Quantum Measurement under AASC

Physical Sources, Finite Detectors, Persistent Records and Actual Events

Abstract

We develop a unified account of quantum measurement under AASC, connecting constructed physical sources, quantum instruments, material amplification, finite record retention, event probabilities and the actuality of outcomes. On a regulated interacting gauge–Higgs–fermion source, explicit bounded receivers yield informative complete instruments and finite calibration bounds without freezing the source Hamiltonian. A separate six-mode cosine circuit realizes resolved registration, with source and pump interfaces, a two-polarization intrinsic-spin receiver, and a finite energy-resolving thermal amplifier. A finite Fourier residual certificate bounds the complete monitored detector instrument throughout its 35.587 ms interval by 0.063241 in restricted diamond norm. Exact interval arithmetic gives seeded amplifier endpoint fidelity above 0.9999235242 after 1024 finite collisions and an explicit conditional survival bound for a subsequent 100 ms quiet hold. Coherent composition retains blank, wrong and unreadable runs and exposes all outstanding physical tolerances. We prove calibrated affine probability uniqueness, sharp coherent calibration limits, adaptive record composition and local-instrument compatibility. A complete instrument admits an exclusive-history completion but does not select one such history through linear dynamics alone. Source-specific preparation and reader analysis, together with a bounded counting-law completion, identifies the additional physical input required for that selection. The resulting account substantially develops the material and probabilistic parts of measurement while isolating the actual-history law and remaining apparatus errors as explicit obligations.

Contents

1 The measurement problem and the scope of the construction

An ideal measurement correlates a source eigenstate with a distinguishable apparatus state,

$$U(|a_r\rangle|b\rangle) = |a_r\rangle|r\rangle.$$

Linearity then takes a coherent input to $\sum_r c_r|a_r\rangle|r\rangle$. The ordinary measurement problem asks how a theory connects this description to definite experimental results and their probabilities. Wave-function completeness, unrestricted linear evolution and one definite outcome are incompatible under the usual eigenstate–eigenvalue reading; specifying the measured observable and explaining persistent records are further parts of the problem [1, 2]. A complete account must identify its physical source, the apparatus interaction, the record carrier, the probability rule, the subsequent conditional state and the status of an actual occurrence.

This paper develops those questions in one connected AASC construction. Its purpose is to exhibit the strongest supported physical and mathematical chain, with each premise located where it enters. The construction includes informative receivers on a specified interacting source, material conversion into distinguishable channels, finite preparation and amplification, a full-duration detector error certificate, a calibrated event-probability theorem and explicit conditional history laws. The distinction between a quantum instrument and a law making one history actual is part of the result.

1.1 Governance, dynamics and events

AASC concerns the necessary governance of non-degenerate determinate construction by admissibility, standing, reference and irreversibility. In this application, standing concerns which specified physical content may persist and be reused under its conditions; reference concerns which occurrence and content a record use retains. Determinate physical carriers and record incidences are already governed by these requirements. The construction does not introduce an optional condition under which physical objects become kernel-governed.

The dependency direction also matters. A theorem about a determinate record's identity does not, simply by being necessary for such an identity, calculate a detector coupling or prove that one record was selected by a given dynamics. The corpus supplies substantial physical source constructions and exact comparison structures. Their source, domain and conclusion must be retained when a new apparatus is added. Quantum dynamics then fixes a complete instrument for a declared preparation and readout. An actual-event interpretation adds a statement about occurrences in the original execution, and its probability law must be identified separately.

We distinguish three output objects throughout. A *complete instrument* contains all completely positive outcome operations, including an unresolved outcome at a finite deadline. A *material record* is a distinguishable state of a physical carrier whose acquisition and retention can be analyzed. An *actual history* concerns which occurrences belong to one original run. The first two objects can be constructed without assuming that a globally coherent unitary evolution has already selected one original-run history. Conversely, admitting an actual event does not by itself determine its probabilities or conditional quantum state.

1.2 The connected physical construction

Two physical targets support the analysis. The first is the corpus's interacting, finite-spatial-regulator quantum source. Its complete unbounded Hamiltonian remains present during the finite receiver pulses. This establishes preparation sensitivity and a controlled number-measurement limit without treating the source as an abstract label. The second is an explicitly parameterized six-mode circuit with full Josephson cosines, positive quadratic circuit matrices, two pumps, losses and distinguishable outputs. It supplies a concrete detector calculation. An exact identification of every intervention on the first target with the second circuit or with the original continuum

Result	Established content and its input
Source and receiver	A specified self-adjoint interacting source admits an informative bounded receiver coupling and a finite-pulse calibration theorem with the full source Hamiltonian retained.
Material registration	A positive circuit construction supplies exact cosine dynamics and a complete monitored detector model; an analytical conversion kernel fixes latency and finite-deadline bias in its stated ideal limit.
Finite amplification	A two-polarization spin receiver, finite Gibbs resources, reversible collision control and a scheduled endpoint reader give a complete apparatus channel with explicit implementation conditions.
Full detector interval	A finite Fourier residual theorem, infinite-tail bounds and validated arithmetic give channel discrepancy < 0.063241 over the full 35.587 ms deadline against an explicitly dressed comparison instrument.
Probabilities and events	Affine preparation consistency and sector calibration fix event weights after actual readout is admitted. Coherent finite-calibration bounds and explicit history models disclose the additional statistical and actuality premises.
Composition and locality	Complete instruments compose with retained failures, time and records. Covariant local operations and source-transfer statements retain the locality and comparison assumptions under which they are proved.

Table 1: One argument with different kinds of input. A physical-model certificate is not an experimental calibration, and an instrument theorem is not an independent actual-history law.

physical source would require an additional source comparison. We preserve that requirement rather than silently identify the targets.

On the circuit target, the sequence is explicit: source-code loading, driven conversion, output registration, removal of the known pump displacement, transfer of the material flag to an intrinsic spin, a finite Gibbs-collision amplifier, an endpoint reader and a quiet hold. The electronic first-display label $e \in \{\varnothing, 0, 1\}$ and the material flag are distinct. A missed count can leave a flag without an electronic display; a dark display can occur without a matching flag. The final calculations retain their joint distribution. No reported original-run probability is obtained by removing blanks or errors and renormalizing an accepted subset.

The finite amplifier has a different role from the source-dependent detector. It amplifies a supplied physical seed into a readable band. Its approximately 99.992% correct endpoint probability is conditional on that seed, while the combined detector/display/material success is about 92.6–92.7% in the specified comparison model. Keeping these denominators explicit prevents an amplifier's fidelity from being mistaken for the performance of the entire measurement chain.

1.3 Main results and their dependencies

The mathematical content is organized by the operations needed for one measurement account, rather than by the order in which the components were developed.

The decisive finite calculation uses one fixed electronic operating point, $s = 0.1$. Its exact cosine-induced energy shifts are included in the effective comparison Hamiltonian. Its original-run probabilities are evaluated with the same deadline, waste linewidths, efficiency and dark-count conventions. The material receiver and amplifier are then specified on the common physical scale $\Omega_0 = 2\pi\, 10^9\,\mathrm{s}^{-1}$. Each remaining physical interface or control error appears in the final residual ledger. An unmeasured term is not assigned zero.

The probability argument has an equally explicit structure. Positivity, density-operator dependence, affinity under classical mixtures and calibration imply the trace probability rule for the declared readout. Approximate sector calibration alone allows coherent deviations of order the square root of the calibration error; stronger linear bounds require additional structure. A complete instrument also determines sequential probability laws and compatible conditional states. To interpret these laws as one actual sequence, an actual-history commitment or a separately constructed source-reader law is still required.

1.4 Conventions and reading guide

Quantum states are normal density operators, and complete positivity always includes an arbitrary idle reference. The trace norm has maximum distance two on states. The diamond norm of the difference of channels likewise has maximum two; a classical total-variation discrepancy is at most half the corresponding complete channel bound. A restricted channel estimate includes its declared source-code embedding and apparatus initialization. Existence of a downstream channel on leaked inputs does not turn that estimate into an unrestricted accuracy bound.

Source Hamiltonians may be unbounded. Every finite-time perturbation estimate retains its operator-domain assumptions. Finite Hilbert-space auxiliaries, finite spatial regulators, finite Fock trial supports and finite gate schedules are different notions of finiteness. A finite Fock trial is used to certify an infinite oscillator model by bounding its exterior residual; it is not substituted for the physical model by definition.

Sections 2–3 establish the source and receiver. Sections 4–10 develop and certify the material chain. Sections 11–13 address probabilities and actual histories, followed by locality and comparison in Sections 14 and 16. Section 15 gives the common error and assumption ledger. The appendices identify source versions and explain reproduction. The accompanying provenance map accounts for every component manuscript; interim project priorities and superseded numerical status reports are not presented as current scientific conclusions.

2 Constructed physical sources and faithful measurement interfaces

The physical starting point is an occupied source with interactions, positive states, response observables and admissible continuations. The corpus constructs such sources at several resolutions. We state the imports used in the new measurement proofs and keep the corresponding targets distinct. The results credited to the source manuscripts are imports; this paper does not independently reprove their field-theoretic existence theorems. Exact inspected versions and theorem locators are recorded in Appendix A.

2.1 An interacting quantum source at finite spatial resolution

Fix the finite connected graph, nontrivial plaquette and coefficient record of CSP-P, Definition 12.2 and Theorems 12.3 and 12.5 [3]. Its gauge group and kinematic space are

$$G = (SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6, \qquad \mathcal{H}_{\mathrm{kin}} = L^2(G^E, dU) \otimes L^2(\mathbb{R}^{4|V|}, dH) \otimes \bigwedge W.$$

The local gauge action has the physical subspace

$$\mathcal{H}_{\mathrm{phys}} = \mathcal{H}_{\mathrm{kin}}^{G^V},$$

with orthogonal projection given by compact Haar averaging. The source includes the specified three copies of chiral matter, one Higgs doublet and their invariant contractions. Although the graph and fermionic factor are finite, the Higgs coordinates remain continuous, so the full physical Hamiltonian is not a finite matrix.

The physical operator H_{src} is the self-adjoint realization of the displayed common form, containing electric and Higgs kinetic energy, a confining quartic Higgs potential, gauge plaquette terms, gauge–Higgs transport, fermionic hopping and the Yukawa interaction $Y = Z_Y + Z_Y^{\dagger}$. Its coefficients belong to the fixed source. Theorem 12.3 proves closedness and semiboundedness of that form, invariance of the physical subspace, compact resolvent, a nonzero finite-dimensional ground eigenspace and a finite heat trace at every positive inverse temperature. The bounded hopping and compact-link terms, and the control of lower-order Higgs terms by the quartic potential, are load-bearing parts of that theorem. The associated unitary dynamics and faithful Gibbs state therefore exist on the represented physical algebra.

Total fermion number N_F is a bounded gauge-invariant physical observable. The physical Schwartz core is preserved by N_F and its finite spectral projections. No commutation between N_F and H_{src} is assumed; the Yukawa interaction generally changes number. A normalized gauge-invariant source vector is

$$\psi_0(U,H) = c_0 e^{-\sum_x |H_x|^2} \otimes |0\rangle, \qquad N_F\psi_0 = 0.$$

This vector establishes physical nonemptiness and lies in the stated Hamiltonian-power domains. It is not identified with the interacting ground state. For nonzero Yukawa data, Theorem 12.5 constructs

$$\chi = Y\psi_0 \neq 0, \qquad \phi = \chi/\|\chi\|, \qquad N_F\phi = 2\phi,$$

with $\phi \perp \psi_0$, and proves

$$\langle e^{-itH_{\mathrm{src}}}\psi_0, N_F e^{-itH_{\mathrm{src}}}\psi_0\rangle = 2\|Y\psi_0\|^2 t^2 + o(t^2). \tag{1}$$

The same source gives nonzero Yukawa feedback in the radial Higgs acceleration. The response is thus preparation-sensitive physical content on a common domain. The receiver in Section 3 uses the bounded number observable to extract this distinction with a fixed ready state and fixed coupling.

Gauge invariance and domain preservation constrain that extraction. A charged elementary field need not preserve the physical subspace, and its compression need not satisfy an independently asserted canonical field algebra. The invariant number construction avoids this substitution. Adding a gauge-singlet receiver is a specified extension of this source; it does not automatically identify that receiver with a microscopic Standard Model detector or preserve a separately proved gravitational attachment.

2.2 Direct physical realization and exact source comparison

CSP-P also supplies a direct physical-source theorem with a different target. Theorem 11.5 constructs the occupied original Standard Model source, physical W, Z and Higgs channels, the color/composite interface and a source-compatible dynamical Einstein realization at one fixed joint point [3, PDF pp. 97–98]. The conjunction uses one actual common-source intersection, retaining the coefficient tuple, state, ports, operation domains and boundary authority. It is stronger than separate nonemptiness statements for unrelated marginal sectors.

The finite-spatial quantum construction is complementary. Identifying a refining regulator family with the original physical source requires its actual regularity, locality, covariance and source-comparison data. Those obligations do not condition the already established direct theorem; they condition a proposed identification with a different construction. Conversely, a direct existence theorem does not identify every newly chosen finite-source intervention with an original physical operation.

Proposition 11.4 provides the relevant standard for exact reception: whole argument records, partial domains, shared operands, first failures and physical authority are retained, and reading back returns those same objects [3, PDF p. 97]. A source-to-detector claim must therefore identify the preparation, interaction, reader and continuation together. Agreement of a Hamiltonian's shape, a particle name or a single expectation value does not establish that comparison.

Further corpus constructions support genuine mixed dynamics while retaining their scope. Proposition 12.7 couples an actual Yang–Mills vacuum source to an auxiliary gauge-singlet fermion pair through a bounded invariant observable and proves short-time entanglement. Theorems 28.1–28.2 provide an entangled stationary ground state and a gap on a specified coupling interval. Theorems 26.1–26.5 give a coupled homogeneous quantum–Einstein development in which the scale responds to quantum expectations of a scale-dependent interacting Hamiltonian [3]. Adding a new receiver to any such model changes its interaction and response terms; the resulting target and constraints require their own comparison.

2.3 Persistence, response structure and the history boundary

The persistence bearer of CSP-R retains localized configurations, boundary comparisons, transport, restoration and response relations, together with participation domains and failures [4, Definition 2.1]. Theorem 3.5 reconstructs native and receiving response quotients as many-sorted partial-operation structures. It preserves and reflects domains, outputs, failures, shared operands and fixed-sibling comparisons. Theorem 4.3 carries the occupied CSP-P state, interaction, physical interfaces and Einstein action under the reconstruction. The bearer permits different configurations and response classes; its commonness does not mean that all physical responses form a singleton.

CSP-P, Theorem 7.10, also proves a non-null actually realized persistence-history quotient with a nonempty stable interior on its stated domain [3, PDF pp. 59–61]. If Hist_D is that domain and r runs through the admitted original readers, then

$$h \sim_{\mathrm{Rec}} k \quad \Longleftrightarrow \quad r(h) = r(k) \text{ for every such } r.$$

Exact reception preserves the resulting quotient, the readers, shared operands and continuation failures. These are positive history witnesses. The theorem does not supply a numerical detector-decay bound or a probability distribution over a newly selected detector reader. Restricting it to an apparatus history family requires preservation of its actual realization antecedents. Section 13 examines precisely the additional reader and preparation data needed for an actual-source event route.

The record and entanglement sources preserve a related distinction. MR admits physical selection as a sufficient route to record fixation while separating selective branches from a nonselective state [5, Section 2.2]. ENT separates witness support, performed-context sampling and joint record compatibility [6, Definition 4.14, Theorem 4.15 and Theorem 6.3]. A unique identity for a record already admitted in an actual execution therefore cannot be used as an unproved selection dynamics for that execution.

2.4 Related coupled constructions and response geometry

The broader source chain supplies coupled classical gauge–Higgs/gravity histories and common quantum/classical interaction data. BCRS-III corrects the coupled constraints before developing nonzero color, weak and hypercharge curvature, a nonconstant Higgs field and a scalar clock [7, Proposition 7.4 and Theorem 7.7]. CSP-C constructs a common coefficient tuple and exact finite Grassmann-valued Dirac–Yukawa fields with nonzero even coupled response [8, Theorems 1.2, 8.4 and Corollary 8.5]. A nilpotent classical response is not thereby a positive quantum detection probability. The common construction supplies compatible dynamics at its declared targets.

The symmetry arc also constructs physically meaningful response distinctions before a chosen coordinate description. SYM-8 obtains the electromagnetic stabilizer and positive response form from the one-doublet kinetic coupling, retaining its fixation and spectral premises [9, Theorems 5.1, 6.2–6.3]. SYM-2 distinguishes active physical changes from passive redescription [10, SYM2-T07–T09]. SYM-11 constructs source-class physical signatures and their compatible role assignments [11, Definitions 8.2–8.4 and Theorems 8.6–8.7]. These imports support a physical account of calibration and covariance. The next sections construct the acquisition and statistical maps that a measurement application still needs.

3 Physical receivers on the constructed quantum source

We use units with $\hbar = 1$. Let $H_{\mathrm{src}} = H_{\mathrm{fin}}$ be the self-adjoint, lower-bounded Hamiltonian on the nonzero physical space $\mathcal{H}$ supplied by the finite source construction in Section 2. Its bounded total fermion number is denoted by N_F. The finite Fock factor makes N_F bounded; the number-preserving gauge action makes it invariant. Its spectral resolution is

$$N_F = \sum_{n=0}^{M} nP_n, \qquad \sum_{n=0}^{M} P_n = I. \tag{2}$$

Some P_n may vanish on $\mathcal{H}$. The integer M can be the dimension of the source one-particle space. The dense physical Schwartz space $\mathcal{D} \subset D(H_{\mathrm{src}})$ is preserved by N_F and every P_n: the latter are polynomials in the finite-spectrum operator N_F. No commutation of H_{src} and N_F is assumed. The constructed Yukawa interaction generally changes number.

3.1 A preparation-sensitive two-channel receiver

Let a gauge-singlet auxiliary receiver have even states $|0\rangle, |1\rangle$, where $|1\rangle$ denotes the occupied two-mode pair. Define

$$Q_r = |r\rangle\langle r|, \quad X = |1\rangle\langle 0| + |0\rangle\langle 1|, \quad H_g = H_{\mathrm{src}} \otimes I + I \otimes \nu Q_1 + gN_F \otimes X, \tag{3}$$

with $\nu > 0$ and real $g \neq 0$. This is a specified new coupling of the source to a finite receiver. It uses the bounded-extension mechanism of CSP-P, Proposition 12.7 [3], with an explicitly chosen physical observable. It does not assert that this auxiliary device is already a microscopic Standard Model apparatus or that its added interaction preserves an independently proved gravitational attachment.

The instrument construction uses the established operational framework of Davies–Lewis and its measuring-process and conditional-state developments [12, 13, 14]. The source-specific coupling and estimates are proved explicitly here.

Theorem 3.1 (Exact receiver instrument)**.** *The operator H_g is self-adjoint on $D(H_{\mathrm{src}}) \otimes \mathbb{C}^2$ and lower bounded. For ready state $|0\rangle$, put*

$$K_r(t) = (I \otimes \langle r|)e^{-itH_g}(I \otimes |0\rangle), \qquad r = 0, 1. \tag{4}$$

The K_r are contractions, $\sum_r K_r^* K_r = I$, and

$$\mathcal{I}_r^t(\rho) = K_r(t)\rho K_r(t)^*, \qquad w_r^t(\rho) = \operatorname{Tr} \mathcal{I}_r^t(\rho) \tag{5}$$

are completely positive operations and normalized nonnegative weights for every density operator ρ. Moreover,

$$\operatorname*{s\!lim}_{t\to 0} \frac{K_1(t)}{t} = -igN_F, \qquad \lim_{t\to 0} \frac{w_1^t(\rho)}{t^2} = g^2 \operatorname{Tr}(\rho N_F^2). \tag{6}$$

The strong limit holds on all of $\mathcal{H}$ and the weight limit on every normal density operator, without a finite-energy assumption.

Proof. The receiver energy and coupling are bounded self-adjoint perturbations of $H_{\mathrm{src}} \otimes I$, so the domain and lower bound follow. The Kraus completeness identity is unitarity evaluated between ready-state vectors, with the sum over the complete invariant receiver subspace. It proves the operation and normalization assertions.

For the stronger short-time statement, set $H_f = H_{\mathrm{src}} \otimes I + I \otimes \nu Q_1$ and $V = gN_F \otimes X$. In the interaction picture,

$$U_I(t) = I - i \int_0^t V_I(s) U_I(s)\, ds, \qquad V_I(s) = e^{isH_f} V e^{-isH_f}. \tag{7}$$

The integral is strong, V_I is strongly continuous and uniformly bounded, and $\|U_I(t)-I\| \le |t|\,\|V\|$. Hence $(U_I(t)-I)/t \to -iV$ strongly. Projecting its off-diagonal receiver block and restoring the strongly continuous free evolution gives the first limit in (6). It also gives $\|K_1(t)/t\| \le \|V\|$ for $t \neq 0$. Write $\rho = \sum_j \lambda_j |u_j\rangle\langle u_j|$ with $\sum_j \lambda_j = 1$. Dominated convergence in $\sum_j \lambda_j \|K_1(t)u_j/t\|^2$ proves the weight limit. □

For nonzero Yukawa data, CSP-P, Theorem 12.5 [3], supplies normalized physical states $\psi_0, \phi \in \mathcal{D}$ with $N_F\psi_0 = 0$, $N_F\phi = 2\phi$ and $\phi = Y\psi_0/\|Y\psi_0\|$. Thus

$$w_1^t(|\psi_0\rangle\langle\psi_0|) = o(t^2), \qquad w_1^t(|\phi\rangle\langle\phi|) = 4g^2t^2 + o(t^2). \tag{8}$$

The weights differ for every sufficiently small nonzero t. This proves preparation sensitivity with the same ready receiver and fixed coupling. A mere nonzero response would not prove that distinction.

At finite t, the effects $E_r(t) = K_r(t)^*K_r(t)$ are the observables actually implemented. They are not asserted to be spectral projections of N_F. In particular, the strong short-time limit measures N_F^2 at its leading weight order; it is not a finite-time Lüders measurement. Its weights become actual-event probabilities only under the physical registration conditions of Section 13.

3.2 A number-calibrated receiver with a controlled projective limit

Choose an integer $d > M$ and a receiver $\mathbb{C}^d$ with labels $\mathcal{R} = \{0,\dots,d-1\}$. Let $S|r\rangle = |r+1 \bmod d\rangle$ and choose a self-adjoint logarithm B with $e^{-iB} = S$. The finite-dimensional spectral theorem supplies such a bounded B. Define

$$V = N_F \otimes B, \qquad W = e^{-iV} = \sum_{n=0}^{M} P_n \otimes S^n. \tag{9}$$

Let H_A be a fixed bounded self-adjoint receiver Hamiltonian. For each pulse duration $\epsilon > 0$ use

$$H_\epsilon = H_{\mathrm{src}} \otimes I + I \otimes H_A + \epsilon^{-1}V, \qquad U_\epsilon = e^{-i\epsilon H_\epsilon}. \tag{10}$$

This family retains H_{src} at every finite duration. Its interaction strength grows as ϵ^{-1}; it is a specified ideal-control limit, not a claim of unlimited laboratory bandwidth or a bounded-energy realization of exact projectivity.

