian product of individually successful input domains. Thus both admissibility and failure of a joint application are preserved.

For a numeric or other ordinary observed result use $E_B = \text{id}$. A defined zero or negative answer then remains that answer, and an unavailable application remains unavailable; undefinedness is not replaced by zero. Original record outputs retain their occurrence, source, and dependency coordinates under their specified record encoding. For any original relation P , put $P_{\text{enc}}(e) = P(D_{\text{in}} e)$. This directly preserves its original truth, including an independently required semantic query that is not generated by finite syntax. A multivalued original relation is preserved as the whole relation, not by selecting one output.

Finite composition follows inductively from (172); if any required intermediate application is undefined, the corresponding encoded application is undefined at the same stage. Ordinary observed Boolean tests select the same branch under a lazy conditional. The proof applies to every original operation and query at its original index, with every standing parameter retained. It therefore does not infer complete coverage from a shorter vocabulary or discard an original question after finding it inconvenient. $\square$

This is the same domain-sensitive conservation expressed by the Kernel's exact partial-algebra results [A11, Lemma B.2 and Theorems B.3-B.4, pp.61-63] and Structure's complete continuation and quotient results [A15, Theorems 5.8-5.9, pp.17-19, and Theorem 10.1, pp.35-36]. Their nontrivial physical input here is the explicit inverse, original metric and clocks in (160)-(167). Merely naming an encoding would not supply that input.

For example, the original insertions $O_d = d^2$, $O_u = u^2$, and the fixed mixed response formulas retain their exact values and derivative profiles on their original admitted events. Proper-time comparisons, null cones and finite transport retain their original metric. Curvature and stress are read from the same metric and source. Source-variation experiments, perturbations, and execution records not determined by this unperturbed tail keep their original external inputs; the compact formula does not compute a new perturbed history or manufacture a missing physical record. No microscopic role match is added by this representation.

13.4. The analytic record does not add an original endpoint

The normalized functions extend to $q=0$; their original readers do not thereby acquire a successful endpoint application. In fact,

$$\begin{aligned} \phi(q) &= 2M d_c - \frac{M}{\alpha} \log q \rightarrow +\infty, \mathcal{R}(q) = -4h^2 u(q)^2 \rightarrow -\infty, \\ R_{ab} R^{ab}(q) &= 16h^4 u(q)^4 \rightarrow +\infty. \end{aligned} \quad (173)$$

Here $u(q) \rightarrow \infty$ follows from $\nu < 1$ and $L(0) > 0$. A nondegenerate $\mathcal{C}^2$ isometric extension assigning a regular endpoint to this approach would have finite continuous curvature invariants there, contradicting (173). An endpoint carrying finite continuous original ϕ contradicts the first limit even if the metric regularity is weakened. The original scalar remains a massless scalar with divergent kinetic stress; normalization has not turned it into vacuum or removed its original source directions.

The exact inverse remains defined for every arbitrarily small positive q . It does not extend the original scalar reader at zero, and changing from proper time to d or q does not change the finite remaining proper time in (162). Nor does extending normalized functions amount to an isometric extension of (163). These representation statements by themselves do not decide metric-only C^0 or C^1 extendibility. For the two globally received positive-shear families, Theorem 27 separately excludes a continuous nondegenerate Lorentzian endpoint for the specified central-strip approach. The exact inverse on every $q > 0$ remains intact; it supplies no such endpoint.

The original graph condition is precise: giving an old undefined application a value changes that operation, whereas adding a genuinely new input point or question is not forbidden merely by graph preservation [A11, Theorem B.4, p.62; A15, Definition 4.6 and Theorem 4.7, p.13]. A new point must still meet every original compatibility duty claimed for it. If endpoint realization is an original mandatory use, Structure's readiness condition prevents a candidate from passing by omitting that operation [A15, Definition 6.1, pp.19-20]. The exact representation preserves the proved failure of the forbidden endpoint uses.

Finally, a residual-role successor is not defined simply as a post-singularity event. Its eligibility concerns the independently specified unresolved burden and original continuation duties. The construction here exactly answers a regular-side reconstruction or asymptotic-profile question when that is the original question. The same data might instead discharge that question during a valid witness subtraction. Neither possibility identifies a nonempty unresolved remainder or receives this representation as its physical successor. Complete source reconstruction is therefore a positive result with a precise domain; it is neither a counterexample to the endpoint obstruction nor a proof of complete residual-support exhaustion.

14. Original continuation duties and the scope of endpoint closure

The physical construction now supplies more than a finite interval of decreasing responses. For the two globally received positive-shear preparation boxes, the same completed scalar source reaches a finite-proper-time singular approach within the original cylinder's domain of dependence. The invariant divergence and the independent local continuous-metric obstruction give exact failures of specified continuation uses. The exterior reconstruction theorem supplies a second result: the entire future exterior can remain identical while an original interior source insertion differs. Neither conclusion, by itself, identifies the complete witness-subtracted burden or residual successor family of the BH endpoint theorem. The passage from these physical results to that theorem must preserve its original target, all genuine witnesses and the stage at which a continuation is required.

14.1. Physical obstructions to specified original uses

Fix a marked comoving timelike curve γ in the received strip of one of the positive-shear histories H . Along its massless tail, at finite τ_* ,

$$\begin{aligned}\mathcal{R}(\gamma(\tau)) &= -4h^2u^2 \rightarrow -\infty, R_{ab}R^{ab}(\gamma(\tau)) = 16h^4u^4 \rightarrow +\infty, \\ \phi(\gamma(\tau)) &= 2Md \rightarrow +\infty.\end{aligned}\quad (174)$$

These are invariants and the original scalar of the completed Einstein–scalar solution. They are not Schwarzschild replacements for the interior. The global statement uses the proved optical bounds on the positive-shear boxes; the corresponding negative-shear endpoint analysis remains a homogeneous result without the same whole-tail global receipt.

Let $\mathcal{E}_0(H, \gamma)$ be the class of extensions of the received smooth development by a smooth isometric embedding into a manifold of the same dimension with a continuous, nondegenerate Lorentzian metric, in which the approached end of γ acquires a point. No symmetry, extension field equation, or extension of the

scalar is required. Theorem 27 excludes the stronger local possibility that any sequence with $\tau_n \rightarrow \tau_*$ and $\gamma_n \rightarrow \gamma_* \in (-0.9, 0.9)$ converges to an extension point. Timelike accessibility of such a point is derived in that theorem, rather than imposed on the proposed extension.

Let $\mathcal{E}_2(H, \gamma)$ be the class of same-dimensional nondegenerate $\mathcal{C}^2$ Lorentzian extensions of the same received smooth development by a smooth isometric embedding, giving γ a regular endpoint. This retains the original metric on its spacetime domain, not merely on the one-dimensional curve. Let $\mathcal{E}_\phi(H, \gamma)$ be the class of joint extensions that give the same approach an endpoint at which the original ϕ is a finite real-valued continuous field. The latter question does not impose $\mathcal{C}^2$ metric regularity. These definitions fix substantive continuation questions at the same approached end; the earlier finite-interval source theorem did not promise positive answers. The continuous-metric class requires less geometric regularity than the $\mathcal{C}^2$ class. The finite-continuous-scalar question is distinct because it retains an original field reader even when the proposed metric is weaker.