Theorem 3.2 (Calibration with the full source Hamiltonian retained)**.** *Every H_ϵ is self-adjoint on the uncoupled domain and lower bounded. Define the isometries*

$$V_\epsilon\psi = U_\epsilon(\psi \otimes |0\rangle), \qquad V_0\psi = W(\psi \otimes |0\rangle) = \sum_{n=0}^{M} P_n\psi \otimes |n\rangle. \tag{11}$$

For $\psi \in \mathcal{D}$, put

$$C_\psi = \sum_{n=0}^{M} \|H_{\mathrm{src}}P_n\psi\| + \|H_A\|\,\|\psi\|. \tag{12}$$

Then

$$\|(V_\epsilon - V_0)\psi\| \le \epsilon C_\psi. \tag{13}$$

In particular, $V_\epsilon \to V_0$ strongly on all of $\mathcal{H}$. With

$$K_{r,\epsilon} = (I \otimes \langle r|)V_\epsilon, \tag{14}$$

the finite instruments are complete for all d labels and converge, for every fixed normal input state, to the N_F-Lüders instrument. Set $P_r = 0$ when $r > M$ and define the labelled operations

$$\mathfrak{I}_\epsilon(\rho) = \sum_{r=0}^{d-1} K_{r,\epsilon} \rho K_{r,\epsilon}^* \otimes |r\rangle\langle r|, \tag{15}$$

$$\mathfrak{L}(\rho) = \sum_{r=0}^{d-1} P_r \rho P_r \otimes |r\rangle\langle r|. \tag{16}$$

For every density operator ρ,

$$\|\mathfrak{I}_\epsilon(\rho) - \mathfrak{L}(\rho)\|_1 \longrightarrow 0. \tag{17}$$

For normalized $\psi \in \mathcal{D}$, the trace-norm error is at most $2\epsilon C_\psi$, and the total-variation distance between the labelled weights is at most ϵC_ψ. If $P_n\psi = \psi$, the wrong-label weight is at most $\epsilon^2 C_\psi^2$.

Proof. Bounded perturbation gives the first assertion. Write $H_f = H_{\mathrm{src}} \otimes I + I \otimes H_A$. For $0 \le s \le 1$,

$$e^{-isV}(\psi \otimes |0\rangle) = \sum_{n=0}^{M} P_n \psi \otimes e^{-isnB}|0\rangle. \tag{18}$$

This vector is in $D(H_f)$ for $\psi \in \mathcal{D}$, and the norm of its H_f image is bounded by C_ψ. Differentiating the mixed propagator on this domain gives the Duhamel identity

$$(e^{-i(\epsilon H_f + V)} - e^{-iV})\xi = -i\epsilon \int_0^1 e^{-i(1-s)(\epsilon H_f + V)} H_f e^{-isV} \xi \, ds \tag{19}$$

for $\xi = \psi \otimes |0\rangle$. It proves (13). Only the bounded-gate orbit must preserve the source domain. Global invariance of $\mathcal{D}$ under e^{-itH} is not needed. Density of $\mathcal{D}$ and the unit norm of the isometries give strong convergence for all ψ.

For a rank-one operator $|u\rangle\langle v|$, strong convergence and uniform boundedness give $V_\epsilon |u\rangle\langle v| V_\epsilon^* \to V_0 |u\rangle\langle v| V_0^*$ in trace norm. Finite-rank approximation and trace-norm contraction extend this to every trace-class input. Apply the completely positive trace-preserving map which reads the receiver sectors into a classical label and retains the corresponding source blocks. This proves (17); its finite-ϵ output includes every receiver label, even when its ideal projector vanishes.

For unit vectors, $\||a\rangle\langle a| - |b\rangle\langle b|\|_1 \le 2\,\|a - b\|$. Contractivity gives the stated labelled trace bound, and classical total variation is half the ℓ^1 distance. When $P_n\psi = \psi$, the ideal isometry has receiver label n with certainty. Projecting the vector error onto all other receiver labels gives squared norm at most $\epsilon^2 C_\psi^2$. □

The bound is state dependent. No unrestricted diamond-norm convergence, uniform all-energy approximation or finite-ϵ exact projection is claimed. The source Hamiltonian can mix the P_n sectors during the pulse; this is precisely why all d outcomes remain part of the finite instrument. The apparatus basis is fixed by the physical number coupling and the specified shift reader. It is not selected afterwards from the observed answer.

The physical coupling, ready state and sector reader define the apparatus basis. Under a passive unitary source presentation T, transporting all these data gives $K_r' = T K_r T^*$ and leaves the weights unchanged. An active change of a prepared state or coupling is a different experiment. The instrument's classical label is a mathematical output coordinate. Its coherent dilation does not, by construction alone, assert one actual original-run occurrence; Section 13 states that commitment.

The number-sensitive source extension and the material converter serve different domains. The former couples the constructed interacting source through the explicitly added N_F interaction. A transfer of a specified bosonic zero–one code to a material memory preserves that code's quantum state, but does not thereby implement an arbitrary N_F-controlled source adapter. Any identification of those inputs must carry its own physical map, preparation and error bound.

3.3 Coherent memories and the record carrier

The receiver instrument admits a further constructive extension: its channels can be correlated with several initially blank memories and retained while the source continues to evolve. This section gives the control Hamiltonians and proofs for any finite receiver alphabet. The memories are finite gauge-singlet model systems. Their introduction, the timed control and their storage isolation are additional apparatus specifications; they are not consequences that CSP-P has already proved for a macroscopic Standard Model detector. No commutation of H_{src} with N_F is used.

3.3.1 A common controlled-copying protocol

Let R be a finite nonempty alphabet and $\mathcal{H}_A = \operatorname{span}\{|r\rangle : r \in \mathsf{R}\}$, with $A_r = |r\rangle\langle r|$. Write $\mathcal{H}_{\text{phys}}$ for the source Hilbert space. Let $U_{\text{acq}}(t)$ be the specified acquisition propagator, with a fixed normalized ready receiver vector $|a_*\rangle$, and put

$$K_r(t) = (I \otimes \langle r|)U_{\text{acq}}(t)(I \otimes |a_*\rangle), \qquad w_r(\rho) = \operatorname{Tr}(K_r(t)\rho K_r(t)^*)\,. \tag{20}$$

Here ρ is a density operator and $\sum_r K_r^* K_r = I$. The binary source construction uses $\mathsf{R} = \{0, 1\}$, $|a_*\rangle = |0\rangle$ and

$$H_{\text{acq}} = H_g = H_{\text{src}} \otimes I_A + I \otimes \nu|1\rangle\langle 1| + gN_F \otimes X, \qquad U_{\text{acq}}(t) = e^{-itH_{\text{acq}}}.$$

The construction below also applies to the finite-alphabet receiver of Theorem 3.2. The acquisition time, subsequent controls and ready states are common to the compared preparations.

Take $m \geq 1$ memories, each with orthonormal basis $\{|b\rangle\} \cup \{|r\rangle : r \in \mathsf{R}\}$. The blank symbol b is distinct from every legitimate outcome, including outcome 0 when present. Set $\mathcal{H}_M = (\mathbb{C}^{|\mathsf{R}|+1})^{\otimes m}$ and $|r^m\rangle = |r\rangle^{\otimes m}$. For each r, let S_r interchange $|b\rangle$ and $|r\rangle$, fixing all other basis vectors. Let $S_r^{(k)}$ act on memory k. For a copying interval $\tau > 0$, define

$$G_{\text{copy}} = \frac{\pi}{2\tau} \sum_{r \in \mathsf{R}} \sum_{k=1}^{m} A_r \otimes (I_M - S_r^{(k)}). \tag{21}$$

During this interval switch off the acquisition coupling and use

$$H_{\text{copy}} = H_{\text{src}} \otimes I_{AM} + I \otimes D_A \otimes I_M + I \otimes G_{\text{copy}}, \qquad D_A = \sum_{r \in \mathsf{R}} \epsilon_r A_r. \tag{22}$$

Thus the full microscopic source Hamiltonian remains active. The receiver's free Hamiltonian is diagonal in the copied basis; it may also be switched off by taking $D_A = 0$. The switching times are externally specified; an autonomous switching clock is not constructed here. All additions to H_{src} are bounded and self-adjoint, so the copying generator is self-adjoint on $D(H_{\text{src}} \otimes I_{AM})$ and bounded below.

Theorem 3.3 (Exact coherent copying)**.** *The copying pulse satisfies*

$$e^{-i\tau G_{\text{copy}}} = C, \qquad C = \sum_{r \in \mathsf{R}} A_r \otimes \prod_{k=1}^{m} S_r^{(k)}.$$

Starting with $\rho \otimes |a_*\rangle\langle a_*|_A \otimes |b^m\rangle\langle b^m|_M$, acquisition followed by copying gives

$$\begin{aligned} \Gamma_{\text{copy}}(\rho) &= \sum_{r,u \in \mathsf{R}} L_r \rho L_u^* \otimes |r\rangle\langle u|_A \otimes |r^m\rangle\langle u^m|_M, \\ L_r &= e^{-i\tau H_{\text{src}}} e^{-i\tau\epsilon_r} K_r(t). \end{aligned} \tag{23}$$

The joint memory law assigns weight $w_r(\rho)$ to r^m and zero to every other string. Every memory therefore has the channel weights $w_r(\rho)$, and their quantum joint law gives agreement with probability one. The physically completed source channel is L_r, including the source evolution during copying.

Proof. Each $S_r^{(k)}$ is a self-adjoint unitary with eigenvalues $1, -1$. Consequently $e^{-i\pi(I-S_r^{(k)})/2} = S_r^{(k)}$. All summands in (21) commute. For fixed r, different summands act on different memories; summands with different receiver labels have orthogonal receiver supports. Exponentiation in each receiver sector gives C, which maps $|r\rangle|b^m\rangle$ to $|r\rangle|r^m\rangle$.

The source, diagonal receiver, and control terms in (22) commute. The propagator is their product. Acquisition sends a source vector ψ to $\sum_r K_r(t)\psi \otimes |r\rangle$. Applying that product proves (23) for pure inputs; linearity and trace-norm continuity give it for every density operator. Projection onto string r^m has trace $\mathrm{Tr}(L_r \rho L_r^*) = w_r(\rho)$. All other strings are orthogonal to the displayed support. This proves normalization and the complete joint agreement statement. □

The protocol copies orthogonal receiver labels. It does not clone an unknown quantum state: superpositions of labels become correlated superpositions. The joint support statement is stronger than equality of memory marginals. Equal marginals permit disagreeing couplings; here every non-code string is excluded by the explicitly constructed support. The binary example requires qutrit memories, whose blank state is not identified with a detector outcome. All registers transform trivially under the source gauge group, so the added control is gauge invariant. This finite model specifies the apparatus algebra and its allowed interactions; a local material realization of those controls is a further physical task. The pulse also has a definite resource scale: $\|G_{\mathrm{copy}}\| = m\pi/\tau$. Indeed, in each receiver sector every $I - S_r^{(k)}$ has eigenvalues zero and two, and the simultaneous maximal eigenvalue is $2m$. Exact copying at fixed m, τ therefore uses a bounded control, but the limit $\tau \to 0$ does not keep that control budget fixed.

3.3.2 Storage while the interacting source continues

For $\boldsymbol{a} \in (\{b\} \cup \mathsf{R})^m$, put $Q_{\boldsymbol{a}} = I_{\mathrm{phys},A} \otimes |\boldsymbol{a}\rangle\langle \boldsymbol{a}|_M$. Let D_M be self-adjoint and diagonal in this memory basis. After copying, use

$$H_{\mathrm{st}} = H_{SA} \otimes I_M + I_{\mathrm{phys},A} \otimes D_M, \tag{24}$$

where H_{SA} is a specified self-adjoint source–receiver Hamiltonian. In particular, it may be the entire original H_{acq}, including $gN_F \otimes X$. The receiver may change its sector during storage; the stored memories need not follow it.

Theorem 3.4 (Exact retention and a perturbation bound)**.** *Under* (24)*, every* $Q_{\boldsymbol{a}}$ *is conserved for all storage times. The weights and joint agreement of Theorem 3.3 remain exact. If a bounded self-adjoint perturbation* R *is added, then for any density operator* Γ *and record projection* $Q_{\boldsymbol{a}}$*,*

$$\begin{aligned} &\left|\mathrm{Tr}\Big(e^{-is(H_{\mathrm{st}}+R)}\Gamma e^{is(H_{\mathrm{st}}+R)}Q_{\boldsymbol{a}}\Big) - \mathrm{Tr}(\Gamma Q_{\boldsymbol{a}})\right| \\ &\qquad \le \min\{1,\ |s|\,\|[R, Q_{\boldsymbol{a}}]\|\} \le \min\{1,\ 2|s|\,\|R\|\}. \end{aligned} \tag{25}$$

Proof. Each record projection reduces H_{st}: it commutes with the source–receiver term by tensor separation and with D_M by diagonality. It therefore commutes with the storage unitary. No stationarity assumption on the source or receiver is needed.

For the perturbed dynamics pass to the interaction picture, with bounded interaction $R_I(u) = e^{iuH_{\mathrm{st}}}Re^{-iuH_{\mathrm{st}}}$. Since $Q_{\boldsymbol{a}}$ commutes with the unperturbed propagator, $\|[R_I(u), Q_{\boldsymbol{a}}]\| = \|[R, Q_{\boldsymbol{a}}]\|$. The Duhamel integral for the Heisenberg projection bounds its change in operator norm by $|s|\|[R, Q_{\boldsymbol{a}}]\|$. Taking a density-operator expectation proves the time-dependent bound. The commutator inequality proves the second, and two probabilities differ by at most one. Boundedness imposes no energy-domain condition on Γ. □

Exact retention is an explicit ideal isolation result. It is reversible and does not establish thermodynamic irreversibility, protection against arbitrary memory interactions, or a material error-correcting mechanism. Equation (25) quantifies a specified isolation error. More generally, source–receiver generators may depend on the memory string, provided the storage generator remains self-adjoint and block diagonal in that string. The same argument then preserves all record projections.

3.3.3 The record state and its unique character measure

Put $Q_r = Q_{r^m}$ and $Q_{\text{code}} = \sum_{r\in\mathsf{R}} Q_r$. The constructed state is supported on the invariant code subspace $Q_{\text{code}}(\mathcal{H}_{\text{phys}} \otimes \mathcal{H}_A \otimes \mathcal{H}_M)$. On that subspace define the finite commutative record algebra

$$\mathcal{A}_{\text{rec}} = \left\{ \sum_{r\in\mathsf{R}} f(r) Q_r : f : \mathsf{R} \longrightarrow \mathbb{C} \right\}.$$

Its unit Q_{code} is the identity on the code subspace. Its characters are the evaluations $\chi_r(f) = f(r)$.

Theorem 3.5 (Character measure and conditional state)**.** *The restriction of the copied state, or its unperturbed stored evolution, to $\mathcal{A}_{\text{rec}}$ has the unique probability representation*

$$\mu_\rho = \sum_{r\in\mathsf{R}} w_r(\rho)\delta_{\chi_r}. \tag{26}$$

On the larger algebra of source–receiver observables with diagonal record labels, the state disintegrates into unique conditional density operators on each positive-weight sector. No conditional state is fixed on a zero-weight sector.

Proof. Let $D_M|r^m\rangle = E_r|r^m\rangle$, and define

$$F_r(s)\psi = e^{-isE_r} e^{-isH_{SA}}(L_r\psi \otimes |r\rangle).$$

The stored global state is

$$\Gamma_s(\rho) = \sum_{r,u\in\mathsf{R}} F_r(s)\rho F_u(s)^* \otimes |r^m\rangle\langle u^m|.$$

Unitarity gives $\mathrm{Tr}(F_r\rho F_r^*) = w_r$. Thus the expectation of a record function f is $\sum_r w_r f(r)$. Any character evaluates exactly one of the orthogonal minimal projections as one and the others as zero: their values are idempotents and sum to one. These are all the characters. Evaluating a representing measure on those projections fixes each mass to w_r, proving existence and uniqueness.

For $w_r > 0$, put $\sigma_r(s) = F_r(s)\rho F_r(s)^*/w_r$. Every bounded block-diagonal observable $O = \sum_r O_r \otimes |r^m\rangle\langle r^m|$ satisfies

$$\mathrm{Tr}(\Gamma_s O) = \sum_{r:w_r>0} w_r \,\mathrm{Tr}(\sigma_r(s) O_r).$$

Testing all bounded O_r determines the positive-weight conditional density operator uniquely. A zero-weight block contributes nothing. □

Immediately after copying the conditional source state is $L_r\rho L_r^*/w_r$: the receiver's selective operation followed by the active source evolution. During storage the appropriate conditional state is $\sigma_r(s)$; its source marginal is obtained by tracing out the receiver. This disintegration concerns a restriction of the quantum state. It does not reweight the channels or replace the global state by a collapsed mixture. For the storage Hamiltonian (24), the reduced memory density operator itself is exactly

$$\mathrm{Tr}_{\text{phys},A}\,\Gamma_s(\rho) = \sum_r w_r(\rho)|r^m\rangle\langle r^m|.$$

To see this, the common source–receiver unitary cancels in $F_u(s)^*F_r(s)$; the remaining receiver inner product vanishes for $r \neq u$. Thus ordinary partial tracing already removes the off-diagonal memory terms in this model. This exact reduced-state statement strengthens the operational record description while leaving the global coherence identified in Theorem 13.3 intact. It does not depend on an approximation in which overlapping wave packets are declared orthogonal.

Code retention and accurate value retention are distinct. With two valid strings, the perturbation $R = \Omega(|0^m\rangle\langle 1^m| + |1^m\rangle\langle 0^m|)$ commutes with Q_{code} yet transfers weight between the labels. The individual commutator bounds in (25) must therefore accompany any leakage bound. When a perturbation permits blank or disagreeing strings, the complete alphabet is $(\{b\} \cup \mathsf{R})^m$; its projections sum to the identity on the full memory carrier. The code algebra alone is not a normalized description of those additional branches.

4 A positive circuit for material registration

The source construction and the detector construction have different jobs. The former supplies a physical state space, Hamiltonian and receiver observable. The latter supplies a material mechanism that responds to a prepared record. We use a six-mode Josephson circuit to expose its couplings, preparation, output ports and finite approximation errors. A photon-conversion detector is a concrete precedent for this mechanism [15]; the two-label network and its source interface below are specified theoretical constructions, not reported experimental devices.

4.1 Retained source, encoded memory and apparatus domain

This stored-record problem is distinct from acquiring the raw, generally nonconserved source observable N_F in the receiver construction above. The projections Q_j below act on the post-acquisition source–memory storage carrier; they are not spectral projections of the raw N_F. The original interacting H_{src} remains active in the declared storage extension. A distinct notation H_{ret} denotes that retained block-diagonal source–memory action, and reduction of Q_j is an interface premise rather than an assertion that $[H_{\mathrm{src}}, N_F] = 0$. Let H_{ret} be the retained self-adjoint source/storage Hamiltonian on $\mathcal{K}$, and let Q_0, Q_1 be orthogonal reducing projections, $Q_0 + Q_1 = I$. Write $U_{\mathrm{ret}}(t) = e^{-itH_{\mathrm{ret}}}$. Reduction means commutation with the spectral measure of H_{ret}, not merely a formal commutator on selected vectors. In a memory oscillator m, the proposed source-preserving encoding is

$$J_{\mathrm{enc}}\psi = \sum_{j=0}^{1} Q_j\psi \otimes |j_m\rangle, \qquad (I \otimes |j\rangle\langle j|)J_{\mathrm{enc}} = J_{\mathrm{enc}}Q_j. \tag{27}$$

It intertwines the retained source evolution. Its physical preparation is an interface obligation: a mathematically specified isometry is not already a source-to-chip interaction. After preparation we retain m and identify the encoded subspace through $J_{\mathrm{enc}}^{\dagger}$. Tracing m would erase cross-sector coherences and is not that identification. A concrete transfer from an already available bosonic receiver is constructed in Section 6; it uses a transferred-carrier identification and does not prove every instance of (27).

Proposition 4.1 (Inherited source domain)**.** *If a finite apparatus extension has interaction $V = \sum_j Q_j \otimes B_j$ with bounded self-adjoint B_j, then $H_{\mathrm{ret}} \otimes I + V$ is self-adjoint on the inherited source domain. Its evolution preserves that domain and factors into the source unitary and finite sector-controlled evolution. Arbitrary source coherences and ancillary entanglement are allowed.*

Proof. Bounded self-adjoint perturbation gives self-adjointness on the original domain. The reducing projections imply strong commutation of the source Hamiltonian with V. Their spectral calculus therefore factors the unitary, while each finite sector evolves by its bounded B_j. □

This proposition concerns the retained finite interface. The exact cosine detector need not preserve the finite memory code; its finite-interval comparison later includes that leakage. Source acquisition, buffer loading and code preparation must precede the prepared-input theorems or be included as preceding instruments.

4.2 Positive quadratic network and nonlinear coefficients

Set $\hbar = 1$. Let φ_i, N_i be canonical node phase and charge, with $[\varphi_i, N_k] = i\delta_{ik}$. Consider

$$H_{\mathrm{cir}} = 4E_C N^T N + \tfrac{1}{2}\varphi^T E_L \varphi - \sum_i E_{Ji} \cos\varphi_i + H_{\mathrm{drive}}(t), \qquad E_C, E_{Ji} > 0. \tag{28}$$

For an orthogonal matrix O and positive frequencies ω_ℓ, choose

$$E_L(s) = O\,\mathrm{diag}\left(\frac{\omega_\ell^2}{8sE_0}\right) O^T - \mathrm{diag}(E_{Ji}), \qquad E_C = sE_0. \tag{29}$$

When $E_L(s) > 0$, the exact quadratic confinement plus bounded cosine potential gives a semibounded self-adjoint Hamiltonian. A smooth linear drive is treated through its exact forced-oscillator transformation. The network permits oriented mutual inductors or transformers; it is not asserted to be a conventional sparse transmon layout.

The normal modes are ordered $(m, b, f_0, w_0, f_1, w_1)$: memory, loaded buffer, two flags and two waste modes. Write

$$\varphi_i = \sqrt{s}\sum_\ell h_{i\ell}(a_\ell + a_\ell^\dagger), \qquad h_{i\ell} = 2\sqrt{E_0/\omega_\ell}\,O_{i\ell}. \tag{30}$$

Participation-based circuit quantization motivates this organization of the physical coefficients [16, 17]. Expanding the cosine to quartic order and retaining number-diagonal monomials gives

$$H_K = -\frac{1}{2}\sum_\ell \alpha_\ell n_\ell(n_\ell - 1) - \sum_{\ell<k}\chi_{\ell k} n_\ell n_k, \tag{31}$$

$$\alpha_\ell = \frac{s^2}{2}\sum_i E_{Ji} h_{i\ell}^4, \qquad \chi_{\ell k} = s^2 \sum_i E_{Ji} h_{i\ell}^2 h_{ik}^2. \tag{32}$$

Scalar and linear frequency shifts are retained in the calibrated frequencies. These coefficients are constrained by the same circuit: Cauchy–Schwarz gives $\chi_{\ell k}^2 \le 4\alpha_\ell\alpha_k$. They are not independently adjustable abstract registration constants.