Corollary 35 (original endpoint-use obstruction). For the histories and curves received by Theorem 26 and the local geometric obstruction Theorem 27,

$$\mathcal{E}_0(H, \gamma) = \emptyset, \mathcal{E}_2(H, \gamma) = \emptyset, \mathcal{E}_\phi(H, \gamma) = \emptyset. (175)$$

Every faithful realization of an original use requiring a member of any displayed class therefore fails that use.

Proof. Theorem 27 makes $\mathcal{E}_0$ empty, including nonsymmetric proposals with no extension Einstein equation or scalar. Every member of $\mathcal{E}_2$ would in particular supply such a continuous Lorentzian endpoint.

Independently, the curvature and original-scalar arguments of Theorem 26 exclude $\mathcal{E}_2$ and $\mathcal{E}_\phi$, respectively; the latter applies even when the metric is weakened. A faithful positive realization of one of these original questions would construct an element of the corresponding empty class, a contradiction. $\square$

The local geometric result rules out continuous nondegenerate Lorentzian endpoint realizations, hence also $\mathcal{C}^1$ and $\mathcal{C}^2$ realizations, for the stated positive-box approach. Its proof uses the expanding longitudinal distance and the finite transverse and longitudinal causal travel, not curvature blow-up as a substitute for a $\mathcal{C}^0$ theorem. It does not exclude every extension through another boundary component, receive the negative-shear homogeneous end globally, or cover a degenerate or non-Lorentzian replacement. Each other proposed use remains subject to its complete original task. A weaker label alone neither supplies the original classical curvature reader nor establishes eligibility for a different residual role.

A compact scalar coordinate illustrates the original-question requirement. A finite limit of $\arctan \phi$ does not supply a finite continuous value of the original ϕ : the original inverse fails at that limiting argument. An extended-real asymptotic answer can correctly describe the approach, but has a different answer type from a finite classical field at a regular endpoint. The partial-domain and joint-use distinctions are required by ATS's original-question, exact-interface and operator results [A14, Definition 2.1, Theorems 3.5 and 6.3, §§7–10], and by the complete-domain results in FDE and AMetric [A13, §§2–5 and Appendices B–C; A12, Theorems 7.2 and 9.2].

The same divergence also gives a representation-independent test for a narrower original reader. If a proposed endpoint is approached by the represented history and its original curvature answer is required to extend as a finite continuous real-valued function, its pullback must tend to that finite value. Equation (174) contradicts this requirement. This conditional reader obstruction needs neither a fundamental classical metric in the code nor a $\mathcal{C}^2$ extension. It does require the original endpoint domain, answer type and continuity duty; regular-side evaluation alone does not create them.

14.2. Necessary governance and faithful preservation

Kernel necessity applies to the actual physical incidences, including unobserved ones. A construction incurs its original bearer and relation identities, constitutive conditions, occurrence and retained dependencies; a later answer claiming that incidence is accountable to them [A11, Proposition 2.4, Theorem 3.1 and Appendix B]. No external observer's approval is needed. These requirements are preserved here at full strength.

The necessity theorem does not assert that every original record remains accessible or every proposed continuation succeeds. Kernel Definition 2.2 permits loss of effective record access. Its complete-profile and partial-algebra results retain the original questions and definedness, while total evaluation is on licensed applications rather than on every raw input [A11, Theorem 2.8, Theorems B.3–B.5]. Structure independently retains required readiness, complete result relations and the actual continuation graph [A15, Definitions 4.6 and 6.1, Theorems 5.2, 6.3 and 10.1]. A new endpoint may be proposed and independently warranted; it is not produced merely by preserving the original history.

The minimality results make the positive preservation requirement precise. ATS identifies the unique minimal complete original-answer interface, up to its commuting equivalence; every adequate description must decode that interface [A14, Theorem 3.3 and Theorem 5.1]. In the present family, Theorem 30 supplies the physical instance: the marked initial tuple, the fixed construction and independently supplied protocol/context inputs recover the complete original regular history. An alternative must therefore recover its original metric comparisons, source, calibrated clocks, transport and joint application domains. It may encode these as derived structure rather than making its code space a manifold. This is not a least differentiability theorem or a new endpoint promise. Current L-metric conservatively receives already proved original uses without adding a physical continuation requirement [A10, Definition 2.13 and section 6, especially p.22].

The original Einstein projection fixes comparison, transport, curvature and closed-source response duties, rather than merely an equation name [A17, Definitions 13.2–14.2 and 20.2–20.3, Theorem 30.1]. Enrichment must preserve those duties wherever it claims the same original task. The native quantum/source interface likewise retains whole original operators, their domains, common inputs and continuation/failure questions [A9, equation 91 and sections 8–11]. A quantum-enriched continuation of this joint task cannot delete an originally required geometric application. Neither statement identifies every abstract quantum description with this task or supplies an occupied quantum continuation of the present singular history.

The exact tail representation constructed above makes this distinction concrete. Its inverse recovers the metric, scalar, source, clock and original readouts at every regular event. Complete preparation and occurrence data remain mathematical inputs to that representation; they are not an asserted physical memory outside the horizon. Independently supplied experiment inputs and records must remain supplied. At the singular limit the inverse still reports the original divergences, and it does not add a successful endpoint operation.

Proposition 36 (transport of required readiness and the greatest safe region). Let T be the actual bijective presentation change of Proposition 34, which already preserves the complete original partial operations, joint domains and supplied history/context data. Suppose an independently fixed safe-use contract has effective local guard A , full promised successor relation R , edge warrants W , and mandatory-operation set $M(x)$. Require the represented contract to preserve all these data in both directions, with the original operation labels identified:

$$\begin{aligned} A_T(Tx) &\Leftrightarrow A(x), \\ R_T(Tx, a, Ty) &\Leftrightarrow R(x, a, y), \\ W_T(Tx, a, Ty) &\Leftrightarrow W(x, a, y), M_T(Tx) = M(x). \end{aligned} \tag{176}$$

Assume complete represented-state coverage by T . Then the greatest safe regions satisfy $\mathcal{S}_T = T[\mathcal{S}]$. In particular, the represented contract cannot evade an original mandatory operation by deleting its failed transition.

Proof. The complete operation and joint-domain preservation is Proposition 34; the additional issue is the original promise. Its effective guard includes the independently required readiness of every mandatory label,

$$\forall a \in M(x) \exists y R(x, a, y). \quad (177)$$

Let $\mathcal{S}$ be the greatest region satisfying the guard and warrant conditions and closed under all promised successors. Equations (176) send every sound original region to a sound represented region: the guard and warrants remain true, and every represented successor is the image of an original successor in that region. Complete state coverage and the inverse give the converse. Maximality on both sides yields $\mathcal{S}_T = T[\mathcal{S}]$. An absent mandatory realization fails (177); hiding a promised failing successor cannot preserve (176). This is the same-task transport criterion of Structure and current L-metric [A15, §§6 and 10; A10, §6]. $\square$

This elementary preservation result does not create an original promised repertoire or an unprovided physical record. It transports the actual one. It preserves a correct negative endpoint answer just as it preserves a positive finite-use answer. Where an original promise requires the impossible endpoint use, that promise fails; a description faithfully reporting the failure is not a successful implementation of the promise.

The complete closures retain three further distinctions. Universal safety of all originally promised finite paths is stronger than existence of one usable branch. Safety along finite prefixes does not by itself establish a limit point, termination or a positive eventual result; an original limit requirement remains an explicit coordinate [A13, §4.6; A15, Theorem 6.6 and Lemma 8.4]. Finally, retained-history non-repair concerns required dependencies and the original occurrences, not every later physical state predicate [A11, Theorems B.11–B.14; A15, Theorem 7.2]. A later obstruction cannot retroactively invalidate an earlier use with a shorter valid promise. A stronger promise already requiring the failed continuation could never have been declared safe.

14.3. The original burden and genuine witness subtraction

The BH contract fixes an exterior reference-standing problem, not a collection of primitive information tokens. In the selected Endpoint formulation its data include

$$\mathcal{S}_{\text{BH}} = (E_{\text{ext}}, H_{\text{BH}}, \mathcal{R}_{\text{ext}}, W_{\text{ext}}, \text{Cont}, \text{Env}, \text{Param}, \text{Therm}), \quad (178)$$

$$\mathcal{B}_H \subseteq \mathcal{R}_{\text{ext}}.$$

The family $\mathcal{R}_{\text{ext}}$ is standing-bearing in the original exterior problem. The witness rule resolves exactly the burdens re-fixed by a lawful exterior carrier, record family or witness algebra, with the original quotient, success/failure and continuation conditions and without hidden interior storage [A16, §15, Definition 25.2 and Theorems 28.1–28.2]. It need not be a human observation, a particular detector or a newly installed memory system.

The exact full-exterior collision proved above establishes a nonempty reconstruction obstruction for the original fine insertion $U = u_i^2$. Every exterior-only reader receives the same input on the displayed preparation fibre but would have to return different original outputs. Because that fibre lies inside each original positive box, a uniform exact reader on the whole box is also impossible. This is an all-readers result for the declared exterior carrier, not an audit of a finite set of detectors.

That reconstruction burden $\mathcal{B}_{\text{rec}}$ is not automatically the BH standing burden $\mathcal{B}_H$. Its receipt as such requires the old standing-positive exterior class, original occurrence and continuation duty, and the complete permitted witness data. An actually retained prior certificate or preparation record cannot be discarded after a horizon is present. Conversely, the analyst's complete initial packet is not automatically an available exterior carrier. The

explicit mass inverse under disclosed fixed preparation data and the conditional Cauchy-data reconstruction show that the witness contract has substantive mathematical consequences. They neither invent a physical recorder nor settle which contract supplies the old BH problem.

After all genuine witnesses are accounted for, set

$$\begin{aligned}\mathcal{B}_H^{\text{res}} &= \mathcal{B}_H \setminus W_{\text{ext}}(\mathcal{B}_H), \\ \mathcal{G}(H) &= \text{Succ}_{D_0}^{\text{res}}(\mathcal{B}_H^{\text{res}}, \text{metric}).\end{aligned}\quad (179)$$

The source denotes this role-ledger residue by R_H ; here $\mathcal{B}_H^{\text{res}}$ avoids confusion with the geometric horizon radius. If every declared burden is re-fixed, $\mathcal{B}_H^{\text{res}} = \emptyset$ and this unresolved-residue endpoint target is absent; successful reconstruction is not then a survivor of a nonempty remainder [A16, §30].

14.4. Standing break and the singular limit are different stages

BHII's residual coherence requires pre-break role ancestry, loss of old standing, absence of prior re-governance or a restoring certificate, a non-null post-break representation, retained ancestral organization and failure of full standing-bearing licensing [A5, pp.25–28]. Its successor relation is post-standing-break role continuation and remains carrier-specific [A5, pp.28,35; A6, §5.3; A16, Definition 16.1]. “Post-standing-break” does not by definition mean “beyond the eventual curvature singularity.” Neither the generic relation nor its complete closure proof uniformly imposes a through-endpoint continuous nondegenerate Lorentzian metric, $\mathcal{C}^2$ endpoint regularity, or finite continuous canonical ϕ on every eligible residual use. The weaker geometric endpoint class now excluded by Theorem 27 enlarges the proved obstruction; it does not make this missing original-duty comparison automatic.

The completed scalar initial data are already trapped. They identify a classical source history and its exterior, but do not supply a selected horizon-dependent old-standing lineage and its standing-transport failure. This is an unreceived application of the BH contract, not an assertion that such lineage is impossible. It requires neither formation from wholly untrapped data nor an extra detector. Old standing is structural; failure of its transport is non-membership in the original operation domain, not merely an observer's ignorance [A5, pp.16–17,24–25].

The exact regular-tail inverse consequently has a limited but positive role. It shows that preserving all originally defined regular-side source questions does not itself demand a new finite endpoint value. It does not establish an element of $\mathcal{G}(H)$. Likewise, a regular use at each $\tau < \tau_*$ does not construct a successor at τ_* , and a complete one-sided germ does not automatically perform a through-endpoint operation. No actual same-burden residual survivor has been received here.

14.5. The conditional full-family lift

Lemma 37 — elementary common-duty lift. Fix the original nonempty witness-subtracted burden, its complete residual metric successor family $\mathcal{G}(H)$, and its original source, occurrence, quotient, joint domains and continuation duties. Suppose an independently established interpretation proves

$$\forall s \in \mathcal{G}(H) \exists e \in \mathcal{E}_0(H, \gamma) \cup \mathcal{E}_2(H, \gamma) \cup \mathcal{E}_\phi(H, \gamma) \quad (180)$$

such that e realizes an original duty of s .

Then $\mathcal{G}(H) = \emptyset$. The same conclusion holds for any independently specified subfamily on which (180) is proved. Any original positive application that necessarily supplies such a successor has no qualifying realization. Faithful same-task recoding cannot restore it.

Proof. If S belonged to the specified family, (180) would supply a member of one of the empty classes in (175), a contradiction. This argument requires neither a finite enumeration nor a choice of a single uniform regularity grade: the union is empty. Restricting the quantifier proves the subfamily statement. The application consequence follows from its independently established support requirement. Exact original-domain and result preservation in Proposition 36, together with the complete faithful-profile criteria [A11, Theorem 2.8; A13, Theorems 3.2–3.3 and 5.9–5.10], prevents a recoding from supplying a false positive realization. $\square$

The $\mathcal{C}^2$ geometric branch is already contained in the continuous-metric branch, but is displayed to retain its independent curvature proof and original duty. The scalar branch is independently meaningful. The logical lift is elementary. Its substantive, unproved premise is (180) for the complete independently fixed BH family. That family cannot be defined as “successors having one of the prohibited endpoints,” nor reduced after examining the obstruction. A positive safe application and an eligible BH successor must be compared at the same burden, occurrence and use; neither direction of that comparison follows just from a shared metric label. Full source, joint-domain, witness, historical and prior-governance conditions remain in force.

Kernel necessity strengthens this test: a supposed weaker continuation cannot erase an original mandatory endpoint duty. It does not manufacture that duty from the bare fact that the continuation is a determinate construction. If a genuinely original residual role entails the prohibited endpoint, Lemma 37 excludes all its faithful realizations. If a different eligible role is actually constructed, its entire original contract must be received. The current regular-tail representation is not such a received counterexample, and a hypothetical weak extension is not one either.

14.6. The complete endpoint target and proof direction

The certified BHIII target has eleven coordinates [A6, Definition 5.6 and Theorems 5.7–5.9, pp.26–29; Theorem 11.1 and Corollary 11.2, pp.43–44]. Keeping them explicit distinguishes results already supplied by the physical model from the original-target premises still needed for its endpoint classification.

Table 16. The complete BH endpoint profile and its present receiving status.