We prescribe an exact two-quadrature force on each flag normal mode,

$$H_{\mathrm{drive}} = \sum_r (\epsilon_r f_r^\dagger + \epsilon_r^* f_r), \qquad \epsilon_r = (\omega_{pr} - \omega_{f_r})\xi_r e^{-i\omega_{pr}t}. \tag{33}$$

With matched initial displacement, the harmonic solution is $\alpha_r(t) = \xi_r e^{-i\omega_{pr}t}$; other normal modes have zero classical displacement. The node force addresses $O_{:f_r}$, and the waste ports address the separate w_r coordinates. This selective control is a declared engineering input. A position-only force with an arbitrary envelope does not give (33) without additional counter-rotating terms and transfer-function calibration.

The resonant quartic conversion is

$$H_{\mathrm{conv}} = \sum_r \left(g_{D,r} e^{-i\omega_{pr}t} b f_r^\dagger w_r^\dagger + \mathrm{h.c.}\right), \qquad g_{D,r} = -s^2\xi_r \sum_i E_{Ji} h_{ib} h_{if_r}^2 h_{iw_r}. \tag{34}$$

This is a physical oscillator interaction; it contains no inserted logical Q_r multiplier. Memory-dependent detuning supplies selectivity. A single-channel pump conversion and waste elimination have an experimental antecedent in [15]; the additional memory and resolved two-channel construction require their own calibration.

4.3 Active star, terminal states and phase-faithful energies

Let A contain one buffer excitation and vacuum flags/waste, let C_r contain flag r and waste r excitations with empty buffer, and let D_r contain flag r with empty waste. An escape state E retains a separate failure label and has no flag. Memory occupation is $j = 0, 1$. In the retained conversion Hamiltonian, $n_b + n_{f_0} + n_{f_1}$ and each $n_{f_r} - n_{w_r}$ are conserved before loss. Hence the active space is $\{A, C_0, C_1\}$ and the terminals are D_0, D_1, E. This invariance is exact for the retained model, not for the full cosine.

In the pump frame, use raw energies

$$\begin{aligned} \alpha_j &= -\chi_{mb} j, & c_{rj} &= \Delta_r - \zeta_r r - (\chi_{mf_r} + \chi_{mw_r}) j, \\ d_{rj} &= -\chi_{mf_r} j, & \zeta_r &= \chi_{mb} - \chi_{mf_r} - \chi_{mw_r}. \end{aligned} \tag{35}$$

A specified escape energy d_{Ej} is also retained. Thus $c_{rj} - \alpha_j = \Delta_r + \zeta_r(j - r)$, but subtracting α_j only inside the active block would lose conditional phases relative to the terminals. The pump settings are fixed by spectroscopy before the unknown input is supplied. We do not tune a different Hamiltonian after learning j.

Independent waste ports have operators $L_r = \sqrt{\kappa_r}|D_r\rangle\langle C_r|$, and loaded-buffer escape has $L_E = \sqrt{\beta_b}|E\rangle\langle A|$. Keeping pumps enabled without replenishing the buffer allows one physical output in this model. Finite-temperature injection, memory/flag decay and leakage enlarge this alphabet and generator; they are included in the exact monitored model of Section 10, not silently set to zero there.

4.4 Controlled full-cosine reduction on a slow interval

The family (29) supplies a controlled existence statement independently of the later finite-point certificate. Its exact forced-harmonic transformation $G_s(t)$ puts every member in one Fock representation. The prepared laboratory code is $G_s(0)$ times the finite code, and the final laboratory frame includes $G_s(T)$. An abrupt drive turn-on from an undisplaced ready state is a different initial-value problem.

In the harmonic interaction frame write $\tau = s^2 t$ and

$$\begin{aligned} H_s(t) &= -\sum_i E_{Ji}\left[\cos(\sqrt{s}X_i(t,\tau)) + \tfrac{s}{2}X_i(t,\tau)^2 - I\right] = s^2 V(t,\tau) + R_s(t), \\ X_i &= \sum_\ell h_{i\ell}(e^{-i\omega_\ell t}a_\ell + e^{i\omega_\ell t}a_\ell^\dagger) + q_i(t,\tau)I, \\ V &= -\sum_i E_{Ji}X_i^4/24. \end{aligned} \tag{36}$$

The bounded real functions q_i contain the prescribed pumps and their slow frequency corrections. Scalar frame phases are retained or consistently removed. For total occupation at most N, let P_N be the Fock projection and set

$$\begin{aligned} x_i(n) &= \|h_i\|_2(\sqrt{n} + \sqrt{n+1}) + \sup|q_i|, \\ r_N &= \sum_i \frac{E_{Ji}}{720}\prod_{k=0}^{5} x_i(N+k), \qquad v_N = \sum_i \frac{E_{Ji}}{24}\prod_{k=0}^{3} x_i(N+k). \end{aligned} \tag{37}$$

Spectral calculus and the scalar cosine Taylor remainder give $\|R_s P_N\| \le s^3 r_N$ and $\|V P_N\| \le v_N$. The six factors in r_N follow because X_i maps the occupation-n space into occupation at most $n+1$. These are bounds on the exact cosine acting on finite-core vectors; $\cos(P_N X P_N)$ is not substituted for $P_N \cos(X) P_N$.

Decompose $V = V_0(\tau) + \sum_{\nu\neq 0} e^{i\nu t}V_\nu(\tau)$ with a positive discarded bare-frequency gap $\nu_{\min}$. Assume bounded coefficients and first slow derivatives on a fixed $[0, \tau_*]$. Choose a Fourier primitive B with fast derivative $\partial_t B = V - V_0$ and anchor $B(0,0) = 0$. Adjoint pairing makes B symmetric on the common finite-particle core. No self-adjoint realization or exponential of this generally unbounded quartic primitive is required. If frequencies are regrouped using physical shifted values, their discarded gap must be checked again; keeping slow phases in $V_\nu(\tau)$ instead charges their derivatives explicitly.

Let J_D embed the twelve memory/active/terminal states, of total occupation at most $N = 3$, and assume $V_0 J_D = J_D H_{\text{eff}}(\tau)$. Set

$$b = \sup \|BJ_D\|, \quad h = \sup \|H_{\text{eff}}\|, \quad b_\tau = \sup \|(\partial_\tau B)J_D\|, \quad c_4 = v_{N+4}b + bh + b_\tau. \tag{38}$$

All are finite-support norms; the Fourier coefficient norms and the nonzero gap bound them without assumptions on the actual state's moments.

Theorem 4.2 (Controlled circuit-to-code limit)**.** *Let U_s be the exact circuit interaction-frame propagator and $u(\tau)$ the finite propagator of H_{eff}. At $T = \tau/s^2$, $0 \le \tau \le \tau_*$,*

$$\|U_s(T,0)J_D - J_D u(\tau)\| \le s^2 b + \tau \left[s r_N + s^2 c_4 + s^3 r_{N+4} b \right]. \tag{39}$$

The bound is $O(s)$ on this fixed slow interval. Embedded channel distance, including arbitrary ancillary input, is at most twice the right-hand side, capped at two.

Proof. Use the finite-core trial $Y_s = (J_D - is^2 BJ_D)u(s^2 t)$. Differentiation gives

$$i\dot{Y}_s - H_s Y_s = \{s^4(\partial_\tau B)J_D + is^4(VBJ_D - BJ_D H_{\text{eff}}) - R_s J_D + is^2 R_s BJ_D\}u. \tag{40}$$

Its norm is at most $s^3 r_N + s^4 c_4 + s^5 r_{N+4} b$. Variation of constants for the exact unitary integrates this to $T = \tau/s^2$. The initial corrector is zero and removing the final one costs $s^2 b$. Both endpoint comparison maps are isometries, yielding the complete channel bound. All residuals act on J_D or BJ_D, of finite occupation support; no global negative-quartic Hamiltonian is evolved. □

For the monitored extension, assume a conservative full oscillator CPTP propagator, a common finite Fock-core generator domain and validity of variation of constants on the trial operators. Scale retained losses and electronic rates as s^2, and include a finite classical first-record register. Write $\mathcal{L}_s = s^2(\mathcal{L}_0 + \mathcal{L}_{\text{osc}}) + \mathcal{R}_s$, where $\mathcal{L}_{\text{osc}} = -i[V - V_0, \cdot]$ and $\mathcal{R}_s = -i[R_s, \cdot]$. With $\mathcal{E}(\rho) = J_D \rho J_D^\dagger$, $\mathcal{L}_0 \mathcal{E} = \mathcal{E}\mathcal{L}_{\text{eff}}$. Put $\mathcal{B} = -i[B, \cdot]$ and

$$\begin{aligned} b_{\mathcal{B}} &= \sup \|\mathcal{B}\mathcal{E}\|_\diamond \le 2b, \\ c_{\mathcal{B}} &= \sup \|(\partial_\tau \mathcal{B})\mathcal{E} + \mathcal{B}\mathcal{E}\mathcal{L}_{\text{eff}} - (\mathcal{L}_0 + \mathcal{L}_{\text{osc}})\mathcal{B}\mathcal{E}\|_\diamond. \end{aligned} \tag{41}$$

The trial $(\mathcal{E} + s^2 \mathcal{B}\mathcal{E})\mathcal{S}_{\text{eff}}(\tau)$ need not be positive. Complete trace-norm contraction and its finite-core residual nevertheless give

$$\|\mathcal{T}_s(T)\mathcal{E} - \mathcal{E}\mathcal{S}_{\text{eff}}(\tau)\|_\diamond \le s^2 b_{\mathcal{B}} + \tau \left[2 s r_N + s^2 c_{\mathcal{B}} + 2 s^3 b (r_{N+4} + r_N) \right]. \tag{42}$$

The last term follows by expanding the double commutator into four products. Every product has finite enlarged support. Reading the register and tracing unused modes are contractions. A fixed finite time partition can be stored too; an arbitrarily refined time-register limit requires uniform bounds. A formal unbounded Lindblad expression alone proves neither these domain conditions nor a physical Markov reservoir.

The scaling also increases the gate as s^{-2}. An omitted decay rate $\gamma(s)$ disappears only if $\gamma(s) = o(s^2)$, or its effect must be included. Existence of this limit does not establish a useful bound at one finite member. Section 10 supplies a separate, full-interval finite certificate using the exact cosine and a dressed comparison instrument.

4.5 One fixed apparatus and one numerical convention

The constructive frequency family uses

$$\omega = (4 + \sqrt{2}/4,\ 6 + \sqrt{3}/7,\ 5 + \sqrt{5}/20,\ 8 + \sqrt{7}/6,\ \tfrac{11}{2} + \sqrt{11}/10,\ 9 + \sqrt{13}/20),$$

$E_0 = 0.05$, $E_J = (8, 1, 12, 1.2, 15, 1.1)$ and the archived seven plane rotations defining O. The exact radical construction gives $E_L(s) > (40/s - 15)I$ for $0 < s \le 1$. Enumerating its 3876 quartic multisets leaves 36 static and four desired conjugate terms; the other bare frequency gaps exceed 0.0044. This establishes a positive example and the finite quartic resonance inventory, not a full-cosine truncation error by itself.

The finite certificate uses the archived binary64 circuit parameters and mode participations as exact serialized numbers; they are close to, but are not silently identified with, ideal radical/orthogonal values.

The rounding distinction has an exact material realization. Let h be that invertible archived matrix, $L = \sqrt{s}h$ and $\Omega = \mathrm{diag}(\omega_\ell)$. In this paragraph use $X = a + a^\dagger$, $P = i(a^\dagger - a)$, so $[X_i, P_j] = 2i\delta_{ij}$. Define the canonical node variables and charging matrix by

$$\varphi = LX, \quad N = \tfrac{1}{2}L^{-T}P, \quad 4\mathsf{E}_C = L\Omega L^T, \quad E_L = \tfrac{1}{2}L^{-T}\Omega L^{-1} - \mathrm{diag}(E_J). \tag{43}$$

Then $[\varphi_i, N_j] = i\delta_{ij}$ and $4N^T\mathsf{E}_C N + \frac{1}{2}\varphi^T(E_L + \mathrm{diag}\,E_J)\varphi = \frac{1}{4}(P^T\Omega P + X^T\Omega X)$ exactly. Thus the nearby matrix-charge network realizes precisely the archived mode participations, with no assumption that their floating-point construction is an exact orthogonal matrix. The ideal scalar-charge family and this frozen matrix-charge realization are distinguished.

Proposition 4.3 (Positive frozen network)**.** *At $s = 1/10$, the exact serialized h, ω, E_J give $4\mathsf{E}_C > 0$ and $E_L > 380I$ in* (43)*. Consequently its full cosine potential has positive quadratic confinement at the operating point used in the finite certificate.*

Proof. Every archived binary64 entry is an exact rational number. Rational Gaussian elimination verifies the inverse of h. Hence $sh\Omega h^T$ is positive because $\Omega > 0$. Exact rational LDL^T factorization of $(2s)^{-1}h^{-T}\Omega h^{-1} - \mathrm{diag}\,E_J - 380I$ has six positive pivots. The replay certificate records those fractions; rounded-down integer lower bounds are respectively $109, 467, 329, 1116, 528, 1477$. The identity and positive pivots are checked without floating arithmetic, proving strict positivity. Bounded cosine terms then preserve a semibounded self-adjoint Hamiltonian. This is a positive network specification, not evidence that its controls and ports have been fabricated and calibrated. □

The operating specification fixes

$$s = \frac{1}{10}, \qquad \kappa_0 = \kappa_1 = \frac{6517034527732037}{9444732965739290427392}, \qquad T_D = \frac{1875694529551769}{8388608}. \tag{44}$$

The quantum efficiency is 19/20, buffer loss $\beta_b = \kappa/1000$, and each electronic dark rate is $\nu_r = \kappa/10000$. Memory and both flag loss coefficients are 10^{-12} and thermal occupation at each of the six positive ports is 10^{-7}. The archived bare pump frequencies plus s^2 times the archived offsets $(-0.00870849782706314, -0.0077173592597816485)$ define the pumps; the printed decimal offsets abbreviate those exact binary64 values. Units are $H/(\hbar\Omega_0)$ and $t\Omega_0$, with the shared physical clock $\Omega_0 = 2\pi\,10^9\,\mathrm{s}^{-1}$. Thus the gate is about 35.5870791 ms. The serialized input and its hashes are part of the reproduction package. Its dressed finite target is defined in Section 10; the quartic star below is an analytical model and is not substituted for that target's calibrated phases.

5 Complete registration, latency and finite blanks

5.1 A two-state analytical benchmark

Before treating cross-talk, a single isolated conversion gives an exact latency law. On $\{A, C\}$, choose the phase of C so $g > 0$ and let

$$\dot{a} = -igb, \qquad \dot{b} = -iga - (\kappa/2 + i\Delta)b, \qquad a(0) = 1,\ b(0) = 0. \tag{45}$$

Define $d = \kappa/2 + i\Delta$ and $\Omega = (g^2 - d^2/4)^{1/2}$. Analytic continuation at $\Omega = 0$ gives

$$a(t) = e^{-dt/2}\left[\cos(\Omega t) + \frac{d}{2\Omega}\sin(\Omega t)\right], \qquad b(t) = -i\frac{g}{\Omega}e^{-dt/2}\sin(\Omega t). \tag{46}$$

The survival and output flux are $S = |a|^2 + |b|^2$ and $f = \kappa|b|^2 = -S'$. For $g, \kappa > 0$, both eigenvalues of the amplitude matrix have strictly negative real part: a non-decaying eigenvector would have $b = 0$, and its second equation then forces $a = 0$. Consequently $\int_0^\infty f = 1$. On resonance $\Omega^2 = g^2 - \kappa^2/16$; critical damping $g = \kappa/4$ gives $a = e^{-\kappa t/4}(1 + \kappa t/4)$, $b = -igte^{-\kappa t/4}$. The same analytic expression includes the overdamped case.

Proposition 5.1 (Exact mean and its material tradeoff)**.** *Within* (45)*,*

$$F(t) = \int_0^t f(v)\,dv = \kappa g^2 t^3/3 + O(t^4), \tag{47}$$
$$\mu = \int_0^\infty S(t)\,dt = \frac{2}{\kappa} + \frac{\kappa}{4g^2} + \frac{\Delta^2}{\kappa g^2}.$$

At fixed g, Δ, μ is minimized by $\kappa_* = 2\sqrt{2g^2 + \Delta^2}$, with $\mu_* = \sqrt{2g^2 + \Delta^2}/g^2$.

Proof. The initial amplitude $b = -igt + O(t^2)$ gives the onset. Put $X = \int |a|^2$, $Y = \int |b|^2$ and $Z = \int a^* b$, with integrals over $[0, \infty)$. Norm balance gives $Y = 1/\kappa$, and integration of $(|a|^2)' = 2g\,\mathrm{Im}(a^* b)$ gives $\mathrm{Im}\,Z = -1/(2g)$. The integrated coherence equation is $(\kappa/2 + i\Delta)Z = ig(Y - X)$, implying $X - Y = (\Delta^2 + \kappa^2/4)/(\kappa g^2)$. Integration by parts gives $\mu = X + Y$. Minimizing its positive κ and $1/\kappa$ terms proves the final claim. □

The optimization is meaningful only where the same physical bath and conversion model remain valid. Cubic onset is not asserted below the bath correlation time. It differs from an exact exponential at arming. Indeed, with $g = \varepsilon g_0$ and fixed κ, Δ, at $t = u/\varepsilon^2$, $u > 0$,

$$S(t) \longrightarrow e^{-\lambda_0 u}, \qquad \varepsilon^{-2} f(t) \longrightarrow \lambda_0 e^{-\lambda_0 u}, \qquad \lambda_0 = \frac{g_0^2 \kappa}{\Delta^2 + (\kappa/2)^2}. \tag{48}$$

The amplitude roots are $-\varepsilon^2 g_0^2/d + O(\varepsilon^4)$ and $-d + O(\varepsilon^2)$, proving the limits; the density limit excludes $u = 0$. The effective rate's reciprocal accounts for only part of the exact mean, $\mu = \lambda_{\text{eff}}^{-1} + 2/\kappa$.

For an ideal sector-controlled benchmark, channel r has amplitudes a_r, b_r only on Q_r. With $A_T = \sum_r a_r(T) Q_r$, the complete reduced no-count map is

$$\mathcal{N}_T(\Gamma) = U_{\text{ret}}(T)\left[A_T \Gamma A_T^\dagger + \sum_r |b_r(T)|^2 Q_r \Gamma Q_r\right] U_{\text{ret}}(T)^\dagger. \tag{49}$$

The output branch is $F_r(T) U_{\text{ret}}(T) Q_r \Gamma Q_r U_{\text{ret}}(T)^\dagger$. Tracing the common A amplitude and orthogonal C_r amplitudes proves these formulas; the effects sum to $\sum_r (S_r + F_r) Q_r = I$. Matched parameters give joint law $w_r f(t)$, $w_r = \mathrm{Tr}(Q_r \Gamma)$; unequal parameters give completed-run proportions $w_r F_r(T)/\sum_j w_j F_j(T)$, even though eventual weights are w_r. The physical two-channel circuit does not have this ideal Q_r control: its finite selectivity must instead be calculated from the next model.

5.2 The detuned star and its coherent instrument

For each memory j, define the amplitude generator

$$\mathsf{A}_j = -i \begin{pmatrix} \alpha_j & g_{D,0}^* & g_{D,1}^* \\ g_{D,0} & c_{0j} & 0 \\ g_{D,1} & 0 & c_{1j} \end{pmatrix} - \tfrac{1}{2}\,\mathrm{diag}(\beta_b, \kappa_0, \kappa_1), \qquad (a_j, b_{0j}, b_{1j})^T = e^{t\mathsf{A}_j}(1,0,0)^T. \tag{50}$$

Define diagonal source operators $A(t) = \sum_j a_j(t)Q_j$, $B_r(t) = \sum_j b_{rj}(t)Q_j$ and $W_r(v) = \sum_j e^{-id_{rj}v}Q_j$, with W_E defined similarly. The full active no-output embedding at deadline T is

$$V_T\psi = \sum_j U_{\mathrm{ret}}(T)Q_j\psi \otimes \left[a_j(T)|A\rangle + \sum_r b_{rj}(T)|C_r\rangle\right]. \tag{51}$$

The terminal output and escape densities have source operators

$$K_r(T,t) = U_{\mathrm{ret}}(T)W_r(T-t)\sqrt{\kappa_r}B_r(t), \qquad K_E(T,t) = U_{\mathrm{ret}}(T)W_E(T-t)\sqrt{\beta_b}A(t). \tag{52}$$

Tensor each with the physical terminal $|D_r\rangle$ or $|E\rangle$ for the joint source–apparatus output. This convention retains memory, source and flag coherences until the specified later reader.

Theorem 5.2 (Complete phase-faithful material instrument)**.** *The maps $\Gamma \mapsto V_T\Gamma V_T^\dagger$ and the terminal maps $\int_0^T K_r\Gamma K_r^\dagger dt$, $\int_0^T K_E\Gamma K_E^\dagger dt$, with their terminal records retained, are CP and sum to a trace-preserving channel. After tracing only the internal detector configurations, the active map is*

$$U_{\mathrm{ret}}(T)\left[A(T)\Gamma A(T)^\dagger + \sum_r B_r(T)\Gamma B_r(T)^\dagger\right]U_{\mathrm{ret}}(T)^\dagger. \tag{53}$$

A wrong physical channel can retain coherent contributions from both j sectors. Its conditional state is determined by the diagonal filter in (52)*; it is generally not $Q_r\Gamma Q_r$.*

Proof. Every displayed branch is a Kraus map or integral of Kraus maps. For $S_j = |a_j|^2 + \sum_r |b_{rj}|^2$, the Hermitian part of (50) gives

$$-S_j' = \beta_b|a_j|^2 + \sum_r \kappa_r|b_{rj}|^2. \tag{54}$$

Since $S_j(0) = 1$, each diagonal entry of the summed effect is one. The effects are diagonal in the orthogonal Q_j, hence their sum is I also for arbitrary coherent or reference-entangled input. Tracing the orthogonal active configurations proves (53). The terminal phases and the two nonzero b_{rj} coefficients prove the conditional-state assertion. Unknown emission times are integrated as CP maps, not summed as coherent amplitudes. □

Proposition 5.3 (Absorption, exact tails and finite selectivity)**.** *If both $\kappa_r > 0$ and either $\beta_b > 0$ or at least one $g_{D,r} \neq 0$, then every A_j is Hurwitz. For $\beta_b = 0$ the two physical output probabilities sum to one at infinite time. At finite detuning, a nonzero wrong coupling cannot give an exact projective detector:*

$$\int_0^t \kappa_r|b_{rj}(v)|^2 dv = \kappa_r|g_{D,r}|^2 t^3/3 + O(t^4). \tag{55}$$

Proof. For an eigenvector with nonnegative eigenvalue real part, (54) forces both intermediate components to vanish, and also its A component when $\beta_b > 0$. Otherwise a nonzero $g_{D,r}$ forces that component to vanish through its C_r equation. No nonzero eigenvector remains, so all real parts are negative. Integration of norm balance proves absorption. The initial expansion $b_{rj} = -ig_{D,r}t + O(t^2)$ proves the last formula. □

The long-time probabilities and mean absorption times are finite-matrix quantities. Let $C_r = \kappa_r|C_r\rangle\langle C_r|$ and $C_E = \beta_b|A\rangle\langle A|$ here denote flux matrices on the active three-space. For any such C,

$$\mathsf{A}_j^\dagger X + X\mathsf{A}_j = -C, \quad X = \int_0^\infty e^{t\mathsf{A}_j^\dagger} C e^{t\mathsf{A}_j}\,dt. \tag{56}$$

The corresponding probability is $e_A^\dagger X e_A$ and its finite-time version replaces X by $X - e^{T\mathsf{A}_j^\dagger} X e^{T\mathsf{A}_j}$. For $C = I$, $P_j = X > 0$ gives the mean $e_A^\dagger P_j e_A$ and

$$S_j(T) \le \frac{e_A^\dagger P_j e_A}{\lambda_{\min}(P_j)} e^{-T/\lambda_{\max}(P_j)}. \tag{57}$$

Indeed the P_j quadratic form has derivative minus the ordinary squared norm. Cross-sector integrated conditional amplitudes are computed by the corresponding Sylvester equations including terminal energy differences. A convergent probability does not imply a stationary raw conditional phase.