Endpoint coordinate	Status for the present scalar histories
1. Parent domain with a licensed metric/source projection	The completed scalar and positive-box global receipt supply the stated classical parent use.
2. A declared horizon-generated standing burden $\mathcal{B}_H$	The original fine insertion supplies $\mathcal{B}_{\text{rec}}$; its receipt as the old exterior standing class remains to be established.
3. The certified exterior-witness rule	The full future exterior algebra and its nonfactorization are constructed. The complete original prior/exterior witness contract for the BH burden remains to be identified.
4. A nonempty post-subtraction $\mathcal{B}_H^{\text{res}}$	The exterior-only reconstruction obstruction is proved. Nonempty BH residue remains conditional on coordinates 2–3.
5. Failure of old standing transport for every retained burden	The fibre contradiction can exclude an old image once that class and its complete original carrier are received; actual old standing transport is not yet instantiated.
6. Residual coherence distinct from standing and hidden carrier	Regular source organization persists. The residual classification still requires its old role ancestry and standing break.

Endpoint coordinate	Status for the present scalar histories
7. Empty residual metric successor support	The continuous nondegenerate Lorentzian endpoint class, and the independently obstructed $\mathcal{C}^2$ and finite-continuous-scalar classes in (175), are empty at the received approach. Complete family coverage (180) is not established.
8. No prior independently established governance of the same residue	No new field or late repair is inserted in the fixed source; the complete same-burden prior-governance condition remains to be received.
9. Closure of metric, temporal, causal, operator, carrier, selector, repair/storage and inherited-law routes	The specified original endpoint uses fail. Their failure does not by itself establish the full route profile.
10. No admitted total vacuity; mute nullity has no positive force	The imported structural result is binding and requires no additional physical experiment. It does not populate the other target coordinates.
11. Independent target gating and no crossing	The imported discipline and boundary conclusion remain binding when the full profile applies; they cannot be used backward to prove that this physical limit has the profile.

The associated AMetric role table separately retains temporal/process, causal, operator, selector, standing-transport, valuation, repair, target-gating and self-authorization failures [A6, Lemma 5.1 and Table 4, pp.23–25]. The complete adversarial closure admits genuine witness resolution, failed specialization, prior independent governance and surviving support as matters that must be decided by their original conditions [A6, §§11–12; A16, Theorems 48.1–50.1 and the subsequent objection closures]. Those clauses do not make the endpoint theorem optional after its certified target is established. The geometric strengthening is already proved for the specified local end; the remaining question is not whether to repeat the old curvature test at lower regularity, but whether every eligible original successor must supply a realization that the stronger theorem excludes.

The proof direction is important. The Endpoint singularity-shadow results classify what remains under the maximum-governance-depth antecedent; they do not derive that antecedent from any invariant divergence [A16, Theorems 39.2–39.5, PDF pp.53–55]. Current Metricity likewise distinguishes a physical singularity, failure of a particular original use and full boundary status [A10, Corollary 15.2 and §§16–17]. The one-sided limits of areal radius and horizon-normalized area are still metric quantities on the received approach. They supply no radius, area or clock to an AMetric boundary.

The remaining target package is therefore concrete: an old standing-positive exterior bearer/quotient and occurrence; its complete actual witness algebra and continuation conditions; the standing-break incidence and retained role ancestry; and the original successor duties, including any mandatory endpoint use and prior governance. The physical obstruction, exterior fibre theorem and exact inverse constructed here can test that package. If the same-burden common-duty premise and the remaining endpoint coordinates are proved, the imported AMetric closure applies. Without that receipt, full closure is neither proved nor disproved by these constructions.

14.7. Optional transverse interface matching

The singular-tail obstruction and a finite interface boundary are different possible failures of original use. For the exact inward reference history, which formally approaches its de Sitter limit, one can instead test an independently matched finite boundary while all source quantities remain bounded. The separately prepared

scalar loop requires its own encounter analysis. Neither history determines this support criterion from its field equations alone [A10, §8 and Appendix C].

Let c denote a source configuration with its original comparison and transport data. Suppose a particular original metric/clock interface has a support margin $m_{\text{lic}}(c)$, defined independently of the desired endpoint, with positive margin on its valid side. A margin could encode a warranted comparison or transport condition, but its definition must come from the receiving source rather than from assigning zero to an inconvenient equation. A genuine validity boundary must distinguish the failure of this interface from the failure of merely one sufficient route to it.

Postulate S — optional transverse interface matching. Assume the effective family receives an actual source curve $c(d)$ and that the selected interface's applicability ends at $d = d_L > 0$, where its original criterion has a simple zero:

$$m_{\text{lic}}(c(d)) = m_1(d - d_L) + O((d - d_L)^2), m_1 > 0. \quad (181)$$

Here the parameterized source curve is the exact monotone reference family used in the conditional timing calculation. A turning history requires its full phase-space and occurrence data rather than a single-valued identification by field amplitude alone. Any faithful alternative supporting that same claimed interface must be addressed by this matching. If m_{lic} is only one sufficient certificate, its vanishing licenses only the narrower conclusion that this certificate no longer supports the calculation. Equation (181) is the local normal form of the stated transverse crossing, not a new universal equation for governance depth. The support margin is not an extra stress component or a hidden sink.

Proposition 38 — a finite support-interface endpoint with finite source limits. Under Postulate S and the exact inward solution with $d_i > d_L > 0$, the metric segment approaches its matched boundary after

$$\Delta t_L = \frac{1}{\gamma h} \log \frac{d_i}{d_L}, N_L = \frac{1}{\gamma} \left[\log \frac{d_i}{d_L} - \frac{d_i^2 - d_L^2}{2} \right]. \quad (182)$$

The one-sided limits of a, H, ρ, p are finite there. No singular stress or discontinuous energy deletion is required to terminate the application of this metric interface.

Proof. Solve (7) for the first approach to d_L and substitute into (8) and (9). Every expression is smooth for $0 < d_L < d_i < 1$. The boundary status follows from the separately assumed original interface criterion; it is not inferred from the smooth formulas. $\square$

For example, choosing $d_L = 1/4$ gives the duration of the worked segment if the receiving source independently has that boundary. This example does not derive the location of a metric boundary from the scalar field. Nor does finite d_L establish complete coalescence: the designated contrasts in (1) remain nonzero there. It instead permits a physically responsive approach whose current metric description ends before its ideal order limit is reached.

The result concerns the selected interface. Lifting it to full metric exhaustion and AMetric closure would still require the complete original-family and endpoint conditions of §§14.3–14.6 [A5–A7, A10–A14]. In particular, a still-supported bearer, one failed interface and full boundary status remain distinct. The effective source postulates introduce new dynamics openly; they do not extract changed original answers from unchanged complete content.

The conclusion is conditional: a smooth, conserved physical source can approach a finite end of its licensed metric application without first producing infinite density or an energy sink. After that application ends, (4) is not a physical clock, pressure law or cross-boundary ledger. The result supplies no dynamics on the other side and no claim that a populated universe follows.

Proposition 38 does not prove Postulate S. A substantive application needs the original criterion and coverage of all faithful alternatives for the claimed interface; defining the boundary as “where the description stops” supplies neither. The global scalar construction and its newly proved singular endpoint obstructions do not identify this separate finite-interface criterion.