In the separate weak-conversion limit $g_{D,r} = \varepsilon\bar{g}_r$, $\beta_b = \varepsilon^2\bar{\beta}_b$, the relative gaps $\delta_{rj} = c_{rj} - \alpha_j$ give slow population rates $\lambda_{rj} = |\bar{g}_r|^2\kappa_r/[(\kappa_r/2)^2 + \delta_{rj}^2]$. The slow amplitude eigenvalue is $-\varepsilon^2[\bar{\beta}_b/2 + \sum_r |\bar{g}_r|^2/(\kappa_r/2 + i\delta_{rj})] + O(\varepsilon^4)$ after the common phase. This proves the limiting rates and branching fractions, but not their accuracy at arbitrary finite coupling. It is distinct from the weak-nonlinearity scaling of Theorem 4.2.

5.3 Inefficiency, electronic dark events and first records

Let $\mathcal{L}$ be the full joint material generator and $\mathcal{J}_r(\rho) = L_r\rho L_r^\dagger$. Quantum efficiency η_r splits a physical output into observed and missed maps. Define $\mathcal{L}_0 = \mathcal{L} - \sum_r \eta_r\mathcal{J}_r$. Its propagator is CP and trace decreasing: it includes missed physical emission, escape and subsequent material evolution, while killing observed emissions. For independent electronic Poisson processes with constant rates ν_r, put $\nu = \nu_0 + \nu_1$ and let ρ_A be the prepared joint input. The instrument recording the *first electronic display* and retaining the material system to T is

$$\begin{aligned} \mathcal{I}_{r,dt}^T(\rho_A) &= e^{-\nu t} e^{(T-t)\mathcal{L}}(\eta_r\mathcal{J}_r + \nu_r\,\mathrm{id})e^{t\mathcal{L}_0}(\rho_A)\,dt, \\ \mathcal{I}_\varnothing^T(\rho_A) &= e^{-\nu T}e^{T\mathcal{L}_0}(\rho_A). \end{aligned} \tag{58}$$

Time-dependent intensities replace νt by their cumulative hazard and use the corresponding propagators. An equivalent finite classical register remains blank until its first jump to 0 or 1; all material dynamics continues after that jump.

Proposition 5.4 (Original-run completeness)**.** *The two time-resolved maps and the blank in* (58) *form a complete CP instrument. A dark event does not project the physical state into its displayed label. In particular, electronic label and later material flag must be retained jointly.*

Proof. The propagators and the observed gain maps are CP. Set $p_0(t) = e^{-\nu t}\,\mathrm{Tr}\,e^{t\mathcal{L}_0}\rho_A$. Trace preservation of $\mathcal{L}$ gives $-p_0'(t) = e^{-\nu t}\sum_r \mathrm{Tr}(\eta_r\mathcal{J}_r + \nu_r\,\mathrm{id})e^{t\mathcal{L}_0}\rho_A$. Integrating from zero to T proves completeness because $p_0(0) = 1$. Post-display propagation has unit trace and retains later physical transitions. The dark gain is $\nu_r\,\mathrm{id}$, not a flag projection, proving the last statements. □

In the absorbing star with no dark events, the blank explicitly contains active survival, escape and missed-emission branches. Multiplying a unit-efficiency output probability by η_r while discarding these branches would not describe the original runs. Similarly, the material flag already exists in C_r: without further flag decay its probability at T in sector j is $|b_{rj}(T)|^2 + \int_0^T \kappa_r|b_{rj}(t)|^2 dt$. The waste-emission probability omits the first term. Reading a flag during conversion specifies a different intervention.

Separate resolved ports are load-bearing. Replacing them by $L_{\text{common}} = \sum_r \sqrt{\kappa_r}|D_r\rangle\langle C_r|$ allows one unresolved count to retain coherent flag alternatives. Mixing output modes before counting also changes the selective Kraus maps, even when the unconditional dissipator is unchanged. The physical output measurement fixes the instrument [18]; a chosen unraveling or unmeasured bath dilation does not establish exclusive actuality. The first-record construction provides a probability-bearing quantum instrument. The argument for its event weights and the additional law of actual history are addressed separately below.

6 Finite physical handoffs and their retained phases

The detector's prepared code and final code are defined in a forced harmonic frame. A laboratory flag occupation is obtained only after the known displacement is removed or the subsequent coupling is transformed consistently. We give finite controls for that removal and a positive quadratic transfer for a specified input oscillator. These are coherent operations on their stated material domains. Their finite control, loss and source-isolation errors remain part of the composed residual.

6.1 Pump-off is an operation on the joint state

At the converter deadline T_D, the two flag displacements are $\alpha_{rD} = \xi_r e^{-i\omega_{pr}T_D}$. For a duration $\tau_H > 0$, let

$$r(u) = 1 - 10u^3 + 15u^4 - 6u^5, \qquad 0 \le u \le 1,$$

so $r(0) = 1$, $r(1) = 0$, $r' = r'' = 0$ at both endpoints and $\sup |r'| = 15/8$. Prescribe

$$\begin{aligned} \alpha_r(t) &= \xi_r e^{-i\omega_{pr}(T_D+t)} r(t/\tau_H), \\ \epsilon_r(t) &= \xi_r e^{-i\omega_{pr}(T_D+t)} \left[(\omega_{pr} - \omega_{f_r}) r(t/\tau_H) + \frac{i}{\tau_H} r'(t/\tau_H)\right]. \end{aligned} \tag{59}$$

Then $i\dot{\alpha}_r = \omega_{f_r}\alpha_r + \epsilon_r$ exactly. The pulse joins the enabled converter force at its beginning and ends with zero displacement and force. Its derivative term is necessary.

Proposition 6.1 (Coherent harmonic undressing)**.** *For the two-quadrature force in* (59)*, write* $D(\alpha) = \prod_r D_r(\alpha_r)$ *and* $R_0(t) = e^{-it\sum_\ell \omega_\ell n_\ell}$*. The forced harmonic propagator is*

$$W_h(t) = e^{i\vartheta(t)} D(\alpha(t)) R_0(t) D(\alpha_D)^\dagger, \qquad \dot{\vartheta} = -\operatorname{Re}\sum_r \alpha_r^* \epsilon_r. \tag{60}$$

Hence $W_h(\tau_H)D(\alpha_D) = e^{i\vartheta(\tau_H)}R_0(\tau_H)$*. It removes the harmonic pump displacement without measuring the flag, including for input entangled with source and reference systems.*

Proof. Differentiate (60), using $D(\alpha)nD(\alpha)^\dagger = (a^\dagger - \alpha^*)(a - \alpha)$. The differential equation for α cancels its linear remainder against the prescribed force. The displayed scalar phase cancels the remaining scalar. At the endpoint $D(\alpha) = I$. The identity is an operator identity and survives tensoring with an arbitrary reference. □

Finite control limits matter. The pulse obeys

$$\sup |\epsilon_r| \le |\xi_r| \left(|\omega_{pr} - \omega_{f_r}| + \frac{15}{8\tau_H} \right). \tag{61}$$

Together with $\sup |r''| = 10\sqrt{3}/3$, differentiation gives a finite slew bound. Specified amplitude and slew ceilings therefore restrict τ_H; no instantaneous undressing is assumed. A driven resonator reset is an experimental precedent [19], but it does not establish this coherent, flag-preserving operation or its nonlinear error. The independent charge and phase forces in (33) are required; a position-only port needs its own carrier approximation and calibrated response.

6.2 Exact diagonal cosine phases and finite ramp error

In the displacement frame set $\Phi_i = \sum_\ell h_{i\ell}(a_\ell + a_\ell^\dagger)$, $q_i(t) = 2\,\mathrm{Re}\sum_r h_{if_r}\alpha_r(t)$ and $q_i^{\max} = 2\sum_r |h_{if_r}\xi_r|$. Up to the known harmonic rotations, the exact nonlinear operator is

$$N_s(t) = -\sum_i E_{Ji}\left[\cos\{\sqrt{s}(\Phi_i + q_i(t))\} + \tfrac{s}{2}(\Phi_i + q_i(t))^2 - I\right]. \tag{62}$$

Let P project onto the twelve memory/material code states. The target keeps its exact number-diagonal matrix $D_s(t) = \sum_{n\in P} d_n(t)|n\rangle\langle n|$, where

$$\begin{aligned} d_n(t) = -\sum_i E_{Ji}\Bigg[&\cos(\sqrt{s}q_i)e^{-s\|h_i\|_2^2/2}\prod_\ell L_{n_\ell}(sh_{i\ell}^2) \\ &+ \frac{s}{2}\left(q_i^2 + \sum_\ell h_{i\ell}^2(2n_\ell+1)\right) - 1\Bigg]. \end{aligned} \tag{63}$$

The formula follows from $\langle n|e^{iz(a+a^\dagger)}|n\rangle = e^{-z^2/2}L_n(z^2)$ and factorization over modes. It evaluates the full cosine, rather than a cosine of a truncated phase matrix. The handoff target is therefore $R_0(\tau_H)\sum_n e^{-i\int_0^{\tau_H} d_n(t)dt}|n\rangle\langle n|$, with the initial displacement and retained source unitary included. Memory–flag Kerr, pump Stark and number phases are not discarded.

For any operator A define

$$\mathrm{off}_P(A) = AP - \sum_{n\in P}\langle n|A|n\rangle|n\rangle\langle n|.$$

This expression includes transitions out of the code. Let r_N be (37) with $\sup|q_i|$ replaced by $q_i^{\max}$, and set

$$C_4(P) = \sum_i \frac{E_{Ji}}{24}\sum_{k=1}^{4}\binom{4}{k}(q_i^{\max})^{4-k}\|\,\mathrm{off}_P(\Phi_i^k)\|. \tag{64}$$

Each displayed polynomial has finite occupation support, and its Frobenius norm is a computable upper bound for the operator norm.

Proposition 6.2 (Full-cosine finite handoff bound)**.** *For the exact cosine circuit and the diagonal-phase target on the specified displaced code, the channel distance obeys*

$$\epsilon_H^{\cos} \le \min\{2,\, 2\tau_H[s^2C_4(P) + 2s^3r_3]\}. \tag{65}$$

It requires neither a rotating-wave approximation during the ramp nor a moment bound on the actual evolved state.

Proof. Taylor remainder gives $N_s = s^2V_4 + R_s$, with $V_4 = -\sum_i E_{Ji}(\Phi_i+q_i)^4/24$ and $\|R_sP_N\| \le s^3r_N$. Removing the quartic diagonal costs at most $s^2C_4(P)$. Removing the remainder diagonal costs at most an additional s^3r_3, so $\|(N_s - D_s)P\| \le s^2C_4(P) + 2s^3r_3$. The ideal trial propagator has range in P. Variation of constants therefore bounds the isometry difference by the time integral of this finite-range residual, using only exact unitarity of the full evolution. Two times the isometry bound controls the channel distance with any reference. Harmonic number rotations preserve all relevant supports and norms. □

Known undisplaced waste and buffer losses can remain in the target: they preserve the flag and map C_r to D_r or A to E. Under the conservative propagator and common-domain assumptions of (42), complete trace-norm contraction gives the same Hamiltonian residual bound for that open target. This does not derive its bath approximation. Residual conversion during the ramp is included in $C_4(P)$; off-diagonal cosine terms after the ramp require a budget for any subsequent idle or routing interval.

A quadrature-force error adds the sufficient term

$$\epsilon_{\text{force}} \le 2 \int_0^{\tau_H} \sum_r |\delta\epsilon_r(t)|(\sqrt{3}+\sqrt{4})\, dt. \tag{66}$$

For an omitted physical jump L, a useful restricted estimate is

$$\|\mathcal{D}[L]\mathcal{E}_P\|_\diamond \le \|LP\|^2 + \|L^\dagger LP\|, \qquad \mathcal{E}_P(\rho) = J_P \rho J_P^\dagger. \tag{67}$$

The gain and two anticommutator terms prove the inequality. It does not assign a bounded all-space norm to an oscillator lowering operator. In this frame a driven flag-loss jump is proportional to $f_r + \alpha_r(t)$, whose displacement contribution must be retained. Natural flag loss changes the physical flag; it is not an electronic dark event. A process already included in the target is not charged again as an omitted error.

The explicit polynomial evaluation gives, approximately, $C_4(P) = 0.1495987865$ and $r_3 = 0.0353254910$ for the archived circuit and pump envelope. At $s = 0.1$ and dimensionless $\tau_H = 1$, (65) evaluates to about 0.003134. This is a floating evaluation of an analytic Hamiltonian-only upper expression. It is not an interval-certified aggregate handoff error, measured control tolerance or proof that a one-unit pulse satisfies physical bandwidth limits. The common composition leaves the full handoff residual explicit.

6.3 Positive quadratic transfer from a specified input oscillator

Assume that a physical receiver oscillator c already carries a qubit in $\text{span}\{|0\rangle, |1\rangle\}$, possibly entangled with an external reference. Let m and all other detector modes start in vacuum. During transfer, retained external evolution commutes with c, m; unwanted source couplings are disabled or independently bounded. This is an explicit input domain, not an identification of every source observable with one laboratory oscillator.

Use normalized quadratures $x_a = (a + a^\dagger)/\sqrt{2}$ and $p_a = (a - a^\dagger)/(i\sqrt{2})$, so $[x_a, p_a] = i$. Choose equal charge and flux matrices

$$K_x(t) = K_p(t) = \begin{pmatrix} \omega & g_{\text{in}}(t) \\ g_{\text{in}}(t) & \omega \end{pmatrix}, \qquad |g_{\text{in}}(t)| < \omega. \tag{68}$$

Then

$$H_2 = \tfrac{1}{2} x^T K_x x + \tfrac{1}{2} p^T K_p p = \omega(n_c + n_m + 1) + g_{\text{in}}(t)(c^\dagger m + c m^\dagger). \tag{69}$$

The pair terms cancel algebraically between the quadratures. With a positive impedance matrix Z, the physical matrices $C^{-1} = Z^{1/2} K_p Z^{1/2}$ and $L^{-1} = Z^{-1/2} K_x Z^{-1/2}$ are positive. The sign pattern can use positive shunt/coupling capacitors and an oriented mutual inductor or transformer. Matched time-dependent controls and selective coupling to normal mode m are declared requirements, not an inferred property of an arbitrary circuit.

Proposition 6.3 (Coherent finite input transfer)**.** *For $g_{\text{in}}(t) = g_* \sin^2(\pi t/\tau_{\text{in}})$, $\tau_{\text{in}} = \pi/g_*$ and $0 < g_* < \omega$, the interaction-picture propagator satisfies*

$$|0_c 0_m\rangle \mapsto |0_c 0_m\rangle, \qquad |1_c 0_m\rangle \mapsto -i|0_c 1_m\rangle. \tag{70}$$

It transfers the entire input qubit and its reference entanglement; a known memory number rotation removes the displayed phase.

Proof. The exchange generator conserves $n_c + n_m$. In its one-excitation block it is a Pauli X with pulse area $\int_0^{\tau_{\text{in}}} g_{\text{in}}(t)dt = \pi/2$; the vacuum block is fixed. The Hamiltonians at different times commute after removing the common harmonic rotation, so this block calculation is exact. Linearity and tensoring prove the reference-entangled statement. □

The operation empties c. It is a transfer, rather than the source-preserving encoding (27). Source/storage dynamics must be transported accordingly. Experimental bosonic exchange and remote microwave state transfer substantiate the kind of operation [20, 21]; they do not establish this exact matched network, its arbitrary-source applicability or its complete error budget. Loading a separate buffer quantum for conversion is another operation.

The exact cosine diagonal changes the vacuum/one-memory energies by $d_0(s) + d_1(s)n_m$, where

$$d_1(s) = -\sum_i E_{Ji} s h_{im}^2 [1 - e^{-s\|h_i\|_2^2/2}]. \tag{71}$$

This follows by setting $q_i = 0$ and using $L_1(z) = 1 - z$ in (63). Tune the input oscillator to $\Omega_s = \omega_m + d_1(s) > 0$ before the unknown state is supplied. The programmed quadratic matrices have diagonal (Ω_s, ω_m) and remain positive when $g_*^2 < \Omega_s\omega_m$. The target, including the exact diagonal shift, is resonant. The same proof as Proposition 6.2, on the detector vacuum/one-memory subspace $P_{\rm in}$, gives

$$\epsilon_{\rm in}^{\cos} \le \min\{2,\, 2\tau_{\rm in}[s^2 C_4(P_{\rm in}) + 2s^3 r_1]\}. \tag{72}$$

The external oscillator can be part of the reference in this estimate; the ideal moving detector support remains within P_1. The finite expressions evaluate approximately to $C_4(P_{\rm in}) = 0.0405440618$, $r_1 = 0.0081070965$. At $s = 0.1$, $g_* = 0.1\omega_m$ and $\tau_{\rm in} \simeq 7.2161574046$, the Hamiltonian-only right-hand side is about 0.006086. Its numerical and physical qualifications are the same as those of the handoff evaluation above.

If actual cross coefficients are g_x, g_p, define $g = (g_x + g_p)/2$ and $\mu = (g_x - g_p)/2$. The extra interaction is $\mu(cm + c^\dagger m^\dagger)$. On total occupation at most one, the exchange generator has norm one and the pair generator applied to that code has norm $\sqrt{2}$. Duhamel therefore gives

$$\epsilon_{\rm match} \le 2\int_0^{\tau_{\rm in}} (|\delta g| + \sqrt{2}|\mu|)dt. \tag{73}$$

Frequency errors add at most $2\int \max(|\delta\omega_c|, |\delta\omega_m|)dt$ on this moving code. An unknown common rotation is not a scalar on a vacuum/one-excitation superposition. A known rotation can instead be kept in the target.

6.4 A complete domain for downstream composition

The handoff must act on the joint source, physical memory and detector state. The retained Hamiltonian includes source storage and inherited conditional phases, such as $-\chi_{mf_r} j$ in a terminal flag sector. During the diagonal undressing target these commute with the physical flag $q_f = n_{f_0} - n_{f_1}$, and flag-preserving buffer/waste loss can continue. Their commutation with a later exchange or receiver coupling must be checked anew; any failed commutation cannot be removed by calling the term a spectator. Section 7 gives the receiver's actual pulse and its residual domain.

A small code-restricted channel error can imply small leakage, but it does not implement a perfect leakage herald. Composition uses an actual downstream CPTP map defined on all possible incoming states. Positive quadratic oscillator networks, bounded cosine potentials and bounded finite-spin terms supply well-defined static Hamiltonians. A linear oscillator force with a bounded spin coefficient is infinitesimally form bounded relative to positive oscillator energy: Cauchy–Schwarz and square completion bound its form by an arbitrarily small energy multiple plus a finite constant. A finite sum therefore defines a closed semibounded form. Its self-adjoint Hamiltonian and finite thermal auxiliary states give a unitary-plus-partial-trace CPTP map on the full incoming oscillator space, without a code projection. For a time-dependent family we additionally require the common-domain propagator conditions used in the stated pulse proofs.

The specific all-state receiver in Section 7 retains its opposite polarization and orbital terms; its ideal exchange also has the full oscillator ladder. The finite-spin amplifier of Section 9 then uses a bounded spin seed and finite thermal collisions. Thus no unproved number-only reduction of an oscillator seed is needed for this selected apparatus. Its control and coupling tolerances still belong to the residual R.

If $\mathcal{H}$ is the full physical handoff and $\widetilde{\mathcal{H}}$ its stated code target, a complete downstream map $\mathcal{A}$ gives

$$\|\mathcal{A}\mathcal{H}\mathcal{I} - \mathcal{A}\widetilde{\mathcal{H}}\mathcal{I}\|_\diamond \le \|(\mathcal{H} - \widetilde{\mathcal{H}})\mathcal{I}\|_\diamond. \tag{74}$$

Here $\mathcal{I}$ retains the full electronic/material output, including blank, escape and mismatched branches. An accepted/rejected handoff can be used instead only if both branches are kept and the accepted carrier, or a downstream extension on its leakage space, is specified. A normalized success-only map is not a replacement for this complete composition. These finite controls and domain statements strengthen the material chain while leaving explicit which laboratory parameters must still be calibrated at the common operating point.

7 A finite spin receiver with both field polarizations retained

The material flag is transferred through harmonic routing into an intrinsic spin-1/2, which supplies a bounded seed for the amplifier. The construction retains both magnetic polarizations and the electron's orbital motion. Its all-state dynamics defines a downstream channel even for leaked converter inputs; accuracy is separately proved on the intended source code. The routing controls extend the source and pump interfaces of Section 6.

7.1 Quadratic routing and the doublet boundary

Balanced capacitive and inductive coupling cancels harmonic pair terms in the circuit Hamiltonian [22]. With $X_j = j + j^\dagger$, $P_j = i(j^\dagger - j)$, a pair of resonant modes has

$$\begin{aligned} H_{jk}/\hbar &= \frac{\omega}{4}(X_j^2 + P_j^2 + X_k^2 + P_k^2) + \frac{v(t)}{2}(X_jX_k + P_jP_k) \\ &= \omega(n_j + n_k + 1) + v(t)(j^\dagger k + jk^\dagger). \end{aligned} \tag{75}$$

For impedance $Z_c > 0$, this is realized at the lumped quadratic level by

$$C^{-1} = Z_c \begin{pmatrix} \omega & v \\ v & \omega \end{pmatrix}, \qquad L^{-1} = Z_c^{-1} \begin{pmatrix} \omega & v \\ v & \omega \end{pmatrix}. \tag{76}$$

Both matrices are positive for $|v| < \omega$. Mutual-inductor orientation fixes the inductive sign. The full renormalized diagonal entries are included. Real tunable couplers must reproduce these matrices during switching; their additional modes and nonlinearities require a residual bound.

A pulse with $\int v\,dt = \pi/2$ transfers an old flag f into a new real mode x, with $|1_f, 0_x\rangle \mapsto -i|0_f, 1_x\rangle$. A 50:50 exchange of x, y, with a known local number phase if needed, then loads

$$a = (x - iy)/\sqrt{2}, \qquad b = (x + iy)/\sqrt{2}, \qquad a^\dagger|0\rangle = (x^\dagger + iy^\dagger)|0\rangle/\sqrt{2}. \tag{77}$$

Mode b is retained in vacuum. A number phase is an exact quadratic control when a positive frequency shift is applied at constant impedance; its area, duration and error belong to the control ledger. A smooth $v_* \sin^2(\pi t/\tau)$ pulse takes π/v_* for a swap and $\pi/(2v_*)$ for a 50:50 exchange. At the declared quadratic design value $v_*/2\pi = 50$ MHz these are 10 and 5 ns. The original detector's cosine residual during this short routing operation is still required. All exchange phases, free rotations, emptied old flags and spectator source/memory factors are retained in the output convention.

Harmonic balance is not an exact superconducting-qubit truncation. If a nonzero finite subspace were invariant under both canonical ϕ, n on their commutator domain, restricting

$[\phi, n] = iI$ would contradict the zero trace of a finite matrix commutator. Moreover, for $H_q = 4E_C n^2 + U(\phi)$,

$$n_{kl} = \frac{i(E_k - E_l)}{8E_C}\phi_{kl}. \tag{78}$$

Balancing the 0–1 transition does not cancel all higher transitions. Sank et al. explicitly find remaining anharmonic leakage; demonstrated charge/flux drive balancing in fluxonium also does not prove a complete two-body receiver channel [22, 23].

7.2 The full two-polarization spin coupling

Write $\sigma_+ = |e\rangle\langle g|$, $Z_s = |e\rangle\langle e| - |g\rangle\langle g|$, and $n_s = (I + Z_s)/2$. Two real modes with equal, perpendicular vacuum magnetic fields of coefficient $B_v(t) = \sqrt{2}g(t)/\gamma_e$ give the Pauli interaction

$$\begin{aligned} H_{\mathrm{mag}}/\hbar &= \frac{g(t)}{\sqrt{2}}(X_x\sigma_x + X_y\sigma_y) \\ &= g(t)(a\sigma_+ + a^\dagger\sigma_- + b^\dagger\sigma_+ + b\sigma_-). \end{aligned} \tag{79}$$

This identity retains the opposite circular polarization. Direct magnetic coupling of localized spins to superconducting circuits is a concrete proposal [24]; polarization-dependent Maxwell modes are also available in cavity designs [25]. These sources do not establish an exact single-spin transfer or justify deleting the opposite polarization. In particular, the analytic magnon model in Ref. [25] uses a rotating-wave approximation.

At $\omega_a = \omega_s$, put

$$A = a\sigma_+ + a^\dagger\sigma_-, \qquad B = b^\dagger\sigma_+, \qquad \Omega = \omega_b + \omega_s. \tag{80}$$

The interaction-frame Hamiltonian is

$$H_I(t)/\hbar = g(t)A + cg(t)(e^{i\Omega t}B + e^{-i\Omega t}B^\dagger). \tag{81}$$

The symmetric real-mode construction has $c = 1$; c below is real and specified. Mode splitting, wrong-mode resonant coupling, polarization error and other modes require additional residuals.

Let W embed the input qubit into $|0_a, 0_b, g\rangle$ and $|1_a, 0_b, g\rangle$. Evolution U_0 under $g(t)A$ transfers

$$|0_a, g\rangle \longmapsto |0_a, g\rangle, \qquad |1_a, g\rangle \longmapsto -i|0_a, e\rangle \quad \text{if} \quad \int g(t)dt = \pi/2. \tag{82}$$

Arbitrary reference entanglement is preserved. The exchange also defines a unitary on every higher-number block, rotating it by $\sqrt{n}\int g dt$. Therefore the downstream channel exists for leaked inputs without an ideal leakage herald; existence does not imply an accurate record for those inputs.

Proposition 7.1 (Finite opposite-polarization bound)**.** *Suppose $g \geq 0$ is continuously differentiable with zero endpoint values. On the input code W, the difference between the channel generated by* (81) *and the ideal exchange channel is at most*

$$\epsilon_{\mathrm{pol}} \leq \frac{2|c|\,\mathrm{TV}(g)}{\Omega} + \frac{2(\sqrt{2}|c| + c^2)}{\Omega}\int g(t)^2 dt \tag{83}$$

in restricted diamond norm, with arbitrary idle reference.

Proof. Define

$$K(t) = \frac{cg(t)}{i\Omega}(e^{i\Omega t}B - e^{-i\Omega t}B^\dagger). \tag{84}$$

Its derivative is the fast interaction plus the envelope derivative. The ideal moving code has b vacuum and at most one shared a/spin excitation. On it, the norms of K, $[gA, K]$ and VK

are bounded by $|c|g/\Omega$, $\sqrt{2}|c|g^2/\Omega$, and c^2g^2/Ω. Indeed $[A,B]=-a^\dagger b^\dagger Z_s$ has restricted norm at most $\sqrt{2}$, the $B^\dagger$ commutator annihilates b vacuum, and $B^2=0$. Integration by parts in the exact Duhamel identity gives half the stated right-hand side as an operator bound for $(U-U_0)W$. The endpoints of K vanish. Twice an isometry difference bounds the induced channel distance; tensoring an idle reference changes none of these norms. The linear oscillator–spin interaction is relatively bounded with arbitrarily small relative bound, and finite Fock trial vectors justify the differentiated identity. No Fock cutoff is used in the estimate. □

For the finite smooth pulse

$$g(t)=g_*\sin^2(\pi t/\tau),\quad \tau=\pi/g_*,\quad I_1=\pi/2,\quad I_2=3\pi g_*/8,\quad I_3=5\pi g_*^2/16, \tag{85}$$

where $I_j=\int g^j\,dt$, and $\mathrm{TV}(g)=2g_*$. Thus

$$\epsilon_{\mathrm{pol}}\le\frac{g_*}{\Omega}\left[4|c|+\frac{3\pi}{4}(\sqrt{2}|c|+c^2)\right]. \tag{86}$$

7.3 A positive orbital model with a finite reduction bound

An intrinsic spin does not eliminate its orbital spectator. We specify a confined electron with coordinate r, momentum p, charge magnitude e, bare electron mass m, and vector potentials

$$A_0=\tfrac{1}{2}B_0\hat{z}\times r,\qquad A_q=\tfrac{1}{2}B_v(t)(X_x\hat{x}+X_y\hat{y})\times r. \tag{87}$$

The full local uniform-mode Pauli Hamiltonian contains

$$\begin{aligned}H_{\mathrm{P}}(t)=&\frac{[p+e(A_0+A_q)]^2}{2m}+\frac{m\Omega_o^2r^2}{2}-\frac{e^2A_0^2}{2m}\\&+H_{\mathrm{photons}}+\frac{\hbar\omega_sZ_s}{2}+H_{\mathrm{mag}}(t).\end{aligned} \tag{88}$$

The external confining potential is positive for $\Omega_o>eB_0/(2m)$. The static compensating quadratic anisotropy is part of the specified trap. At $A_q=0$, the orbital Hamiltonian is an isotropic oscillator plus $(eB_0/2m)L_z$; its exact ground state is an isotropic Gaussian with variance $s_r^2=\hbar/(2m\Omega_o)$ in each coordinate. The kinetic square, positive trap and photon terms define a semibounded quadratic form and its Friedrichs realization; the bounded-spin linear magnetic field is controlled by the photon energy. We use this all-state Hamiltonian realization, not a projected two-level orbital model. For the stated smooth control family we require its unitary propagator and the Duhamel identities on the Gaussian/finite-Fock trial domain. Instantaneous semiboundedness alone is not invoked as a theorem for arbitrary time-dependent controls.

The linear orbital quantum-field term $(e/2m)B_q\cdot L$ annihilates the Gaussian. The residual orbital terms $e^2A_0\cdot A_q/m$ and $e^2A_q^2/(2m)$ remain. Put $\kappa_e=e/(m\gamma_e)$; for the bare-mass model with $g_e\ge 2$, $\kappa_e\le 1$. A semiconductor effective mass cannot be substituted without re-evaluating these factors and the orbital Hamiltonian. On the ideal Gaussian/code trajectory their norm in angular-frequency units is at most $a_0g+b_0g^2$, with

$$\begin{aligned}a_0&=\frac{\sqrt{6}\kappa_e^2\omega_s}{4\Omega_o}, & b_0&=\frac{\kappa_e^2(\sqrt{120}+\sqrt{3})}{4\Omega_o},\\ a_1&=\frac{\sqrt{10}\kappa_e^2\omega_s}{4\Omega_o}, & b_1&=\frac{\kappa_e^2(\sqrt{312}+3)}{4\Omega_o}.\end{aligned} \tag{89}$$

The second row will control the trajectory after applying K.

Proposition 7.2 (Uniform-mode Pauli transfer bound). *For the admitted propagator of* (88)*, the initial Gaussian orbital ground and code W, and the pulse* (85)*, the restricted diamond distance to the ideal spin transfer, retaining the orbital/mode spectators, is bounded by*

$$\epsilon_{\mathrm{P}} \le 2\left\{\frac{|c|\,\mathrm{TV}(g)}{\Omega} + \frac{\sqrt{2}|c| + c^2}{\Omega} I_2 + a_0 I_1 + b_0 I_2 + \frac{|c|}{\Omega}(a_1 I_2 + b_1 I_3)\right\}. \tag{90}$$

The bound also applies after any common CPTP readout or spectator trace.

Proof. The orbital cross term is proportional to $-r_z(r_x X_x + r_y X_y)$. Gaussian moments give $\|r_z r_x \psi_o\| = s_r^2$, while on $U_0 W$, $\|X_i U_0 W\| \le \sqrt{3}$. The quadratic term is proportional to

$$X_x^2(r_y^2 + r_z^2) + X_y^2(r_x^2 + r_z^2) - 2X_x X_y r_x r_y. \tag{91}$$

Here $\|(r_y^2 + r_z^2)\psi_o\| = \sqrt{8}s_r^2$, $\|X_i^2 U_0 W\| \le \sqrt{15}$, and $\|X_x X_y U_0 W\| \le \sqrt{3}$. Substitution of $B_v = \sqrt{2}g/\gamma_e$ gives a_0, b_0. Integrating by parts with the *full* propagator also produces $R_o K$, which must not be omitted. After K, the photon vector lies in the one- or two-photon sectors, with no vacuum component. The even moment operators have no matrix elements between these two sectors; within each fixed-number sector their largest moments give $\sqrt{5}, \sqrt{39}, 3$. These bounds give a_1, b_1 and $\|R_o K U_0 W\| \le |c|g(a_1 g + b_1 g^2)/\Omega$. Adding these two terms to the exact identity in Proposition 7.1 proves (90). The polynomial orbital terms act on a Gaussian times finite Fock vectors, so the trial-vector norms and differentiated products are defined. □

This is an explicitly specified Pauli model with finite orbital errors. It is not asserted to be the exact Maxwell mode profile and confinement of a fabricated hybrid chip. A spatial, control or omitted-material residual with trial bounds $r_0(t)$ and $r_1(t)$, respectively before and after the normalized fast primitive, contributes

$$2\int \left[r_0(t) + \frac{|c|g(t)}{\Omega} r_1(t)\right] dt. \tag{92}$$

This norm includes off-orbital and off-code output components. Phase and exchange-area errors can additionally be bounded by twice their absolute errors on the transfer code. An unmeasured residual is not assigned zero.

7.4 Finite preparation and the spin seed

A fresh diagonal thermal spin auxiliary can reset an arbitrary old receiver, including a receiver entangled with an idle reference, by the finite exchange

$$H_R(t)/\hbar = j(t)(\sigma_+ s_- + \sigma_- s_+), \qquad \int j(t)dt = \pi/2. \tag{93}$$

This gate is an iSWAP. Nevertheless, for a diagonal auxiliary Gibbs state τ_s, tracing the old-state auxiliary gives exactly $\rho_{RS} \mapsto \rho_R \otimes \tau_s$: in each auxiliary energy sector, the receiver ends in that sector and the input phases remain only in the discarded system. The ground-state preparation error is $2/(1+e^{\beta\omega_s})$ in trace norm, and an exchange-area error $\delta\theta$ adds at most $2|\delta\theta|$. A balanced oscillator swap likewise replaces a loading mode by a fresh thermal oscillator, with vacuum error $2\bar{n}/(1+\bar{n})$. These are a finite number of finite operations; no infinite-rate Poisson reset is invoked. Fresh thermal resources, their temperature and finite gate-error envelopes are explicit inputs. The construction does not derive a Gibbs resource from arbitrary unknown matter. The loaded receiver is not reset during decision or storage.

Contribution to restricted diamond bound	Receiver 0	Receiver 1
Opposite polarization	$2.84294\,10^{-6}$	$2.49201\,10^{-6}$
Orbital cross and quadratic terms	0.00098342363	0.00112191191
Orbital/fast-primitive cross term	$2.79412\,10^{-10}$	$2.79412\,10^{-10}$
Closed uniform Pauli-model transfer	0.00098626685	0.00112440419

Table 2: Finite model bounds. The orbital and opposite-polarization terms are retained. Spatial-mode, switching, loading, loss and actual seed-coupler errors are additional conditions.

After both transfers, let

$$q = n_{s_0} - n_{s_1} = (Z_0 - Z_1)/2, \qquad M = \sum_{i=1}^{N} Z_i. \tag{94}$$

The finite seed target is

$$H_{\text{seed}} = -\hbar h q M = -\frac{\hbar h}{2} \sum_i Z_0 Z_i + \frac{\hbar h}{2} \sum_i Z_1 Z_i. \tag{95}$$

It has norm at most $\hbar h N$ and commutes with both receiver Z's. There is no bosonic seed curvature or flag-pair process in this bounded spin Hamiltonian. A physical current-doublet implementation can use mutual inductance; sign- and magnitude-tunable flux-qubit couplers have been constructed [26]. Their existence does not prove the doublet reduction, tunnelling suppression or hybrid adapter for the present device. Ordinary magnetic dipole coupling of true spins also contains transverse terms, which require control or a residual bound. The finite spin-seed implementation is therefore a separate specified primitive.

Both receivers excited gives $q = 0$ and remains in the complete channel. The bosonic input may leak, yet its unitary transfer followed by a bounded spin apparatus has a CPTP action on every input. This removes the need for an ideal leak herald as a condition of downstream channel existence. Accuracy on the intended source code still uses the stated error bounds. Neither this composition nor the thermal reset selects an actual history; the realization commitment of Section 13 remains a separate input.

7.5 One declared finite parameter example

Use the common clock $\Omega_0 = 2\pi \times 10^9$ s^{-1}, the actual archived detector flag frequencies $\omega_0/\Omega_0 = 5.111803398874989$ and $\omega_1/\Omega_0 = 5.83166247903554$, understood as the exact stored binary64 values, $g_*/2\pi = 3$ kHz, $\Omega_o/2\pi = 10$ THz, $c = 1$, and $\beta\hbar\Omega_0 = 2$. The smooth transfer lasts $166.6667\,\mu$s; the transfers can run in parallel if their cross-coupling residual is bounded. For $\gamma_e/2\pi = 28$ GHz/T, the static fields are about 0.1825644 and 0.2082737 T, and each real mode has vacuum coefficient $0.1515\,\mu$T. These are declared Hamiltonian parameters, not a joint experimental fit.

The two transfer bounds sum to less than 0.00211067104. Independent Gibbs preparation of the two receiver spins and their four loading modes contributes

$$2\left[1 - \prod_{r=0}^{1} \frac{(1 - e^{-\beta\hbar\omega_r})^2}{1 + e^{-\beta\hbar\omega_r}}\right] < 0.000269423831. \tag{96}$$

Thus their combined receiver/preparation allowance is less than 0.00238009487. An orbital Gibbs resource has a further positive error at most $6e^{-\beta\hbar(\Omega_o - eB_0/(2m))}$ per electron; a different orbital preparation requires its actual envelope. A large orbital gap is not a substitute for such a preparation statement.

The remaining finite physical conditions concern mode functions and their spatial residual, loading/switching and phase envelopes, actual confinement and material structure, loss and dephasing over the same transfer interval, and the spin-seed primitive. Inferred large local single-spin couplings in ensemble experiments [27] cannot be combined with unrelated coherence measurements to fill these entries. The calculation is a useful conditional receiver certificate; it is not a fabricated-device certificate. The independent validation records 74 checks of the identities, finite reset, orbital polynomial bounds and toy two-polarization evolution. Separately, 256-bit Arb ball arithmetic certifies five strict decimal upper bounds, including both individual transfers and the combined allowance above, from the exact frequency values, represented as exact binary rationals. The nominal design origins $5+\sqrt{5}/20$ and $11/2+\sqrt{11}/10$ are not substituted for the archived values. This certifies evaluation of the analytic expressions. The analytic bounds do not depend on the toy numerical Fock cutoffs.

8 A finite thermal collision and its coherent control circuit

A specified finite Gibbs register supplies an exact thermal collision channel and a finite preparation protocol. Repeated finite thermal interactions and their coherent phases have precedents in [28]; Ising-dependent transition frequencies require a physical energy-resolving interaction [29]. The present ladder and circuit are a new driven apparatus choice. They do not establish that a passive bath implements the operation.

For the 32-spin reference Hamiltonian $H_S = -M^2/64 - qM/4$, write $z_i = 1$ for an up spin and define

$$K_i = \sum_{j\neq i}(2z_j - 1) + 8q, \qquad \ell_i = -K_i.$$

The down-to-up gap is $\ell_i/16$, and K_i is an odd integer in $[-39, 39]$. It is unchanged by that spin flip. Nine bath qubits have Hamiltonian

$$H_B = \sum_{j=0}^{8} \frac{2^j}{16} b_j = \frac{n}{16}, \qquad 0 \le n < 512.$$

Their independently specified state is $\tau_B = e^{-2H_B}/Z_B$. It factorizes with excited populations $p_j = (1 + e^{2^{j-3}})^{-1}$. Put $r = e^{-1/8}$.

For each fixed spectator configuration, W_i exchanges $|\downarrow, n\rangle$ with $|\uparrow, n-\ell_i\rangle$ whenever both numbers are allowed. The paired subspaces are disjoint, so $W_i = W_i^\dagger$, $\|W_i\| \le 1$, $W_i^2 = P_i$ and $[W_i, H_S + H_B] = 0$. With physical arming $a = q^2 = n_{s_0} \operatorname{xor} n_{s_1}$, the selected collision is

$$U_i = e^{-i\pi a W_i/2} = I - aP_i - iaW_i. \tag{97}$$

A five-qubit Gibbs register with zero Hamiltonian, hence state $I/32$, controls the site uniformly and independently. It is not a measured source coordinate. Tracing the fresh registers after the controlled unitary defines the complete collision, including all cross-q coherences. On the $q = 0$ space, both 00 and 11 receiver states receive identity coherently; the nominal balanced pointer is not armed. The $q = \pm 1$ channels are unchanged from the always-armed ladder.

For an uphill gap $\ell/16 > 0$, the exact finite probabilities are

$$a_\uparrow(\ell) = \frac{r^\ell - r^{512}}{1 - r^{512}}, \qquad a_\downarrow(\ell) = \frac{1 - r^{512-\ell}}{1 - r^{512}}, \qquad a_\uparrow = r^\ell a_\downarrow. \tag{98}$$

These follow by summing allowed bath occupations. No target switching probability is programmed. Uniform site selection gives the finite magnetization chain. The channel requires no continuous-time or secular limit.

8.1 Finite synthesis without measuring the local field

Seven work qubits compute K_i mod 128 by exact Fourier phase accumulation and inverse QFT. Each controlled phase decomposes into 31 spin and two flag terms. After routing the selected spin to a fixed target, two ten-qubit work registers compute

$$R = (n + \ell_i z_i) \bmod 1024, \qquad D = (R - \ell_i) \bmod 1024.$$

The represented signed values are in $[-39, 550]$, so both bath states exist precisely when their top bits are zero. The activity bit is the AND of that condition and the physical arm bit. Conditional on activity, XOR the low nine bits of R and D into n, flip z_i, and apply phase $-i$. This maps each paired basis state to the other with the phase in (97). K_i, R, D are invariant through that exchange. Reversing the arithmetic and routing therefore erases every work bit. In particular the circuit does not pinch the seed or retain an unreported measurement of it.

The gate count includes exact phases: a Toffoli uses five two-qubit primitives by [30, Lemma 6.1]; a controlled SWAP uses seven. The always-armed construction costs 2048 arbitrary one- or two-qubit primitives, including routing and preparation. Physical XOR arming adds four CNOTs and two Toffolis, giving **2062 primitives per collision**. The selected circuit uses 30 reusable clean work qubits. Every collision consumes nine bath and five selector qubits plus fourteen purifying qubits. Both the Gibbs and maximally mixed selector preparations are thereby accounted for. An exact integer audit checks all 40960 signed-field/spin/bath basis cases and workspace erasure; it separately checks identity in both unarmed flag sectors.

8.2 Preparation, finite clock and error obligations

A finite independent bank in the product Gibbs state of $16 \sum_j n_j$ at inverse temperature two has per-qubit excitation

$$p_c = (1 + e^{32})^{-1} < 1.27 \times 10^{-14}.$$

The bank contains $28m$ fresh bath/selector and purifying qubits, 30 reusable work qubits, and 32 initial pointer spins. Its trace-norm error relative to the complete zero-state preparation reference is at most $2(28m + 62)p_c$. With the native diagonal spin Hamiltonian disabled, apply X to a fixed set of sixteen pointer spins. This finite word maps the ideal zero product exactly to a specified balanced product with $M = 0$. Its action on the finite-temperature input retains the same trace-norm error by unitary invariance. Two commuting switching ramps of duration $100\Omega_0^{-1}$ each can disable and restore the diagonal terms; their phases are retained. The product pointer preparations are diagonal during these ramps, so the ramps do not require a measurement or a choice of magnetization sector. A fixed $R_y(2 \arcsin \sqrt{p_j})$ and CNOT prepares each bath purification; a Hadamard and CNOT prepare each selector purification. The angles depend solely on the bath Hamiltonian and temperature. The initial finite equilibrium law is an apparatus preparation input, not an equilibration theorem from an arbitrary unknown state.

The compiled circuit assumes programmable one- and two-qubit controls. General arithmetic does not commute with H_S at intermediate times. We explicitly disable diagonal Ising and auxiliary free terms during the gate word, using finite commuting ramps and retaining their phases. The net operation conserves the reference energy; its driven implementation has switching work. This is a materially stronger control specification than the existence of an abstract many-spin projector.

For a primitive $G = e^{-i\pi A}$ with $\|A\| \le 1$, a pulse

$$H(t) = g \sin^2(\pi t/\tau_g) A, \qquad \tau_g = 2\pi/g,$$

has finite peak norm g, derivative at most $g^2/2$, and implements G exactly. Two diagonal ramp intervals of $100\Omega_0^{-1}$ give the complete clock

$$\tau_{\text{col}} = \frac{2062(2\pi/g) + 200}{\Omega_0}.$$

At $g = 0.1$, $\Omega_0 = 2\pi\, 10^9\,\mathrm{s}^{-1}$, this is approximately 20.651831 μs; 1024 collisions take 21.147475 ms. These are specified-control timings, not observed device performance.

Pointer setup adds sixteen primitives and the two finite ramps, hence

$$\tau_{\mathrm{setup}} = \frac{16(2\pi/g) + 200}{\Omega_0} = 160\,\mathrm{ns} + \frac{100}{\pi}\,\mathrm{ns} \approx 191.831\,\mathrm{ns}.$$

It is performed before admitting the source. The amplification word still contains 2111488 primitives; including pointer setup gives 2111504. The cold resource contains 28734 qubits at $m = 1024$. The separate setup certificate checks the fixed sixteen-bit flip, total magnetization, resource count and directed arithmetic bounds. It does not assume that an arbitrary unknown pointer spontaneously starts in the balanced state.