14.8. Support encounters versus physical events

The optional support premise S must be independently received on the original metric-use domain and cover the promised repertoire and all applicable faithful alternatives. On the actual scalar loop of Theorem 9, write d_T for the field minimum and d_E for the incoming acceleration-entry field. If the support premise says that the relevant authority terminates on the incoming crossing of $d = d_L$, then:

- $d_T < d_L < d_E$ gives an attained support encounter during scalar decreasing-response braking, because the field is strictly inward there.
- $d_L > d_E$, if encountered after preparation, cuts off application before the proved acceleration entry. Equality $d_L = d_E$ gives a tie and no positive-length admitted interval after that entry.
- $d_L < d_T$ is not reached on the complete scalar loop. Equality $d_L = d_T$ is a temporal touch, not a transverse crossing, because the field has a quadratic minimum.

On a compact monotone incoming segment, $0 < v_{min} \leq v \leq v_{max}$ yields $(d_a - d_L)/v_{max} \leq \tau_L - \tau_a \leq (d_a - d_L)/v_{min}$. These timing inequalities have content only once the independently specified original-source criterion and physical match are supplied. Defining support loss as “where this description stops” would make its support conclusion tautological.

With components or another fixed response family, order the actual support encounter, first response-rate/application failure, and first acceleration exit on that same solution. Equalities are ties. The first applicable event ends the claimed joint regime. This is a classification rule; the nonempty scalar and component cases above supply actual realizations for two different orders. An optional threshold is not a derived physical density scale, and the theorem supplies no universal ordering for every possible source.

If authority is lost first, later turns and exits of the formal ODE are mathematical continuations, not licensed predictions made by transporting the old physical clock and energy ledger across the boundary. A later outgoing return to the same field threshold is not by itself a receipt across an unsupported interval. Earlier valid finite-use records remain valid for their original promises; no later event retroactively repairs or erases them.

14.9. Tests of the receiving interpretation

The receiving interpretation has concrete conditional tests. They concern the declared source and match; they do not promise that an exterior observer directly measures the interior quantities.

Table 17. Tests of the proposed physical receiving interpretation.

Receiving question	Prediction and possible failure of the proposed match
Joint response calibration	On the exact reference match (1), $C_\alpha / c_\alpha = (C_\beta / c_\beta)^{p_\alpha / p_\beta}$ while both applications survive. Incompatible independent contrast histories reject that single-coordinate match and may motivate a multifold description. Away from the reference curve, the broader two-port theorem uses its declared derivative-error bounds rather than this exact power law.

Receiving question	Prediction and possible failure of the proposed match
Source and pressure relation	Equations (3), (7) and (9) connect the differentiation amplitude, pressure, density and scalar rate. In the open class (19), the actual potential and constraint give corridor inequalities rather than exact tracking. Loss of the designated contrasts without sufficiently negative pressure rejects this constitutive match; defining reduced differentiation by negative pressure would therefore be circular.
Geometric response	The flat accelerating branch has increasing $ H $ and density, decreasing comoving energy, no bounce and growing comoving Hubble length. Opposite behavior requires changed source assumptions or an additional contribution. The spherical result separately excludes an areal bounce while trapped; it does not import flat mean-Hubble monotonicity or assert a general volume-bounce theorem.
Preparation and retained components	The matter, shear and charge budgets and inward separation remain part of the prediction. Indefinitely sustained inward capture under unrestricted perturbations is inconsistent with this canonical stability structure. A longer controlled approach needs its own mechanism, conservation and stability analysis.
Approached geometric endpoint	For either received positive-shear box, a same-dimensional continuous nondegenerate Lorentzian extension cannot receive a sequence with $\tau_n \rightarrow \tau_*$ and $\mathcal{Y}_n \rightarrow \mathcal{Y}_* \in (-0.9, 0.9)$, even without symmetry or extension source equations. A proposed counterexample must satisfy that exact embedding and approach contract; another boundary, a degenerate metric, or an unreceived homogeneous future is a different claim.
Original interface support	Continued metric applicability beyond an independently proposed d_L rejects that boundary match while leaving the scalar solution intact. Original comparison, transport and all-alternative conditions decide this test.

The relations distinguish this source mechanism from kinetic-dominated loss of designated contrasts, anisotropy control by a sufficiently stiff contracting component [P2], a genuinely multifield contrast history, or loss of metric application before a vacuum-like response develops. None is ruled out as a different physical realization by the present receiving hypothesis.

The full CSP source provides a useful accounting comparison. Where its homogeneous construction is legitimately received, it has

$$H_a = a^{-3} T_H + a^3 (V_4 + V_2) + a^{-1} K_{\text{CSP}} + a W_H + Y, K_{\text{CSP}} = T_G + W_G + F_{\text{hop}},$$

with pressure determined by the scale derivative and with the original work identity [A9, equations 404–407, Theorem 26.5]. Its acceleration diagnostic contains all appropriate source channels:

$$\mathcal{V}(\rho + 3p) = 4a^{-3} \langle T_H \rangle - 2a^3 \langle V_4 + V_2 \rangle + 2a^{-1} \langle K_{\text{CSP}} \rangle + \langle Y \rangle. \quad (183)$$

The W_H term cancels only from this combination, not from the complete source. Equation (183) is not identified term by term with the scalar potential in (3). It states what a future matching to that richer source would have to reproduce, together with its dynamics, preparation data and original domains. The scalar model has its own complete stress tensor; it does not claim authority for an unperformed truncation of the CSP construction.

A controlled reduction from the native interacting common source is still needed to identify its collective coordinate with this scalar action, coefficient ratios and fixed effective insertions. Such a reduction must control moments, backreaction, interaction errors and original-response derivatives along the occupied history.

The native source already has coupled source/metric dynamics; the missing step is this particular reduction, not the existence of native dynamics. Negative pressure alone would not establish return of every original role.

15. Physical interpretation and conclusions

The central physical result is a conserved inward source response with an attained beginning and end, followed in a specified black-hole family by a local obstruction even to continuous nondegenerate Lorentzian continuation. It gives the persistence-bearer proposal a concrete test without replacing its unproved source matching. Reduced designated contrasts can coexist with negative-pressure braking. Braking need not reverse contraction, and its end need not be the end of the metric solution. In the received positive-shear families both mean and areal braking end strictly before the later invariant singular approach, with no subsequent braking re-entry.

15.1. What the construction says about decreasing differentiation

The exact flat branch and the open-potential corridor demonstrate finite decrease of independently defined effective responses under quantified assumptions. The preparation and separation estimates also explain its cost: the contracting reference is not an attractor established by these results. Different fixed-potential preparations attain a positive-field turn, and retained components or signed spherical shear can end one kind of braking before another. A turn or exit is a physical event of the effective source; it does not itself establish loss of the original comparison or metric roles.

The global construction removes a separate geometric uncertainty. Its recorded KS experiments really occur inside a black-hole region of the stated completed model. The later optical bound then receives the entire positive-box singular approach in that same development. At this approach, Theorem 27 excludes any same-dimensional continuous nondegenerate Lorentzian extension reached by a sequence with $\tau_n \rightarrow \tau_*$ and $\mathcal{Y}_n \rightarrow \mathcal{Y}_* \in (-0.9, 0.9)$. It permits arbitrary extension coordinates and nonsymmetric metrics and imposes no extension Einstein equation or scalar field. The proof derives the accessible boundary chart and its incompatible distance bounds; it does not infer continuous-metric inextendibility from curvature divergence. This is stronger than a local trapped chart or an extrapolated homogeneous trajectory. The negative-shear boxes still have their full received finite experiments, but their actual global late futures remain unresolved by the present estimate.

The horizon calibration answers how small the recorded spheres are relative to a specified external reference. It also proves why that number cannot be intrinsic to the local differentiation mechanism: changing an allowed outer completion preserves the experiment while changing the actual model horizon. The vanishing area ratio at the positive-box end is meaningful on the regular approach. It is neither a distance to the horizon nor an intrinsic area of an AMetric boundary. Proper length, volume and elapsed time require their stated region, foliation and continuing metric application.

The model does not establish that compression undoes the lawful role differentiation of reheating. Its postulated source coordinate decreases only during the specified inward response interval; it turns at positive field, and the completed positive-shear solutions later develop a divergent scalar and curvature. These are definite consequences of the chosen dynamics, not a general obstruction to every possible inverse-reheating mechanism. A successful bearer-recomposition proposal would still need a controlled realization of the complete original role family, including its joint incidences, source accounting and any lawful withdrawal of applications.

15.2. Energy, original answers and the limit of the metric account

There is no separate metric-energy reservoir in the canonical closure. On each licensed segment the stress tensor is conserved, and the energy of a comoving material cell changes by pressure work. The spherical source ledger retains shear and curvature in the geometric equations without misidentifying them as an independently adjustable matter drain. The completed scalar tail becomes kinetic dominated and divergent; its source load is not deleted at the endpoint. No source equation or pressure law is continued as a physical ledger after the relevant application has ended.

The exterior reconstruction results clarify what can remain describable. Distinct interiors can share their complete future exterior while differing in an original source insertion. That is an exact obstruction to every reader of that record algebra. It does not mean every possible original exterior witness, preparation record or governed carrier has already been removed. With independently declared preparation data, an exterior mass readout can instead determine the remaining parameter on a narrower family. On the regular side, a compact coordinate and exact inverse also preserve the original metric, source, clocks and all supplied partial operations. This representation does not add an endpoint operation. The continuous-metric theorem excludes the specified geometric endpoint, while the invariant and original-field divergences independently exclude its C^2 and finite-continuous-scalar versions.

These statements preserve the direction of the exhaustion question. The physical theorem excludes whole classes of through-endpoint realizations, not merely a chosen coordinate description. Faithfulness transports that impossibility whenever the original duty requires the prohibited operation. Full residual exhaustion would require proof that every eligible successor of the actual witness-subtracted burden has such a duty, together with the remaining BH closure premises. An exact regular-side representation is not automatically an eligible residual successor; a singular endpoint is not automatically complete support exhaustion. Section 14 identifies the missing receiving structure while retaining the full Kernel and fixed-domain machinery.

15.3. Structural dependencies and remaining research

The structural results determine the source and use conditions of the construction. They do not choose the scalar potential or receive the effective ports as the entire original role family. The accompanying manifest identifies the selected local editions and reading scope; no corpus-wide latest-edition survey is asserted.

Table 18. Structural dependencies and their receiving duties.

Imported dependency	Required distinction and receiving duty
Early transport closure [A1], §§9–11, 26–28, 44, 46	Structural closure stress differs from stress-energy; reheating preserves original roles and ledgers; onset thinness differs from terminal emptiness.
Persistence bearer and carrier/particle analyses [A2–A4]	Dependence is logical rather than temporal fragmentation. Original source directions, identities, shared inputs, histories and operation domains must survive a receiving map.
Native common source [A9], especially equations 404–407 and Theorem 26.5	Interacting source/metric dynamics already exists; the unproved step is its controlled reduction to this particular effective action, coefficients and responses.
BH sequence [A5–A7] and selected Endpoint companion [A16]	Standing loss, residual successor support, invariant singularity and full boundary closure have different premises. The old ledger does not cross closure by default.

Imported dependency	Required distinction and receiving duty
Higgs and envelope analysis [A8]	High density or a scale ratio alone supplies no structural-order reversal or physical GUT threshold.
Current L-metric and faithful reconstruction [A10, A14]	The complete original profile has a minimal faithful interface. Preserve all original readers, application/joint domains and promised operations; neither the code nor finite-path safety creates a new endpoint. A failed sufficient certificate is not the absence of every safe implementation.
Einstein comparison/source projection and native quantum/source receipt [A17, A9]	The same task retains its original comparison, transport, curvature, source and operator duties under enrichment. This is not a universal demand that every quantum object carry a classical endpoint.
Continuous-extension boundary tools [P11, P12]	Their local chart and distance methods are proved applicable to the received scalar tail in Theorem 27; no vacuum or globally bounded-mass theorem is substituted for this source. The result is local to the stated positive-box approach.
Kernel, AMetric, fixed-domain exhaustion and Structure [A11–A13, A15]	Retain necessity, readiness, identity, full original incidence and all applicable adversarial closures. Labels, finite audits and faithful regular-side recoding do not by themselves prove a full endpoint claim.

The remaining obligations are specific.

1. **Receive the effective source and complete role family.** Derive a collective map, coefficients and controlled error bounds from the native coupled source, including interactions, moments, backreaction and original-response derivatives. Establish the full intended role and history coverage before claiming complete bearer return or an inverse reheating map.
2. **Explain preparation and stability.** The open finite preparation sets prove existence, not generic capture or natural selection. The global construction starts with trapped data. Formation from initially untrapped collapse, constraint-compatible spatial or nonspherical stability, and transverse or entropy-mode stability of a multifield receiver are separate stronger claims.
3. **Extend only the model classes actually received.** The completed real-scalar spherical construction does not automatically receive uncertain local potentials, every profile, fluids or angular charge. The negative-shear actual global futures require further control beyond the established finite experiments. Metric-only C^0 and therefore C^1 endpoint realizations are now excluded for the precise positive-box approach of Theorem 27. Extension at other boundary incidences, degenerate or non-Lorentzian replacement geometries, and globally received negative-shear endpoint claims are not settled by that result.
4. **Instantiate the original residual duty and universal bridge.** Receive the old-standing lineage and occurrence, actual exterior/prior witness algebra, standing-transport failure, successor ancestry and carrier-specific duties. Compare the standing stage with the physical end, and prove the required endpoint duty over the complete independently fixed successor family. Preserve all remaining metric, temporal, causal, operator, carrier, selection, repair, storage and null-route closures.

Postulate S remains a distinct possible receiving route: if an independently warranted interface ends transversely at a finite point of the exact reference family, its approach has finite timing and finite one-sided

source limits. The divergent positive-KS end neither proves that premise nor invalidates that conditional result. A future receiving source may establish one route, another, or neither.

The paper therefore supplies a physical progression from finite response reduction to an actually received invariant singular approach and local continuous-metric continuation obstruction, and a precise limit on promoting that failure to full exhaustion. It establishes no intrinsic geometry beyond metric license and no populated successor cosmos. Fresh governance-status existence, full bearer return, inverse reheating and full AMetric closure remain distinct claims whose additional premises must be met in their own right.