With primitive diamond errors bounded by ϵ_g, each ramp error by ϵ_r, and all other complete channel errors retained, telescoping yields

$$\epsilon_{\mathrm{impl}} \le 2(28m + 62)p_c + (2062m + 16)\epsilon_g + (2m + 2)\epsilon_r + \epsilon_{\mathrm{ambient}} + \epsilon_{\mathrm{adapter}} + \epsilon_{\mathrm{reader}}. \tag{99}$$

A primitive operator-norm error 10^{-10} gives a channel bound 2×10^{-10}, hence 0.0004222976 for amplification alone and 0.0004223008 including pointer setup at $m = 1024$. This is a stringent required tolerance, not demonstrated attainable precision. An unspecified contribution cannot be entered as zero.

Quiet storage is a separate declared schedule: fresh collisions stop, the seed is removed by a commuting ramp, and a residual per-up-spin downward hazard is bounded by γ_h. Under a diagonal single-spin population model, independent dominating death clocks give readable-band survival at least

$$\Pr\{\mathrm{Binomial}(k, e^{-\gamma_h T_h}) \ge 24\}$$

from an upper-band state with k up spins. Upward restoration can only improve this lower bound. This requires the actual conditional hazard cap and excludes unbounded or unaccounted coherent spin-changing dynamics. It is not full microscopic state preservation, and requires the stated quiet schedule with collisions disabled.

9 Validated finite amplification and quiet record retention

Let k be the number of up spins and $M = 2k - 32$. Prepare a specified balanced product state, $k = 16$, without using the detector input. The upper readable band is $k \ge 24$, the lower band is $k \le 8$, and intermediate values are unreadable. The reader is gated until collision $m = 1024$. The electronic detector's first-display register is a separate variable and retains its original definition. In particular the amplifier is not judged by its first excursion across a band boundary.

For $q = +1$, the full finite chain has upward gap index $\ell_+(k) = -(2k - 31) - 8q$ and downward gap index $\ell_-(k) = 2k - 33 + 8q$. If $F(\ell)$ is the finite Gibbs probability that an energy increase $\ell/16$ can be supplied, then

$$F(\ell) = \begin{cases} (r^\ell - r^{512})/(1 - r^{512}), & \ell > 0, \\ (1 - r^{512+\ell})/(1 - r^{512}), & \ell < 0, \end{cases}$$

and the tridiagonal transition matrix has

$$P_{k,k+1} = \frac{32 - k}{32} F(\ell_+(k)), \qquad P_{k,k-1} = \frac{k}{32} F(\ell_-(k)), \qquad P_{k,k} = 1 - P_{k,k+1} - P_{k,k-1}. \tag{100}$$

Boundary terms vanish by their site-count factors. The $q = -1$ chain is the reflected chain. The $q = 0$ channel is identity in the selected coherently armed protocol, not the always-armed zero-field thermal chain.

Proposition 9.1 (Finite arithmetic certificate)**.** *For the declared collision model and endpoint reader, either seeded input has correct readable probability between* 0.99992352423742848 *and* 0.99992352423742849*. Its wrong readable probability is below* 0.000062942257*, and its unreadable probability is below* 0.000013533507*. An unarmed input remains ready with probability one in the nominal model.*

Proof. The calculation encloses $r = e^{-1/8}$ by exact rational alternating Taylor sums of orders 61 and 60. Every power, ratio and transition is rounded outwards on an integer grid with denominator 10^{70}. Starting from the exact point mass at $k = 16$, 1024 applications of the interval transition matrix enclose every exact probability. Nonnegativity, stochasticity, detailed balance and reflected symmetry are checked independently. Summing the appropriate final components gives the stated enclosures. The full machine-readable intervals have widths below 10^{-64}; this precision certifies the mathematical model and carries no implication about laboratory parameter precision. □

The distinction between readers is consequential. Under the same seeded dynamics, the probability that the first reached band is correct is only 0.994167124355; the first wrong-band probability is 0.005832875646. Continuing the controlled amplifier corrects many initial excursions. Reporting the endpoint result as an irreversible first-display success would therefore be false. The chosen endpoint gate is part of the apparatus protocol.

Retention depends on the storage schedule. The protocol turns the thermal collisions off and detaches the seed through a finite commuting ramp before the quiet hold. Continued thermal collisions would define a different channel and cannot inherit the following survival bound.

For the selected quiet interval $T_h = 0.1\,\mathrm{s}$, require a conditional downward hazard no greater than $\gamma_h = 10^{-12}\Omega_0$ for each currently up spin, with the reflected condition for a lower record. Population dynamics are diagonal with single-spin transitions. Any coherent transverse, correlated multi-spin or reader fault requires an additional bound. Upward restoration may be arbitrary within this population model because it improves upper-band retention. Couple all losses to independent death clocks at the maximum rate. Starting from $k \geq 24$, the number surviving those clocks is binomial with survival parameter $p = e^{-\gamma_h T_h}$. Its endpoint band event equals its first-exit survival event because the dominating process is monotone.

Proposition 9.2 (Whole-hold readable survival)**.** *Under this residual-hazard condition, average over the actual seeded amplifier endpoint distribution. The conditional probability that a correct readable band is never exited during the entire quiet interval is at least* 0.9999999794474568*. The original seeded run is correct at the endpoint and remains readable throughout the hold with probability at least* 0.9999235036864570*.*

Proof. For each endpoint $k \geq 24$, sum

$$B_k(p) = \sum_{j=24}^{k} \binom{k}{j} p^j (1-p)^{k-j}.$$

Use the preceding interval distribution in $\sum_{k=24}^{32} v_k B_k(p)$, and divide only to report the separately labelled conditional retention. The original-run joint probability is never renormalized. Here $\gamma_h T_h = \pi/5000$. Machin's identity $\pi = 16\arctan(1/5) - 4\arctan(1/239)$ and alternating rational series enclose π; orders 25 and 24 enclose the decreasing exponential. Positive polynomial products are rounded outwards on the same integer grid. Monotone death-clock domination proves that the computed binomial enclosure is a lower bound for the actual specified population process throughout the interval. □

For an unarmed balanced pointer, a false quiet record requires at least eight flips. If both directions have the same per-spin hazard cap, the total flip count is dominated by a Poisson variable of mean $\lambda = 32\gamma_h T_h < 0.021$. Thus, without conditioning on a record,

$$\Pr(\text{false band during the hold}) \le \frac{\lambda^8}{8!}\frac{1}{1-\lambda/9} < 6.639 \times 10^{-19}. \tag{101}$$

This bound concerns residual spin flips in the declared hold model; it is not a bound on an unspecified reader's false electronic displays.

9.1 A coherent instrument bound and its comparator

Endpoint probabilities alone do not specify a conditional quantum channel. Let V be the actual coherent Stinespring isometry of the selected finite amplifier, including the seed, fresh registers and endpoint reader. Let P select the correct readable band in each $q = \pm 1$ sector and the ready outcome in the $q = 0$ sector. The seed is conserved, so $A = V^\dagger P V$ is block diagonal and $A \ge (1-\epsilon)I$, where $\epsilon < 0.000076475763$. The normalized isometry

$$V_{\text{good}} = PVA^{-1/2}$$

defines a particular correct-record comparison channel. For any input, including a reference, $V^\dagger V_{\text{good}} = A^{1/2} \ge \sqrt{1-\epsilon}I$. The pure-state trace-distance formula and contractivity give

$$\|\mathcal{A} - \mathcal{A}_{\text{good}}\|_\diamond \le 2\sqrt{\epsilon} < 0.017490085. \tag{102}$$

An outward integer square root certifies the numerical upper bound. The polar-normalized comparison channel has the actual normalized successful conditional state on each fixed-q input. On coherent superpositions of sectors with different success probabilities it is a specified linear comparison isometry: it preserves the input sector weights, whereas conditioning the original output on success reweights them. These two operations are not identified. It is not an independently imposed Lüders update, and the bound does not turn a channel decomposition into a unique actual history. The much smaller classical band-error probability and this coherent channel bound answer different questions.

10 A certified detector bound over the full interval

The full cosine converter admits a nonvacuous finite-interval comparison at the specified operating point. The result concerns the complete first-display instrument, including blank runs and the retained quantum material state. Its comparison instrument includes the finite cosine-induced energy shifts. It is consequently distinct from the resonant quartic approximation in Section 5, even when their electronic probabilities are close.

10.1 The frozen model and the comparison target

The full cosine Hamiltonian and its six-mode circuit are specified in Section 4; here the mode order is $(m, b, f_0, w_0, f_1, w_1)$. In the forced harmonic frame the nonlinear remainder is

$$N(t) = -\sum_i E_i \left[\cos(\sqrt{s}[\Phi_i + q_i(t)]) + \frac{s}{2}[\Phi_i + q_i(t)]^2 - 1\right], \tag{103}$$

with $\Phi_i = \sum_\ell h_{i\ell}(a_\ell + a_\ell^\dagger)$ and $q_i(t) = 2\sum_r h_{if_r}\xi_r \cos(\Omega_{pr} t)$. The archived six-mode circuit specifies every $E_i, h_{i\ell}, \omega_\ell, \xi_r$ and bare pump frequency. We retain $s = 1/10$ and the fixed pump offsets $s^2(-0.00870849782706314, -0.0077173592597816485)$. No physical pump is retuned to fit the comparison. The exact serialized convention is given in the detector archive's `operating_point.json`: stored binary64 values define the displayed $T = 223600212.28215325$ and $\kappa = 6.900178704228631 \times 10^{-7}$, while $\eta = 19/20$, $\beta_b = \kappa/1000$ and each electronic dark rate $d_r = \kappa/10000$ are exact ratios. Memory and each flag have positive loss $\gamma = 10^{-12}$, and every material port has thermal occupation 10^{-7}. Evaluation-order differences in rounded rate expressions are included in the arithmetic allowance. The specified T is approximately 35.587 ms at the common scale.

The twelve-dimensional bare carrier has, for $j = 0, 1$,

$$\begin{aligned} A_j &= (j,1,0,0,0,0), & C_{0j} &= (j,0,1,1,0,0), & C_{1j} &= (j,0,0,0,1,1), \\ D_{0j} &= (j,0,1,0,0,0), & D_{1j} &= (j,0,0,0,1,0), & E_j &= (j,0,0,0,0,0). \end{aligned}$$

Its pump indices are $k_A = k_E = (0,0)$, $k_{C_0} = k_{D_0} = (1,0)$ and $k_{C_1} = k_{D_1} = (0,1)$. Write $J(t)$ for the corresponding bare occupation embedding with phases $e^{-ik_a\cdot\Omega_p t}$. Waste channels are independent and resolved; their target lowering operators are $l_r = \sqrt{\kappa}\sum_j |D_{rj}\rangle\langle C_{rj}|$, and buffer escape has $l_b = \sqrt{\beta_b}\sum_j |E_j\rangle\langle A_j|$. The first-display register retains the first observed waste or independent dark label; subsequent material evolution continues.

A finite Fourier trial supplies real quasienergies $\Lambda = \mathrm{diag}(\lambda_a)$ and an exactly block-orthogonal matrix U. The target Hamiltonian is $H_{\mathrm{eff}} = U\Lambda U^T$. Each active (A_j, C_{0j}, C_{1j}) block and each terminal state is separately preserved by U; exact Gram–Schmidt of the stored principal trial coefficients defines it. Arb enclosures determine its rounded matrix elements. The physical converter therefore has a precisely specified comparison operator, rather than a set of phases adjusted after measuring a desired outcome.

The intended-channel detunings in its two active blocks are $8.5202326696 \times 10^{-8}$ and $5.8148561135 \times 10^{-8}$; the four conversion couplings are about 8.264×10^{-8}. The small induced cross-conversion terms, about -2×10^{-14}, are retained. These energy shifts matter for conditional coherent states over the long gate. The next section computes the new target probabilities independently; the resonant two-state formulas are not substituted for this three-state calculation.

10.2 A finite Fourier residual theorem

The argument uses a finite trial, not an infinite Floquet spectral-gap assumption. Let

$$W(t) = \sum_{k\in K} e^{-ik\cdot\Omega_p t} W_k, \qquad R_H(t) = H(t)W(t) - i\dot{W}(t) - W(t)\Lambda. \tag{104}$$

Every column of W_k has finite Fock support. The coefficients and λ_a are exact stored binary64 numbers defining this trial; no eigenvalue-solver tolerance is used as a proof. Let $V(t) = J(t)U$ and suppose, uniformly in $t \in [0,T]$,

$$\|W(t) - V(t)\| \le \delta, \qquad \|W(t)\| \le w = 1+\delta, \qquad \|R_H(t)\| \le r_H. \tag{105}$$

For a physical Lindblad operator L and its target operator $\ell = U^T l U$, set

$$a_L = \sup_t \|LW(t) - W(t)\ell\|, \tag{106}$$

$$b_L = \sup_t \left\|L^\dagger L W(t) - W(t)\ell^\dagger \ell\right\|, \qquad c_L = \sup_t(\|LW(t)\| + \|W(t)\ell\|). \tag{107}$$

For a physical port omitted from the target, take $\ell = 0$.

Theorem 10.1 (Complete finite-interval detector comparison)**.** *Assume the specified full oscillator Markov model has a conservative CPTP propagator and the common finite-core generator-domain property. Then the complete original-run instrument on the prepared binary input, including its retained bare material output, satisfies*

$$\|\mathcal{I}_{\mathrm{full}} - \mathcal{I}_{\mathrm{eff}}\|_\diamond \le 2\delta(2+\delta) + T\left[2wr_H + \sum_L (a_L c_L + b_L w)\right]. \tag{108}$$

The same estimate holds uniformly on the full twelve-dimensional target carrier. Resolved observed/unobserved output splitting and the first-display register do not increase the stated sum.

Proof. For a positive target state ρ of unit trace, the Hamiltonian defect of $W\rho W^\dagger$ is bounded by $2wr_H$. Expanding the two jump products gives $a_L c_L$, and expanding the two anticommutator products gives $b_L w$. These are norms on finite-support vectors; no global bounded norm is assigned to an oscillator annihilation operator. Every estimate is unchanged by an idle reference.

The observed and missed versions of one channel have coefficients $\sqrt{\eta}$ and $\sqrt{1-\eta}$, whose squared contributions sum to one. Routing an observed jump into a classical register preserves the trace-norm estimate. Independent electronic dark transitions act identically on the material trial and target, so their intertwining defect is zero. They remain in the target dynamics. Duhamel's formula and the full propagator's trace-norm contraction bound the integrated defect. Initial and final comparison with the bare isometry V each cost at most $\delta(2+\delta)$. This proves the complete instrument bound, with no conditioning or discarded alphabet symbol. □

This is an operator-intertwiner estimate valid for arbitrary reference systems, so no extra input-dimension factor is required. A certificate based solely on one normalized binary Choi trajectory would instead require the usual factor two. The residual principle extends the finite-support state-estimation method of Etienney, Robin and Rouchon [31]; the present finite Fourier and monitored-intertwiner construction is the additional argument used here.

10.3 Infinite tails and arithmetic

For each oscillator matrix element we evaluate the infinite displacement operator $D(i\sqrt{s}\,h_i)$ through its Laguerre formula. The cosine's pump Fourier coefficients involve

$$\mathrm{Re}\Big[i^{k_0+k_1}\langle m|D(i\sqrt{s}\,h_i)|n\rangle\Big]\, J_{k_0}(2\sqrt{s}\,h_{if_0}\xi_0)J_{k_1}(2\sqrt{s}\,h_{if_1}\xi_1). \tag{109}$$

The quadratic counterterm is evaluated exactly on its finite ladder support. In particular, no exponential of a truncated annihilation matrix replaces a compressed infinite cosine.

Let P_N denote total occupation at most N, $a=\sqrt{s}\,\|h_i\|$, $K_*=M-N+1$ and $r_*=a\sqrt{N+K_*+1}/(K_*+1)<1$. Collective-mode normal ordering gives

$$\big\|(1-P_M)D(i\sqrt{s}\,h_i)P_N\big\|\le \frac{e^{-a^2/2+a\sqrt{N}}a^{K_*}}{K_*!}\sqrt{\frac{(N+K_*)!}{N!}\,\frac{1}{1-r_*}}. \tag{110}$$

The finite trial's separate occupation-shell norms weight these bounds. For pump amplitudes $x_r=2\sqrt{s}|h_{if_r}\xi_r|$, $X=x_0+x_1$, the omitted harmonics of total order greater than L have absolute sum bounded by

$$e^{(x_0^2+x_1^2)/4}\,e^X\,\frac{X^{L+1}}{(L+1)!}. \tag{111}$$

Each source harmonic uses its own remaining order L. The final trial has source occupation at most 12 and pump radius 6. Evaluation through occupation 14 and pump radius 8 contains every polynomial image; the two displayed bounds control the remaining infinite cosine images.

The first finite calculation used 924 physical states and 13 pump harmonics. Its tiny internal Ritz residual did not certify the result: the external residual was 2.3×10^{-4}. Two explicit diagonal corrections and a subsequent residual evaluation reduce the external defect to the scale needed here. The correction construction is not itself assumed convergent. The final residual is recomputed for the fixed trial without a further unaccounted Rayleigh transformation.

Arb at 192-bit precision encloses all displacement and Bessel factors. Every zero-containing factor ball is checked to be exactly zero. The largest relative rounding errors of a displacement factor and Bessel coefficient are respectively below 1.10×10^{-16} and 1.09×10^{-16}. Matrix contractions use at most 32 inner products per block followed by an explicit pairwise summation tree. Their real/imaginary structure permits an IEEE forward-error bound; it is not inferred from agreement with an ordinary matrix product. The bound uses the source row-norm sum to avoid multiplying entrywise errors by an artificial infinite dimension. It also includes finite polynomial accumulation, scalar and pump rounding, norm reductions, subnormal underflow, and exact-U corrections. These finite arithmetic hypotheses and the frozen coefficient data are part of the reproducible certificate.

Contribution over the complete interval	Outward channel-norm bound
Hamiltonian: finite residual, infinite tails and arithmetic	0.0284140252
Both endpoint comparisons with bare code	0.0007745472
Retained waste and buffer jump defects	0.0247117153
Positive memory and flag loss	0.0076571879
Thermal increments at all six ports	0.0016107986
Displaced flag-loss Hamiltonian	0.0000726413
Complete instrument	$< 0.063241 < 0.10$

Table 3: The certified full-interval ledger. The final Hamiltonian term includes an arithmetic residual allowance 5.16613×10^{-11}; the smaller unrounded diagnostic is not used as the certificate.

The uniform Hamiltonian residual is at most 6.35253×10^{-11}, including the infinite Fock tail 7.94495×10^{-13} and pump tail below 7.75×10^{-23}. The final bound uses $w > 1$ and the exact output-unitary correction. This establishes the full-interval comparison for the stated conservative monitored model and its explicitly dressed comparison instrument.

10.4 Interface and reproduction

The theorem's final state is compared with the *bare* embedding $J(t)$, not left in an unspecified Floquet-dressed carrier. Both dressing discrepancies have already been paid. The effective converter flags therefore have their declared zero/one occupations in the converter displacement frame. The later physical pump-undressing operation, its phases and its control error remain a separate interface obligation; this theorem does not erase the pump displacement or certify an unmodelled switching pulse.

The archive pins the circuit, the exact-number operating point, the losslessly stored finite trial, its target matrices and all residual ledgers. A replay begins with the supplied trial; it need not repeat a potentially platform-dependent eigensolver to define the certified object. The scripts then rebuild exact displacement/Fourier actions, the rounded enclosures and the whole-interval sum. Standard NumPy, SciPy and python-flint are external runtime dependencies. The certified finite action took about 729 seconds on the present host; this is a reproduction observation, not a complexity or hardware-speed claim. It avoids propagation through millions of fast oscillations.

The result does not identify measured material lifetimes, derive the declared reservoir law from a fabricated device, or infer that a single original-run history is actual. It supplies a finite, full-interval physical-model comparison with positive noise, coherent conditional states and stable target output occupations. Those are the precise detector obligations discharged here.

10.5 Validated original-run probabilities of the finite target

The dressed target permits a small exact calculation of the complete electronic outcome distribution. For fixed memory j, its active carrier is A, C_0, C_1 and its absorbing carrier states are D_0, D_1, E. Each electronic first-display block therefore requires nine active density entries and three absorbing populations. Three such blocks give an exact 36-component representation for the specified preparation. Missed waste counts retain their physical flag; a dark display acts identically on the carrier and does not create a flag. The material dynamics continues after the first display.

We evaluate the finite matrix exponential using 256-bit Arb ball arithmetic. The rigorous matrix-exponential routine includes arithmetic and truncation errors; the FLINT documentation

Memory	Correct display	Wrong display	Blank at T
0	0.926594883904	0.007260512358	0.066144603738
1	0.927505664566	0.006841778146	0.065652557288

Table 4: Original-run finite-target probabilities, rounded to twelve decimal places. Each displayed value differs from every point of its certified interval by less than 6×10^{-13}. The saved certificate contains outward twenty-place endpoints. These arithmetic intervals do not include the separate full-cosine, receiver or hardware errors.

distinguishes this routine from methods explicitly marked as approximate.[1] The computation uses the exact binary64 centre of each supplied relative 3×3 Hamiltonian block. If its entry radii have Frobenius bound h_j, CPTP contraction and Duhamel give a channel error at most $2Th_j$ and a single-event probability allowance Th_j. The single-event allowances Th_j are below 1.852×10^{-14} and 4.721×10^{-14} for $j = 0$ and $j = 1$, respectively; the corresponding channel allowances are twice as large. The exact parameter convention is the specified binary64 T, κ, with $\eta = 19/20$, $\beta_b = \kappa/1000$ and each dark rate $d = \kappa/10000$.

An independent reduction checks the first-display routing. Let $\rho_a(t)$ be the unnormalised active density under the no-jump generator, and define

$$I_r = \int_0^T \kappa \langle C_r|\rho_a(t)|C_r\rangle \, dt, \qquad J_r = \int_0^T e^{-2dt} \kappa \langle C_r|\rho_a(t)|C_r\rangle \, dt, \qquad z = e^{-2dT}. \tag{112}$$

At most one waste jump occurs on this carrier. Therefore

$$\begin{aligned} P_\varnothing &= z[1 - \eta(I_0 + I_1)], \\ D &= \tfrac{1}{2}[1 - z - \eta(J_0 + J_1 - z(I_0 + I_1))], \\ P_r &= \eta J_r + D. \end{aligned} \tag{113}$$

Here D includes dark displays before a later physical flag, and dark displays after unobserved waste or buffer loss. Two independent augmented 11-component exponentials evaluate the integrals and agree with the complete register calculation. Conservation of trace and positivity of the endpoint populations are also checked.

The full joint table retains each displayed label together with the physical flag 0, flag 1, or absence of a flag. Composing that table with the declared finite amplifier uses the physical flag as its seed. It retains all nine display/band combinations. With the ideal phase-correct interface, the probabilities of both a correct electronic display and a correct final amplifier band are enclosed by

$$\begin{aligned} j = 0: &\quad [0.92622207828679944441,\ 0.92622207828683646610], \\ j = 1: &\quad [0.92714201122237865558,\ 0.92714201122247307430]. \end{aligned} \tag{114}$$

Exact rational interval arithmetic performs this composition. Keeping only the correctly seeded contributions and applying the declared quiet hold condition gives lower bounds 0.92622205857372363691 and 0.92714199156216410040 for a correct display, correct band, and no subsequent exit during the hold. These are unconditional probabilities in the comparison model; no blank or failed run is discarded. The physical receiver/preparation bounds, the full-cosine comparison and all remaining implementation errors must be added separately.