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Appendix A. Full nonlinear corridor proof

This appendix proves the quantitative claim in Theorem 6. All constants refer to the openly perturbed scalar source (19). Set $\kappa=1/5$, $\eta=0.005$, $\eta_1=0.01$, $\delta=0.0009$, and $f=1-d^2$. Assume temporarily that $|\Delta| \leq \kappa d^2$. The constraint gives

$$v^2 = d^2 + 3f\Delta + \frac{3}{2}\Delta^2 - \frac{\Delta W}{2}, m^2 d^2 \leq v^2 \leq z^2 d^2, \quad (A1)$$

$$m^2 = 1 - 3\kappa - \eta/2, z^2 = 1 + 3\kappa + \eta/2.$$

For the upper estimate, $3f\kappa + (3/2)\kappa^2 d^2 = 3\kappa + [-3\kappa + (3/2)\kappa^2]d^2 \leq 3\kappa$. For the lower estimate, discard the nonnegative square term and use $f \leq 1$. In particular, v stays real and strictly positive in the strip. From (24), the upper Dini derivative satisfies

$$D_{\ell_d}^+ |\Delta| \leq \frac{6f + 3\kappa d^2}{(1+m)d} |\Delta| + \frac{\eta d}{1+m}. \quad (A2)$$

This also holds at $\Delta=0$. Put

$$a_0 = \frac{6}{1+m}, b_0 = \frac{6-3\kappa}{1+m},$$

$$G(d_i, d) = \left(\frac{d_i}{d}\right)^{a_0} \exp\left[-\frac{b_0}{2}(d_i^2 - d^2)\right]. \quad (A3)$$

The coefficient in (A2) is positive. Its propagator therefore increases as the field decreases, and is at least one. Variation of constants gives

$$|\Delta(d)| \leq G(d_i, d) \left[\delta + \frac{\eta(d_i^2 - d^2)}{2(1+m)} \right]. \quad (A4)$$

To maximize G before halving, first use its monotonicity in the field distance to set $d = d_i/2$, then maximize over the initial interval. This yields $G \leq 2^{a_0} \exp[-3b_0(0.495)^2/8]$. Maximizing its two factors separately at incompatible points would not establish this bound.

There is an elementary strict closure. Since $0.63 < m < 0.631$, one has $a_0 < 15/4$, $b_0 > 3.3$, and $c = 3b_0(0.495)^2/8 > 0.301$. Also $2^{15/4} < 13.5$, while $e^c > 1 + 0.301 + 0.301^2/2 + 0.301^3/6 > 1.35$. Hence $G < 10$. The bracket in (A4) is at most $\delta + 3\eta(0.505)^2/[8(1+m)] < 0.0012$. Thus

$$|\Delta| < 0.012 < \kappa(0.495/2)^2 = 0.01225125. \quad (A5)$$

The initial state lies strictly within the strip. Inequality (A5) excludes its first exit before halving. On the closed candidate band, $Q \geq 1 - (1+\kappa)d^2 > 0$ and $v \geq md > 0$; the smooth field-coordinate system cannot end at a turning or square-root boundary. Its solution extends to the half crossing. Since $-d'/d \geq m$, that crossing occurs in finite proper time. This completes the existence and continuation argument, rather than merely estimating a trajectory assumed to reach the target.

The force $g(d) = -W_{,d}(d)/4$ satisfies

$$g - 3Qv \geq d \left[4 - \frac{\eta_1}{4} - 3z + d^2 \{ 3(1-\kappa)z - 3 \} \right] > 0.19980d. \quad (A6)$$

Both the bracket's constant term and its d^2 coefficient are positive: their approximate values are 0.1998033296 and 0.0381573363. Their signs follow directly by squaring positive rational bounds on $z = \sqrt{641/400}$. Therefore $v' < 0$, $Q' > 0$, and v/Q decreases. Potential increase also follows from $W_{,d} < 0$.

For $\mathcal{A} = Q^2 - 2v^2 = W - 2Q^2$, the initial root condition gives the rational lower bound

$$\mathcal{A}_i \geq [1 - (0.505)^2]^2 - 2(0.505)^2 - \eta(0.505)^2 - 4[1 - (0.495)^2]\delta - 2\delta^2 = 0.040943095625 > 0.040. \quad (\text{A7})$$

Equation (23) gives $\mathcal{A}' = 4v(g - 2Qv) > 0$ using (A6). The kinetic, pressure and acceleration claims follow. Integrating $md < v < zd$ and $[1 - (1 + \kappa)d^2]/(zd) \leq Q/v \leq [1 - (1 - \kappa)d^2]/(md)$ proves (22).

For the response endpoint estimates, the much tighter initial condition must be used instead of the whole strip. With $e_i = Q_i - (1 - d_i^2)$,

$$\begin{aligned} \frac{v_i^2}{d_i^2} &> 1 - \frac{3(0.754975)(0.0009)}{0.245025} - 0.0025 > 0.9891807, \\ v_f^2 &< \frac{1.6025}{4} d_i^2. \end{aligned} \quad (\text{A8})$$

Thus $v_f^2/v_i^2 < 0.406$. The value-error budget 0.001, with $d_i > 0.495$, gives the conservative matched ratios $C_{d,f}/C_{d,i} < 0.256$ and $C_{u,f}/C_{u,i} < 0.412$. The positivity of both denominators follows from the same lower bounds. Scripts reproduce these arithmetic estimates; the explicit comparisons above carry the proof.

Appendix B. General-potential comparison criteria

These criteria show how a wider preparation interval can be obtained over a shorter prescribed excursion. They assume a $\mathcal{C}^2$, positive, strictly decreasing potential on a compact positive-field band and the flat canonical scalar closure. Write $H_{\text{abs}} = |H| = h Q$, $\xi = -\dot{\phi}/(\sqrt{6} M H_{\text{abs}}) > 0$, $\lambda_V = -M V_{,\phi}/(\sqrt{6} V) > 0$, and $\mathcal{Y}_\phi = (\phi_i - \phi)/(\sqrt{6} M)$, with final field width Y_ϕ . The exact equations are

$$\begin{aligned} V &= 3 M^2 H_{\text{abs}}^2 (1 - \xi^2), \mathcal{Y}_{\phi,N} = \xi, (\log H_{\text{abs}})_N = 3 \xi^2, \\ \xi_N &= 3(1 - \xi^2)(\xi - \lambda_V), \epsilon_H = 3 \xi^2. \end{aligned} \quad (\text{B1})$$

The boundary $\xi = 0$ permits a field turn; $\xi = 1/\sqrt{3}$ is the acceleration boundary. The two are distinct. Contracting antidamping is retained in (B1).

Potential-ratio criterion. Put $\mathcal{R}_V = V_f/V_i > 1$. While $\xi > 0$,

$$\begin{aligned} \frac{d \operatorname{atanh} \xi}{d \mathcal{Y}_\phi} &= 3(1 - \lambda_V/\xi) < 3, \\ \xi^2 &= 1 - V/\rho \geq 1 - \mathcal{R}_V(1 - \xi_i^2). \end{aligned} \quad (\text{B2})$$

The second bound uses nondecreasing scalar density during contraction. Therefore every initial speed satisfying the nonempty interval

$$\sqrt{1 - \mathcal{R}_V^{-1}} < \xi_i < \tanh[\operatorname{atanh}(1/\sqrt{3}) - 3 Y_\phi] \quad (\text{B3})$$

has a strictly positive lower speed and stays below the acceleration boundary through the field crossing. The constrained Hubble rate is bounded on this compact strip, so continuation and $\mathcal{Y}_{\phi,N} > 0$ establish an actual finite crossing. If (B3) is empty, only this sufficient criterion has failed.

For width $M/10$ and $1 < \mathcal{R}_V < 26/25$, all $\xi_i \in (1/4, 9/20)$ work. Uniform bounds are $m_A = 1/\sqrt{40} < \xi < b_A$, where $b_A = \tanh[\operatorname{atanh}(9/20) + \sqrt{3/2}/10] \simeq 0.5421353 < 1/\sqrt{3}$. The reference potential on $\phi/M \in [0.2, 0.3]$ has ratio about 1.035010, so the class is nonempty. No decreasing kinetic-density or potential-fraction conclusion is implied by this first criterion alone.

Potential-slope criterion. Suppose $0 < c < \lambda_V < L$. Choose $0 < m < a < b < \min(c, 1/\sqrt{3})$ with

$$(1 - a^2) e^{6(L-m)Y_\phi} < 1 - m^2. \quad (\text{B4})$$

Then all $\xi_i \in (a, b)$ cross with $m < \xi < b$ and $\xi_N < 0$. To prove this, the upper boundary points inward by $b < c$. On a candidate strip, $z_0 = \xi^2$ satisfies $(z_0)_{\mathcal{Y}_\phi} = 6(1 - z_0)(\sqrt{z_0} - \lambda_V) \geq -6(L - m)(1 - z_0)$. Hence $z_0(\mathcal{Y}_\phi) \geq 1 - (1 - \xi_i^2) e^{6(L-m)Y_\phi} > m^2$, excluding a first lower exit. Compact-strip continuation proves finite crossing. The decreasing ξ makes the potential fraction grow but does not alone make dimensional kinetic density decrease.

On width $M/20$, take $m = 0.10$, $a = 0.21$, $b = 0.30$, $c = 0.305$, $L = 0.36$. Here $6(L - m)Y_\phi = 0.078/\sqrt{6} < 13/400$; using $e^z < 1/(1 - z)$ for $0 < z < 1$, the left side of (B4) is below $0.9559/0.9675 < 0.99$. For the reference potential on $\phi/M \in [0.50, 0.55]$, λ_V increases from approximately 0.309837 to 0.351223, strictly within the required interval. Its derivative is positive because the numerator is $(8 - 18 d^2) W_0 + 2(8 d - 6 d^3)^2 > 0$ on the stated band. Both examples have strict local potential margins.

For either criterion, with $m \leq \xi \leq b$, uniform duration bounds are

$$N_- := Y_\phi / b \leq N_f \leq Y_\phi / m =: N_+,$$

$$\frac{1 - e^{-3b^2 N_-}}{3b^2 H_{\text{abs},i}} \leq \Delta t \leq \frac{1 - e^{-3m^2 N_+}}{3m^2 H_{\text{abs},i}}. \quad (\text{B5})$$

These follow by integrating the bounds on $(\log H_{\text{abs}})_N$, then $d t = d N / H_{\text{abs}}$. The first example gives $0.075304 \lesssim N_f \lesssim 0.258199$; the second gives $0.068041 \lesssim N_f \lesssim 0.204125$. The exact formulas, rather than rounded table values, specify the guarantees. Openness concerns these local potential and constrained initial-data sections. It is not a measure of how often a cosmological or black-hole source supplies them.

Appendix C. Reproducibility and scope of the evidence

The analytic inequalities, numerical certificates and ordinary illustrations have different jobs. The complete nonlinear corridor proof is retained in Appendix A and the two shorter-band comparison criteria in Appendix B. Their formulas and continuation arguments carry the guarantees; sampled curves do not replace them. Sections 10–14 include the complete additional horizon, endpoint, reconstruction and conditional receiving proofs within the article.

For the flat uncertain-potential examples, the supplied scripts evolve the full homogeneous equations rather than projecting the Friedmann constraint. Seven potential/preparation cases reproduce their event times and response ratios, together with the separately backreacting scalar–dust–radiation–shear example. Rational square-root enclosures and bounded elementary-function series check the corridor constants. Ordinary Runge–Kutta trajectories and small constraint/work residuals remain numerical evidence, not validated ODE enclosures.

The exact reference halving has dimensionless duration 0.693147181, contraction count 0.599397181 and kinetic ratio 0.25. Across the seven supplied numerical examples, halving times range approximately from 0.670028669 to 0.720747523 and kinetic ratios from 0.187819 to 0.310424. Those sampled ranges are not the uniform theorem bounds. The exact-reference, nonlinear-corridor and component-budget figures retain their respective exact, analytic-plus-illustrative and necessary-bound meanings.

The component and KS certificates enclose every continuous solution step for all data in their declared initial boxes. Their strict Picard inclusions, Taylor remainder bounds, outward elementary-operation rounding and nearest-rounding/square-root premises are part of the computer-assisted statements. The component step is 1/128 and the KS step is 1/256; their finite record horizons and event extractors are separate. The KS record contains eight runs—four centre cases and four full boxes—with 2,214 continuous steps. The original independent reproduction checked the full record and certified summary byte for byte. Subsequent extraction from archived enclosures checks a processing layer and is not another independent solver execution.

The five inherited scientific figures retain their original plotted data and assets. Ordinary trajectories and dashed centre-event locations remain separate from the certified whole-box event bands. The trapped-interior figure preserves its signed-shear labels, distinct panel windows, mean/areal distinction and symmetric-log ordinate. Its finite optical bounds support the global domain-of-dependence argument; a plot of local trapping alone would not establish membership in a black-hole region. The argument diagram is a dependency diagram, not an additional numerical experiment.

For fixed horizon units, the exact completion and finite mass integrals are proved in Section 10. General whole-box horizon bounds use rational monotonicity; the narrower analytic normalization enclosures refer to the four central preparations. The twenty whole-box relative-area intervals are rational quotients of extrema from 61 continuous-step tubes intersecting the certified event brackets. The fixed-unit event figure instead plots ordinary numerical centre values from the same explicit completion. Its separate panel ranges and unchanged denominators are intentional. Neither agreement of independent ordinary quadratures nor a plotted decimal is promoted to a validated narrow absolute-area interval.

The endpoint proof uses the original completed potential, finite endpoint enclosures and an analytic transition to the exact zero-potential tail. Its future time and positive-box optical bounds are independently reproduced with exact fractions and the explicit positive logarithm-series remainder. Stored binary64 endpoint values are interpreted as their exact dyadic rationals before that processing. The continuous-metric obstruction uses the same exact tail and the same received positive-box strip. Its accessibility, local causal confinement, timelike homotopy, boundary-chart localization and slice-distance contradiction are proved in Section 11. The imported boundary graph is [P11, Proposition 2.2]; the local homotopy adapts [P12, Proposition 4] with its angular

speed estimate given explicitly. Neither a whole-spacetime Schwarzschild theorem nor the full warped-product hypotheses are substituted for the present construction. No new validated integration across the cutoff is asserted. The separately reproduced central estimates of endpoint times and negative-shear optical reaches are diagnostics, not proofs of whole-box receipt or of the actual negative-box global futures.

The exterior collision and mass inverse use exact algebra. The derivative of the original kinetic readout has a strict rational sign on the entire admissible one-parameter interval, which establishes nonreconstruction for every reader of the common exterior. Numerical sampling is not the argument. The complete classical profile is proved through the fixed Cauchy-data construction, marking and uniqueness. Its supplied preparation and experiment inputs are mathematical data; their existence in a proof is not a claim that a corresponding physical exterior record survives.

The compact tail has exact inverse formulas and analytic normalized functions. Its metric and original-time derivative operator receive every specified original regular-side operation on its actual domain. The whole-domain conjugacy is an elementary semantic consequence of the constructed inverse. It is not a new physical extension theorem, an executed infinite computation, or proof of an eligible same-burden residual successor. The common-duty lifting lemma is likewise elementary once its universal premise has been independently established; the premise remains the substantive obligation.

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