Memory conservation makes the electronic effects diagonal in j. Thus an arbitrary coherent binary input ρ has label probability $\rho_{00}P(r|0) + \rho_{11}P(r|1)$. This marginal calculation does not pinch the source state or certify the entire postmeasurement instrument from basis statistics. The uniform channel comparison supplies that separate control. Neither the matrix exponential nor the interval composition derives an exclusive actual history.

[1] FLINT, *Matrices over the complex numbers*, `https://flintlib.org/doc/acb_mat.html`.

11 Event probabilities and conditional quantum states

The quantum instrument supplies normalized weights. Their identification with probabilities of exclusive original-run events requires a statistical law for those events. We first prove exactly what an affine, calibrated law determines. The existence and scope of actual records are analyzed in Section 13.

11.1 Affine preparation consistency and exact calibration

A sole actual record value does not determine a distribution across runs. In particular, a branch-relative conditional label cannot silently replace the declared exclusive original-run event in the following assumption.

For a general finite register, let $\mathcal{K}$ be its quantum carrier, with orthogonal projections Q_j, $j \in \mathcal{J}$, summing to $I_{\mathcal{K}}$. Zero projections are allowed. An actual readout law assigns probabilities $p_j(\sigma)$ to normal density operators on this declared carrier. Two further operational conditions have a precise consequence.

Assumption 11.1 (Affine preparation law and calibration)**.** The readout has one actual label per original readout execution. Its probabilities depend on the prepared density operator, rather than on a distinct ensemble decomposition producing the same operator. They are nonnegative, sum to one, and satisfy

$$p_j(a\sigma + (1-a)\tau) = ap_j(\sigma) + (1-a)p_j(\tau), \qquad 0 \le a \le 1. \tag{115}$$

Calibration is exact on each register sector: for every unit $u \in \operatorname{ran} Q_k$,

$$p_j(|u\rangle\langle u|) = \delta_{jk}. \tag{116}$$

Dependence only on the density operator is a preparation-noncontextual statistical assumption for this specified readout, not a claim that the density operator is the complete physical ontology. Its application to the whole declared preparation class is substantive. The corpus's positivity of a quantum source is not by itself a proof of (115) for actual events.

Theorem 11.2 (Uniqueness of a calibrated affine event law)**.** *Under Assumption 11.1,*

$$p_j(\sigma) = \operatorname{Tr}(\sigma Q_j) \tag{117}$$

for every normal density operator. No separate normality assumption on p_j is required. Applied to the ideal copied or stored code state, this gives exactly $p_r = w_r(\rho)$, including the weights of every finite-pulse receiver label.

Proof. For a positive trace-class $T \neq 0$, set $\ell_j(T) = \operatorname{Tr}(T)p_j(T/\operatorname{Tr} T)$ and set $\ell_j(0) = 0$. Affinity proves additivity on the positive cone and positive homogeneity. Extending by differences of positive operators gives a well-defined real-linear functional on self-adjoint trace-class operators. Since $0 \le \ell_j(T) \le \operatorname{Tr} T$ for $T \ge 0$, its absolute value on a self-adjoint T is at most $\|T\|_1$, by the positive/negative decomposition. Its complex extension is continuous and is represented by a bounded effect $0 \le F_j \le I$ under trace-class duality.

For unit $u \in \operatorname{ran} Q_j$, calibration gives $\langle u, (I - F_j)u\rangle = 0$. Positivity implies $(I - F_j)^{1/2}u = 0$, so $F_j u = u$. For $u \in \operatorname{ran} Q_k$, $k \neq j$, it similarly gives $F_j^{1/2}u = 0$, so $F_j u = 0$. The finite orthogonal decomposition therefore forces $F_j = Q_j$. If $Q_j = 0$, all sectors give zero and the same conclusion holds. The final application uses the record projections and Theorem 3.5. ☐

This is a conditional derivation of the event weights from mixture consistency and calibration. It does not derive actual record occurrence, or establish those statistical assumptions from the unification theorem. It is stronger than merely demanding agreement with the Born weights as a single unexplained formula: it exposes exactly the preparation and calibration requirements that fix them. An always-select-a-supported-label rule can satisfy exclusivity and ideal eigenstate calibration while violating affinity; it is therefore not a counterexample to this theorem. Conversely, a deterministic affine rule on all preparations cannot obey nontrivial two-sector calibration, since a half-mixture of two calibrated sector states must give half-weights.

11.2 Conditional states and the exact compatibility requirement

Fix a copied or ideally stored state Γ on its code subspace. Write $w_r = \mathrm{Tr}(\Gamma Q_r)$ and, for $w_r > 0$, $\sigma_r = Q_r \Gamma Q_r / w_r$, understood on its source–receiver sector. The record measure alone does not determine a physical conditional quantum state after an event. The required joint condition can be stated without concealing it in the probability calculation.

Proposition 11.3 (Classical–quantum compatibility)**.** *Suppose actual labels have probabilities p_r and conditional normal quantum states τ_r on the corresponding sectors $\operatorname{ran} Q_r$ whenever $p_r > 0$. Their predictions agree with Γ on every bounded block-diagonal observable if and only if*

$$p_r = w_r, \qquad \tau_r = \sigma_r \quad \textit{when } w_r > 0. \tag{118}$$

Zero-weight conditional states are undetermined.

Proof. Agreement means $\sum_r p_r \mathrm{Tr}(\tau_r O_r) = \mathrm{Tr}(\Gamma \sum_r Q_r O_r Q_r)$ for every bounded sector family. Taking one $O_r = I$ and all others zero fixes $p_r = w_r$. Testing arbitrary bounded O_r then determines the positive-weight trace-class state uniquely. Substitution proves the converse. □

Equation (118) is an exact equivalence, not an independent proof that physical registration implements sector conditioning. At the end of copying, its source marginal is $L_r \rho L_r^* / w_r$, with L_r from (23). The copying-time source evolution must not be omitted. If acquisition has already produced the joint state, registration acts with its record projections; applying K_r to that state again would duplicate acquisition.

The equivalence also identifies a domain of agreement. The coherent state and its block-dephased completion agree on record-compatible observations. They need not agree on the global coherence witness of Theorem 13.3. No claim that a registration completion preserves every possible coherent laboratory operation follows from record compatibility.

12 Coherent calibration and finite-sample event probabilities

The exact affinity and calibration theorem in Section 11 identifies an event effect from its preparation law. Its actual-event, density-operator and mixture premises remain in force here. Imperfect calibration requires a stronger analysis than replacing exact probabilities by nearby frequencies: definite-input calibration can conceal a larger error on a coherent preparation, and a readout matrix does not specify a selective state update.

Let Q be a projection on the admitted separable source–memory Hilbert space. The affine representation of Theorem 11.2 gives an effect $0 \le F \le I$ with $p(\rho) = \mathrm{Tr}(\rho F)$ for every admitted normal density operator. Suppose its uniform calibration errors obey

$$\langle u, Fu \rangle \le \varepsilon_0 \quad (u \in \operatorname{ran}(I - Q)), \qquad \langle v, (I - F)v \rangle \le \varepsilon_1 \quad (v \in \operatorname{ran} Q), \tag{119}$$

for unit sector vectors. No finite sample of one vector in a higher-dimensional sector is presumed to establish this uniform condition.

Theorem 12.1 (Sharp robust binary calibration)**.** *Under* (119)*, set $\varepsilon = \max(\varepsilon_0, \varepsilon_1) \le 1$. Then*

$$\|F - Q\| \le \sqrt{\varepsilon}, \qquad |p(\rho) - \mathrm{Tr}(\rho Q)| \le \sqrt{\varepsilon} \tag{120}$$

for every normal density operator. If $[F, Q] = 0$, the sharper bound is $\|F - Q\| \le \varepsilon$. Both bounds are sharp.

Proof. Write $D = F - Q$. Since $F^2 \le F$,

$$\begin{aligned} D^2 &\le F - FQ - QF + Q \\ &= (I-Q)F(I-Q) + Q(I-F)Q \le \varepsilon I. \end{aligned} \tag{121}$$

Thus $\|D\|^2 \le \varepsilon$, and trace duality gives the probability bound. If F commutes with Q, the two diagonal blocks of D have norms at most ε_0 and ε_1, giving the sharper result. For sharpness take $Q = |1\rangle\langle 1|$ on $\mathbb{C}^2$ and

$$F_\pm = \begin{pmatrix} \varepsilon & \pm\sqrt{\varepsilon(1-\varepsilon)} \\ \pm\sqrt{\varepsilon(1-\varepsilon)} & 1-\varepsilon \end{pmatrix}. \tag{122}$$

These are rank-one projections with both calibration errors ε and $\|F_\pm - Q\| = \sqrt{\varepsilon}$. The commuting effect $F = \varepsilon(I-Q) + (1-\varepsilon)Q$ attains the second bound. $\square$

The two effects in (122) have the same complete binary calibration matrix on the definite inputs $|0\rangle, |1\rangle$. Nevertheless, for $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ their probabilities are

$$p_\pm(+) = \tfrac{1}{2} \pm \sqrt{\varepsilon(1-\varepsilon)}. \tag{123}$$

For $\varepsilon = 0.01$ these are approximately 0.599499 and 0.400501, while the diagonal effect gives 0.5. Thus a one-percent calibration error on definite inputs alone does not certify one-percent accuracy on coherent inputs. The operator bound in (120) is 0.1. Finite tests of one state in a higher-dimensional sector also do not establish the uniform condition (119); sector universality requires a proved device structure or additional preparation/tomographic control.

12.1 When the detector really has a classical calibration matrix

Let $Q_1, \ldots, Q_k$ be orthogonal stored-memory projections summing to I on the declared carrier, and let E_z be the actual readout POVM, $z = 1, \ldots, m$. All terminal outcomes, including blank and failure, are retained. Define $w_j(\rho) = \mathrm{Tr}(\rho Q_j)$.

Proposition 12.2 (Exact diagonal criterion and a controlled defect)**.** *For a column-stochastic matrix C, the identity*

$$p_z(\rho) = \sum_j C_{zj} w_j(\rho) \quad \textit{for every normal } \rho \tag{124}$$

holds if and only if

$$E_z = \sum_j C_{zj} Q_j. \tag{125}$$

If instead $\|E_z - \sum_j C_{zj} Q_j\| \le \eta_z$, then $|p_z(\rho) - (Cw(\rho))_z| \le \eta_z$ and $\|p(\rho) - Cw(\rho)\|_1 \le \sum_z \eta_z$.

Proof. Substitution proves sufficiency and the defect bounds. Necessity follows because normal density operators separate bounded self-adjoint operators: equality of all vector-state expectations of $E_z - \sum_j C_{zj} Q_j$ forces that operator to vanish. $\square$

Commutation $[E_z, Q_j] = 0$ alone does not force the scalar value C_{zj} throughout a multidimensional sector. Equation (125) is the stronger relevant property. For a detector with memory-controlled amplitudes, suppose every observed and blank instrument component is a diagonal function of the memory projections, up to a common output unitary. Summing its adjoint products then gives (125) with coefficients derived from the full detector dynamics. A Kraus family $M_{z\alpha} = U \sum_j m_{z\alpha,j} Q_j$, for example, gives $C_{zj} = \sum_\alpha |m_{z\alpha,j}|^2$. Continuous latent times replace the sum by its integral. Orthogonality of the Q_j removes cross terms in each effect even when a conditional state retains coherences between sectors.

This is a positive consequence of the specified apparatus structure. The ambiguity in (122) does not change an instrument whose full operators have already been fixed. It shows what definite-input calibration data alone cannot establish about an unknown apparatus. Laboratory transfer requires warranted bounds on model defects, preparation errors and drift. If calibration reference states are not exactly supported in their declared sectors, their induced systematic probability errors must be included in those bounds.

In applications to the occupied AASC gauge–Higgs–fermion source, the stored memory Q_j remains distinct from the source's fermion number: preservation of a copied memory does not make a number-changing Yukawa source QND. Likewise C fixes event probabilities, not the complete selective state update. Conditional states require the resolved instrument; population correction below is not a replacement for it.

12.2 Finite-sample confidence with every output retained

Assume independent repetitions of a fixed calibration and test protocol, with a constant actual readout law during the data collection. In calibration column j, take n_j trials in a declared sector reference state; let $\widehat{C}_{zj}$ be each output frequency. Take n_* test trials of the fixed unknown preparation and let $\widehat{p}_z$ be their frequencies. Let C denote the true reference probabilities and assume the test probabilities obey

$$|p_z - (Cw)_z| \le \eta_z \tag{126}$$

for the target weight vector w in the simplex. This explicitly separates systematic transfer error from sampling error.

Put $M = m(k+1)$ and, for $0 < \delta < 1$, choose

$$b_j = \sqrt{\frac{\log(2M/\delta)}{2n_j}}, \qquad b_* = \sqrt{\frac{\log(2M/\delta)}{2n_*}}. \tag{127}$$

Larger conservative radii are also allowed. The elementary two-sided concentration bound $\Pr(|\widehat{q}-q| > b) \le 2e^{-2nb^2}$ for independent Bernoulli indicators is Hoeffding's bound.[32, Theorem 1, Eq. (2.3)]. The inequality is applied separately to each output indicator; independence between different frequencies in a multinomial sample is not required.

Theorem 12.3 (Simultaneous calibration and population confidence set)**.** *With probability at least $1-\delta$, all calibration and test entries satisfy*

$$|\widehat{C}_{zj} - C_{zj}| \le b_j, \qquad |\widehat{p}_z - p_z| \le b_*. \tag{128}$$

On this same event the true w belongs to the closed polytope

$$\begin{aligned} \mathcal{W}_\delta &= \{v \in \Delta_{k-1} : g_z(v) \le 0 \text{ for all } z\}, \\ g_z(v) &= \left| \widehat{p}_z - \sum_j \widehat{C}_{zj} v_j \right| - b_* - \sum_j b_j v_j - \eta_z, \end{aligned} \tag{129}$$

where $\Delta_{k-1} = \{v \ge 0 : \sum_j v_j = 1\}$ is the probability simplex. Consequently minimizing and maximizing any linear population quantity over this set gives a simultaneous confidence interval with the same coverage guarantee.

Proof. There are M scalar frequency estimates. Each has failure probability at most δ/M by the displayed concentration inequality, so the union bound proves (128). This does not require independence between different frequency estimates. For the true w, the triangle inequality gives

$$|\widehat{p}_z - (\widehat{C}w)_z| \le |\widehat{p}_z - p_z| + |p_z - (Cw)_z| + \sum_j w_j |C_{zj} - \widehat{C}_{zj}| \le b_* + \eta_z + \sum_j b_j w_j.$$

Thus $w \in \mathcal{W}_\delta$. The absolute-value inequality is a pair of linear inequalities in v, and a linear functional's extrema contain its value at every feasible vector. □

The confidence statement is conditional on the stated repetition, preparation and transfer premises. Increasing the number of trials shrinks the b terms, not the independent η_z. Under those premises an empty set has probability at most δ and is a statistically qualified inconsistency signal, not permission to discard inconvenient outputs. A nonempty set does not prove the physical assumptions or remove the coherent-input ambiguity of an uncharacterized apparatus.

12.3 Identifiability and stable population correction

Proposition 12.4 (Linear inverse bound)**.** *A column-stochastic matrix C identifies every simplex vector w from Cw if and only if it has full column rank. If the observed $\widehat{C}$ has a left inverse L, the raw estimate $\widetilde{w} = L\widehat{p}$ obeys, on* (128)*,*

$$\|\widetilde{w} - w\|_1 \le \|L\|_{1\to 1}\left(mb_* + m\max_j b_j + \sum_z \eta_z\right). \tag{130}$$

A computed approximate inverse with residual $R = L\widehat{C} - I$ adds $\|R\|_{1\to 1}$ to this bound. The inverse may be selected from the observed calibration data; the simultaneous event makes the bound valid for that selection.

Proof. Full rank gives a left inverse. Conversely, if $Cv = 0$ with $v \neq 0$, column stochasticity gives $\sum_j v_j = 0$. A sufficiently small positive and negative displacement of an interior simplex point along v gives two distinct probabilities with the same image. For the error bound, write $\widetilde{w} - w = L(\widehat{p} - p) + L(p - Cw) + L(C - \widehat{C})w$. Use $\|w\|_1 = 1$, the maximum column-sum norm, and (128). If $L\widehat{C} \neq I$, add Rw. □

A large inverse norm correctly reports poor identification. A raw linear estimate can have negative entries; the confidence polytope respects probability constraints directly. An arbitrary projection is not silently assumed to preserve the same error constant. In the binary case a particularly simple nonexpansive correction is available.

Choose any one retained output, for example observed label 1, and set $a_j = C_{1j}$ for prepared memory $j = 0, 1$. With $\theta = w_1$, $p_1 = a_0 + (a_1 - a_0)\theta$ in the exact model. Write $\widehat{a}_j = \widehat{C}_{1j}$ and $\widehat{d} = \widehat{a}_1 - \widehat{a}_0$. For $\widehat{d} \neq 0$, define

$$\widehat{\theta} = \operatorname{clip}_{[0,1]}\left(\frac{\widehat{p}_1 - \widehat{a}_0}{\widehat{d}}\right). \tag{131}$$

On (128),

$$|\widehat{\theta} - \theta| \le \min\left\{1, \frac{b_* + \max(b_0, b_1) + \eta_1}{|\widehat{d}|}\right\}, \qquad |a_1 - a_0| \ge |\widehat{d}| - b_0 - b_1. \tag{132}$$

To prove the first inequality, substitute the true θ into the numerator and bound its residual by $b_* + (1-\theta)b_0 + \theta b_1 + \eta_1$. Divide by $|\widehat{d}|$; clipping to $[0,1]$ cannot increase distance from θ. The second inequality is the reverse triangle inequality. A strictly positive right-hand side certifies a true nonzero response contrast.

The estimate concerns the population of stored sectors. It neither corrects the value of each individual record nor proves that a sole actual event occurred. It also retains the unconditional frequency of label 1. Discarding blanks would replace it by a conditional ratio; when detection efficiency depends on the input sector, that ratio is not the same linear model.

12.4 Using the current detector without confusing statistical and physical error

For the dressed finite detector with the declared apparatus ready state fixed, the admitted input is the two-dimensional memory code. Its one-dimensional memory sectors reduce the complete instrument, so the effects pulled back to that initialized input code satisfy (125), with the original-run blank retained as a third row. The numerical columns and their directed enclosures are those of the current common-deadline calculation, rather than independently fitted success-conditioned frequencies. A classical correction may use those columns while the joint material state continues to require the full instrument.

The unprojected monitored cosine model is related to that target by the complete initialized-input bound $\delta_D < 0.063241$. Each terminal event probability can consequently differ by at most $\delta_D/2 < 0.0316205$, before additional source/preparation or interface errors. This is an available conservative systematic allowance for a comparison on that same admitted input domain, not an estimate of the actual discrepancy and not a confidence radius that shrinks with sample size. Exact centrality of the finite comparator is not thereby established for the unprojected model. If calibration reference states, controls or incoming preparations differ from the specified comparison, their transfer errors must also be bounded.

Within a stipulated simulation of the exact finite comparator one may choose $\eta_z = 0$. Such a convention describes the simulation, not a measured apparatus. The confidence theorem then predicts coverage over repeated simulated data; it does not establish an observed physical preparation law. Conversely, actual calibration data may constrain a response more sharply than this conservative model bound, provided the coherent-input, sector-uniformity and drift obligations are met. The central-effect criterion states exactly which missing operator claim a basis-only calibration would otherwise leave open.

These results strengthen the conditional probability analysis: imperfect calibration has a sharp coherent bound, a full response matrix has a precise operator criterion, and population inference has a simultaneous finite-sample certificate. They neither infer the outcome of an individual trial from its corrected frequency nor establish that a unique original-run event exists.

13 Actual records and a complete history law

The physical construction specifies response operators, time-dependent weights, conditional states and stable record tests. These outputs must be distinguished from one exclusive actual result of the original run. Every non-degenerate determinate source carrier, apparatus incidence and retained record is necessarily kernel-governed, as Section 2 explains. The separate question is which occupied history carries all of a run's actual value-bearing occurrences. Necessary governance is retained in full; it does not supply an unstated occurrence or stochastic-law premise.

13.1 Original-run exclusivity and the record algebra

The finite algebra makes the original-run quantifier explicit.

Definition 13.1 (Determinate-registration premise, REG)**.** For each original run in the declared completed-registration scope, at least one owned actual value-bearing record occurrence exists. All such occurrences are covered by one complete physical record valuation. There is no additional competing actual code value for that same run. Its fixed, representative-independent readout is a map $v : \mathcal{A}_{\mathrm{rec}} \to \mathbb{C}$ that is complex-linear, unital, multiplicative and star-preserving.

Actual occurrence and original-run exclusivity in REG are substantive. A branch-local profile selected from several actual successors would not satisfy this complete-run premise. Representative independence may be justified by source-preserving physical-record comparison; the existence of the profile and the dispersion-free character of its readout require their own physical justification. A named certificate would not prove them.

Proposition 13.2 (Unique value conditional on REG)**.** *REG determines exactly one $r \in \mathsf{R}$, with $v = \chi_r$. All memories then have that same encoded value. Conversely, a sole actual code* *value r covering every owned actual value-bearing occurrence of the original run, supplied with evaluation of its record functions, gives REG.*

Proof. Idempotence gives $v(Q_r)^2 = v(Q_r)$, so every value is zero or one. Since the projections sum to the unit, exactly one value is one. Linearity gives $v(f) = f(r)$ for every record function. Each code string is constant. Conversely, evaluation at any code value is a unital multiplicative star-preserving linear map. □

This equivalence exposes REG as a definite-value premise, not a derivation of definite values from a mixed state. A unique quotient identity for an already realized physical record does not imply that the quotient has only one possible record or that a record occurs in every run. AASC or CSP-R comparisons may constrain its lawful identity and readout; their application must not silently insert occurrence. Even REG for every run does not identify the distribution of its characters with μ_ρ. That identification needs an actual-event probability theorem or an explicitly adopted operational Born postulate.

13.2 A complete instrument and its record algebra

Let $\mathcal{H}$ and $\mathcal{K}$ be separable Hilbert spaces. The finite alphabet Z contains every declared output, including unresolved, wrong-output and failure statuses when present. For each $z \in Z$, let $K_{z\alpha} : \mathcal{H} \to \mathcal{K}$ be bounded operators, with a finite or countable auxiliary index, such that

$$\sum_{z,\alpha} K_{z\alpha}^* K_{z\alpha} = I_{\mathcal{H}} \quad \text{in the strong operator topology.} \tag{133}$$

They define the normal quantum instrument

$$\mathcal{I}_z(\rho) = \sum_\alpha K_{z\alpha} \rho K_{z\alpha}^*, \qquad p_z(\rho) = \operatorname{Tr} \mathcal{I}_z(\rho), \qquad E_z = \sum_\alpha K_{z\alpha}^* K_{z\alpha}. \tag{134}$$

Here ρ is a normal density operator and the positive output sum converges in trace norm. The finite-outcome restriction does not truncate the physical source Hilbert space. A finite time-bin protocol can be treated by its complete integrated instrument, whenever a countable Kraus representation on the stated separable carriers is available. No claim of a finite Kraus rank for an integrated detector channel is needed.

Let $\mathcal{E}$ have orthonormal basis $\{|z, \alpha\rangle\}$, put

$$V\psi = \sum_{z,\alpha} K_{z\alpha}\psi \otimes |z, \alpha\rangle, \quad P_z = I_{\mathcal{K}} \otimes \sum_\alpha |z, \alpha\rangle\langle z, \alpha|, \quad \Omega_\rho = V\rho V^*, \tag{135}$$

and define $\Delta_Z(X) = \sum_z P_z X P_z$. The record-compatible algebra consists of all bounded A satisfying $[A, P_z] = 0$ for every z. It contains the experimentally relevant tests

$$A = \sum_z A_z \otimes \sum_\alpha |z, \alpha\rangle\langle z, \alpha|, \qquad A_z \in \mathcal{B}(\mathcal{K}). \tag{136}$$

Declaring this algebra fixes a scope of comparison; it is not a proof that coherent recombination is impossible in every larger laboratory.

Theorem 13.3 (Exact record agreement and distinct completions)**.** *The map V in* (135) *is an isometry. The coherent state Ω_ρ and the block-dephased state $\Delta_Z(\Omega_\rho)$ give the same expectation for every record-compatible observable. In particular,*

$$\operatorname{Tr}(\Omega_\rho A) = \sum_z \operatorname{Tr}\big(\mathcal{I}_z(\rho) A_z\big) \tag{137}$$

for (136)*. There is also a single-event completion with exactly one classical label z, probability $p_z(\rho)$, and conditional extended state*

$$\omega_z = \frac{P_z \Omega_\rho P_z}{p_z(\rho)}, \qquad \mathrm{Tr}_{\mathcal{E}}\, \omega_z = \frac{\mathcal{I}_z(\rho)}{p_z(\rho)} \quad (p_z(\rho) > 0). \tag{138}$$

Its average is $\Delta_Z(\Omega_\rho)$, so it agrees with the coherent account on the same algebra. If the input is pure and two distinct labels z, u have positive weight, there is a bounded self-adjoint W, with $\|W\| = 1$, for which

$$\mathrm{Tr}(\Omega_\rho W) = 2\sqrt{p_z p_u}, \qquad \mathrm{Tr}(\Delta_Z(\Omega_\rho) W) = 0. \tag{139}$$

Thus this agreement does not extend to all coherent observations.

Proof. Equation (133) gives $\|V\psi\|^2 = \sum_{z,\alpha} \|K_{z\alpha}\psi\|^2 = \|\psi\|^2$; therefore the defining orthogonal series converges and $V^*V = I$. If A commutes with every P_z, then $\sum_z P_z A P_z = A$, and trace duality gives $\mathrm{Tr}(\Delta_Z(\Omega_\rho)A) = \mathrm{Tr}(\Omega_\rho A)$. Taking the partial trace of $P_z \Omega_\rho P_z$ gives $\mathcal{I}_z(\rho)$ and proves both (137) and (138). Positivity and normalization make the specified classical distribution and conditional states a valid mathematical completion. No conditional state is constrained at a zero-probability label.

For $\rho = |\psi\rangle\langle\psi|$, write $\Psi = V\psi$ and $\Phi_z = P_z\Psi/\sqrt{p_z}$, $\Phi_u = P_u\Psi/\sqrt{p_u}$. The two vectors are orthonormal. Set $W = |\Phi_z\rangle\langle\Phi_u| + |\Phi_u\rangle\langle\Phi_z|$, zero on their orthogonal complement. Its eigenvalues there are $1, -1$, and $\langle\Psi, W\Psi\rangle = 2\sqrt{p_z p_u}$. Every diagonal sector of W is zero, proving the second equality in (139). □

The theorem is an exact sufficiency test for the stated quantum data. Those data permit a coherent account with correlated record sectors; adjoining a unique actual label gives another account with the same specified record functionals. The unique-label claim is not a consequence of their equality. The coherent comparator does not thereby satisfy the exclusive original-run requirement REG: relative record content and one exclusive owned value are different targets. Conversely, the displayed single-event completion is not claimed to preserve all global coherent interventions. A richer theory that retains a universal coherent state while adding an actual variable requires its own analysis; the theorem does not identify every such theory with physical dephasing.

The quantum instrument already contains the numerical Born functional $p_z = \mathrm{Tr}(\rho E_z)$. Calling this the frequency law of actual original-run events requires either a physical event law reproducing that functional or explicit assumptions that imply this compatibility. The theorem does not derive actuality by treating the word "outcome" in the definition of an instrument as an occurrence axiom.

13.3 A deterministic representation of the added event law

Proposition 13.4 (Seed representation and finite adaptive histories)**.** *Order the finite alphabet and define $c_0(\rho) = 0$ and $c_j(\rho) = \sum_{i \le j} p_i(\rho)$. If an additional variable U is uniformly distributed on $[0, 1)$, the rule*

$$R(\rho, U) = j \quad \Longleftrightarrow \quad c_{j-1}(\rho) \le U < c_j(\rho) \tag{140}$$

has exactly one value and distribution $p_j(\rho)$. Together with the conditional state in (138)*, it realizes the single-event completion of Theorem 13.3.*

For a finite adaptive instrument tree of depth at most n, independent uniform variables $U_1, \ldots, U_n$, the same rule at each reached node, and the specified conditional quantum updates give every complete history $h = (z_1, \ldots, z_k)$ probability

$$\Pr(h) = \mathrm{Tr}\big(\mathcal{I}_{z_k}^{h_{<k}} \circ \cdots \circ \mathcal{I}_{z_1}^{\varnothing}(\rho)\big). \tag{141}$$

The history is deterministic given the initial preparation, the controls and all the seeds. Early terminal statuses may be padded with an absorbing stop symbol.

Proof. The half-open intervals in (140) partition $[0, 1)$; a zero-width interval is empty. Their lengths are $p_j(\rho)$, proving the one-step claim. At a reached node of positive weight, normalization gives the trace-normalized conditional input state. Multiplying its next conditional probability by the prefix probability cancels that normalization and yields the trace of the composed instrument. Induction proves (141). Zero-weight prefixes are never reached except on a null set; their stipulated continuation does not alter the measure. The complete alphabet at each node, including terminal failures, makes the total history law normalized. □

This is a constructive representation of an already specified probability kernel, not a physical derivation of its seed, preparation law or actuality. Its interval lengths were defined using the desired instrument weights; it would be circular to present those lengths as an independent derivation of the Born rule. Equally, the representation shows that the word "stochastic" is not forced by the event statistics alone: the same finite law has a deterministic enlarged-state representation. Establishing that such variables are occupied physical source coordinates, with an independently justified preparation measure and appropriate dynamics, is further work.

13.4 The precise linearity obstruction

Theorem 13.5 (Calibrated linear evolution cannot produce only one record sector)**.** *Let $|0\rangle, |1\rangle$ be orthogonal input states, $|a\rangle$ one common ready state, and V the isometry of a declared complete quantum description, including the apparatus and environment. Suppose normalized vectors Φ_0, Φ_1 obey*

$$V(|j\rangle \otimes |a\rangle) = \Phi_j, \qquad Q_k\Phi_j = \delta_{kj}\Phi_j, \tag{142}$$

where Q_0, Q_1 are orthogonal record projections. For nonzero α, β with $|\alpha|^2 + |\beta|^2 = 1$, the output on $(\alpha|0\rangle + \beta|1\rangle) \otimes |a\rangle$ lies in neither record sector alone. Therefore the following three requirements cannot all hold on this input family: complete quantum-state description; the same calibrated linear isometry; and a final complete quantum state supported in exactly one of these sectors for every input.

Proof. Linearity gives $\Psi = \alpha\Phi_0 + \beta\Phi_1$. Orthogonality gives $\|Q_0\Psi\|^2 = |\alpha|^2 > 0$ and $\|Q_1\Psi\|^2 = |\beta|^2 > 0$. Neither projection fixes Ψ. This contradicts the third requirement while retaining the first two. A phase-adjusted version of the witness in (139) distinguishes the coherent vector from its sector mixture by $2|\alpha\beta|$. □

The conclusion concerns support of the *complete quantum state*. A physical variable beyond that state, modified dynamics or an explicitly relative outcome target changes a declared premise. The theorem does not exclude those possibilities. Tracing out an environment changes the state description and may remove access to the witness; it is not a proof that the original complete run acquires one exclusive actual value. Perfect calibration and irreversible-looking reduced dynamics do not change this quantifier.

A related finite obstruction makes a deterministic selection requirement transparent. If fixed data D are invariant under a relabeling g that exchanges two candidate labels, no deterministic label-valued map s can both obey $s(gD) = gs(D)$ and choose one of those two labels. Indeed $D = gD$ would require the chosen label to be fixed by their exchange. An additional actual asymmetry or a changed domain can remove this obstruction. An invariant probability distribution is also possible, but a distribution alone is not one actual draw. BTF supplies precisely a conditional positive route: its Theorem 6.1 requires an independently defined stable, outcome-independent boundary trace with exactly one positive candidate (E4). It does not derive that singleton condition merely from the presence of a bath. [33, Defs. 4.7–4.8, 5.1 and Thm. 6.1; PDF pp. 10–14].

13.4.1 Imperfect calibration does not remove the obstruction

The exact-support argument above is a familiar part of the measurement problem. Bassi and Ghirardi treat imperfect reliability in a more general formulation of macroscopic configurations [34]. The quantitative corollary below uses an explicitly declared orthogonal record partition. Here a direct norm estimate connects that issue to any explicitly declared nonideal instrument.

Corollary 13.6 (A quantitative obstruction from noisy calibration)**.** *Let $\Phi_j = V(|j\rangle \otimes |a\rangle)$ as above, with the same complete linear isometry and common ready state. Let $\{P_z\}$ be a mutually orthogonal record partition including labels $0, 1$ and every declared blank or failure cell. Suppose*

$$\|P_j\Phi_j\|^2 \geq 1 - \varepsilon_j, \qquad 0 \leq \varepsilon_j \leq 1. \tag{143}$$

For $\Psi = \alpha\Phi_0 + \beta\Phi_1$ with $|\alpha|^2 + |\beta|^2 = 1$,

$$\|P_0\Psi\| \geq \left[|\alpha|\sqrt{1-\varepsilon_0} - |\beta|\sqrt{\varepsilon_1}\right]_+, \tag{144}$$

$$\|P_1\Psi\| \geq \left[|\beta|\sqrt{1-\varepsilon_1} - |\alpha|\sqrt{\varepsilon_0}\right]_+, \tag{145}$$

where $[x]_+ = \max(0, x)$. In particular, if $|\alpha| = |\beta| = 1/\sqrt{2}$ and $\varepsilon = \max_j \varepsilon_j < 1/2$, each of these two sector probabilities is at least

$$L(\varepsilon) = \frac{1}{2} - \sqrt{\varepsilon(1-\varepsilon)} > 0. \tag{146}$$

For every density operator τ supported in a single cell of this same partition,

$$\big\||\Psi\rangle\langle\Psi| - \tau\big\|_1 \geq 2L(\varepsilon). \tag{147}$$

The conventional trace distance, half this norm, is therefore at least $L(\varepsilon)$.

Proof. Since $P_0 \leq I - P_1$, $\|P_0\Phi_1\| \leq \sqrt{\varepsilon_1}$. The reverse triangle inequality gives the first bound, and interchanging the labels gives the second. For balanced coefficients, the squared lower bound is $(\sqrt{1-\varepsilon} - \sqrt{\varepsilon})^2/2 = L(\varepsilon)$. If τ is supported in cell 0, its weight in cell 1 is zero; if it is supported in any other single cell, its weight in cell 0 is zero. In either case, one of the two projections distinguishes $|\Psi\rangle\langle\Psi|$ from τ by at least L. The variational formula for trace distance gives (147). A coarse cell combining 0 and 1 is excluded by the stipulated partition. □

13.5 Preparation and response on an occupied source carrier

The common-source construction contains more physical structure than a finite instrument. CSPP constructs complete native quantum sources, exact source operations and physically non-null history reconstruction, and a reference-resolved source with central classical observations [3, Secs. 7.4–7.5 and 26.3–26.4]. Its reference construction provides a concrete candidate carrier for an event extension. The following results identify what that carrier supplies and what remains to be attached to the detector. They concern the displayed source family; they do not assert a universal obstruction to a physical event law.

The supplied source. For the admitted periodic cubic member, let

$$\Gamma = \mathbb{T}_L^3 \rtimes \widetilde{C}, \qquad \widehat{\mathcal{H}} = L^2(\Gamma; \mathcal{H}),$$

where $\widetilde{C}$ is the 48-element cubic spin group and μ is normalized Haar measure. CSPP Proposition 26.4 constructs a faithful algebra of retained strong-star continuous bounded sections, including central continuous reference functions. Each reference evaluates to its native algebra, state and operator graphs. All normal preparations have the form

$$\omega_\rho(A) = \int_\Gamma \operatorname{Tr}[\rho(\gamma)A(\gamma)]\, d\mu(\gamma), \quad \rho(\gamma) \geq 0, \quad \int_\Gamma \operatorname{Tr}\rho(\gamma)\, d\mu = 1. \tag{148}$$

Conversely every such measurable trace-class density field is an admitted normal preparation. Point-evaluation states are positive states of the retained algebra and are generally singular in this Haar representation. The operator is decomposable, rather than the average of the native Hamiltonians. At a prescribed homogeneous metric, constant or co-transported material words evaluate to the same native word at every reference. Central reference observations and fixed ambient tests can still distinguish the preparations. These are source results, not interpretive additions.

Proposition 13.7 (Different reference preparations with one native projection)**.** *Fix this prescribed homogeneous source geometry and a normalized admitted native vector* ψ. *Write* $\theta = 2\pi y_1/L$ *for one translation coordinate. For every* $a \in [-1, 1]$, *the density field*

$$\rho_a(\gamma) = (1 + a\cos\theta)\,|\psi\rangle\langle\psi|$$

is a normal source preparation. All these preparations agree on every constant or co-transported native material word. Their central reference distributions are nevertheless different. In particular, the Borel reference event $B = \{\cos\theta \geq 0\}$ *has weight*

$$\mu_a(B) = \frac{1}{2} + \frac{a}{\pi}. \tag{149}$$

Proof. The density is nonnegative and the Haar integral of $\cos\theta$ is zero, so (148) applies. For a native word with constant section $A(\gamma) = A_0$, its expectation is $\langle\psi, A_0\psi\rangle \int(1 + a\cos\theta)d\mu = \langle\psi, A_0\psi\rangle$. The restriction to the central continuous functions has density $1 + a\cos\theta$, and therefore determines the displayed Borel reference distribution. Integrating over $[-\pi/2, \pi/2]$ gives (149). $\square$

At $a = \pm 1/2$ the weights are approximately 0.6591549431 and 0.3408450569. This is a family of distinguishable complete source states with one restricted native projection. It is not a hidden classifier beneath an unchanged complete AASC source class. Nor does the mathematical central event B by itself construct a detector: a physical threshold reader and its channel remain a separate operation to be supplied. The source permits central reference observations, but it does not identify this bin with the detector's registered sector. The proposition holds at the prescribed source-family geometry. CSPP Theorem 26.5's coupled isotropic solution also fixes a reference prescription; nonuniform densities can change its all-test metric source and cannot be inserted while holding that entire coupled solution fixed.

Proposition 13.8 (The reference marginal is conserved)**.** *For the supplied decomposable unitary propagator, every central reference function is fixed by Heisenberg evolution. In* (148), $\operatorname{Tr}\rho_t(\gamma) = \operatorname{Tr}\rho_0(\gamma)$ *almost everywhere. In particular this dynamics does not drive arbitrary normal reference preparations to the uniform Haar preparation.*

Proof. The propagator acts fiberwise by $(\widehat{U}\Psi)(\gamma) = U_\gamma\Psi(\gamma)$. Multiplication by a scalar function $f(\gamma)$ commutes with it. Equivalently, $\rho_t(\gamma) = U_\gamma\rho_0(\gamma)U_\gamma^*$ preserves the trace in each fiber. Distinct central marginals therefore remain distinct. $\square$

Haar uniqueness would fix a preparation if invariance under all reference translations and rotations were independently required of that preparation. Indeed, for an invariant probability ν on a compact group and a continuous f, averaging $\int f(g\gamma)d\nu(\gamma)$ over Haar g gives $\int f\,d\nu = \int f\,d\mu$. Such an invariance requirement concerns the prepared state. Faithful representation and covariance of a source family do not impose it on every state in (148).

An actual reference and an actual detector sector. Let $\{P_q\}$ be the detector's physically specified orthogonal sector partition, with the ready and failure sectors retained. A point-evaluation state at an actual reference can still give a nontrivial distribution on this native quantum partition.

Proposition 13.9 (Point reference does not force a pointer character)**.** *If $\omega_{\gamma,\rho}(P_r) > 0$ and $\omega_{\gamma,\rho}(P_s) > 0$ for $r \neq s$, then the restriction of this state to the commutative sector algebra is not a character. Thus identifying a reference point γ does not, by itself, supply a definite sector value for that state. If instead the prepared state is supported entirely in the ready sector A, its restriction to that algebra is the character δ_A.*

Proof. Orthogonality gives $\omega(P_r P_s) = 0$ while $\omega(P_r)\omega(P_s) > 0$, contradicting multiplicativity of a character. For ready-sector support, $\omega(P_A) = 1$ and the other projection values vanish; the sector-algebra restriction is evaluation at A. $\square$

The latter observation removes an initial lottery for an actual-sector model with this ready preparation, even for coherent memory input. It does not supply the future actual history after the quantum state has populated several sectors. A source operation R assigning the actual sector must be constructed or postulated with its own state/history dependence and law.

The roles of symmetry and continuation. Proper gauge symmetry cannot supply the missing event coordinate. For a source-admitted quotient $\pi : X \to X/G$ and a physical reader $R = \overline{R}\pi$, one has $R(gx) = R(x)$ for every admitted gauge arrow. Consequently sampling g on a fixed orbit cannot randomize R. This follows by substitution, independently of the sampling measure. It is the same quotient distinction retained by the electroweak fixation analysis [9, Thms. 4.2–4.3] and by CSPP's invariant neutral-port construction [3, Sec. 7.2]. Physical boundary or global actions are separate cases, not covered by this statement. CSPP's central reference observations must likewise not be silently identified with quotient-trivial gauge coordinates.

UC supplies an explicit complete upstream inhabitant $x = (\vartheta, q, r)$ with $r \in \mathbb{R}$ and

$$\alpha_t x = (\vartheta + t, q, r + t), \qquad \Pi_H x = R_\vartheta q, \qquad \Pi_{\mathrm{rec}} x = [r]_{\mathrm{MR}}$$

on its declared local clock slab [35, Defs. 24.1, 24.3, 24.5 and Thm. 24.7]. This is genuine nonidentity record continuation. Its supplied record translation is not a detector sector reader. It also does not supply a normalized all-real translation-invariant preparation on r: disjoint intervals $[n, n+1)$ would all have one common mass, which is either positive and gives infinite total mass, or zero and gives zero total mass. The local UC construction requires no such measure. Finite-window preparations or a new compact phase carrier remain possible specified inputs.

Even a uniquely invariant probability would not determine an event history. For example, each continuous circle flow $u_t = u_0 + vt \pmod 1$, $v > 0$, has the unique invariant probability du: averaging a continuous test over a complete period proves the assertion for any invariant probability. With the same uniform preparation and binary reader $R(u) = \mathbb{1}_{[0,1/2)}(u)$, all these models have one-time probabilities 1/2. Nevertheless

$$\Pr\{R(u_t) \neq R(u_0)\} = 2\ \mathrm{dist}(vt, \mathbb{Z}), \tag{150}$$

where the distance lies in $[0, 1/2]$. The formula is the length of the symmetric difference of two half-circle intervals. At $t = 1/4$, speeds 1 and 2 give probabilities 1/2 and 1. Every orbit has time-occupation fraction 1/2, but correlations and residence times differ. These are explicit mathematical comparison models, not alternative solutions of the fixed detector Hamiltonian. They show why invariant measure or typicality alone does not supply detector timing, independent preparations or irreversible storage.

13.5.1 The source-reader construction that would replace an event postulate

An actual-source derivation must specify an occupied state space X, preparation measures $\nu_{\rho,c}$ for independently declared preparation and control data c, physical continuation α_t, the quantum comparison, and a physical reader R_c. It must prove existence through the declared deadline or retain a failure status; cover all owned actual value-bearing occurrences of the original run; exclude competing values for that same run; derive the complete pushforward $(R_c)_*\nu_{\rho,c}$; and preserve the conditional continuations tested in subsequent operations. An initial fibre defined as "the states yielding the desired result" supplies neither physical occupation nor an independent preparation law. The seed coding in Proposition 13.4 is therefore not such a derivation.

CSP-R's complete-source criterion also excludes an independent consequential classifier beneath an unchanged complete standing-response class. A new physical coordinate must enter the actual source's response or domain distinctions [4, Lemma 2.3 and Thm. 3.5]. This condition allows richer source models while requiring their claimed completeness to be assessed on the enlarged state. Unique continuation conditional on an initial complete state does not specify which initial state is actual or how preparations populate its fibres.

AER at its observable-complete Hamiltonian target is forced by the source construction. GRQM Theorem I.1 requires outcome-relevant stochastic data to have an enlarged physical-state representation, or equivalent state-level structure, when those data are not fixed by the reduced state [36, App. I, PDF pp. 46–47]. Its displayed Markov kernel uses a transition map and noise measure; it does not independently derive them. Theorem K.2 leaves open a Born measure fixed by reduced-space invariants [36, App. K, PDF pp. 48–49], without exhibiting that measure or a detector-sector path.

The positive source imports of Section 2 remain intact. CSP-P's physically non-null realized-history theorem preserves furnished readers and failures but explicitly supplies no probability measure [3, Thms. 7.8 and 7.10]. CSP-R preserves actual operational graphs without selecting a value of an unselected relation; a unique common-bearer identity is not singleton cardinality [4, Thm. 3.5 and Cor. 5.3]. BTF fixes the record after a predeclared stable trace has exactly one positive candidate [33, Def. 5.1 and Thm. 6.1]. MR permits physical selection as a stronger realization when its dynamics or ontic/boundary structure is supplied [5, Thm. 2.13, PDF pp. 22–24]. These theorems constrain and support a proposed occupied event model. They cannot acquire a new detector reader, singleton or preparation measure by changing their input scope during application.

13.6 A bounded counting completion linked to detector dynamics

Let $H(t)$ be self-adjoint and let $L_c(t)$ be a finite family of matrices, all piecewise continuous on the declared finite detector carrier, including memory and terminal sectors. Write

$$C(t) = \sum_c L_c(t)^\dagger L_c(t), \qquad \partial_t V(t,u) = \left[-iH(t) - \tfrac{1}{2}C(t)\right] V(t,u), \quad V(u,u) = I.$$

For a normalized conditional state ρ at u, define

$$S(t \mid u,\rho) = \mathrm{Tr}[V(t,u)\rho V(t,u)^\dagger], \tag{151}$$

$$f_c(t \mid u,\rho) = \mathrm{Tr}[L_c(t)V(t,u)\rho V(t,u)^\dagger L_c(t)^\dagger], \tag{152}$$

$$\lambda_c(t \mid \rho_t) = \mathrm{Tr}[L_c(t)^\dagger L_c(t)\rho_t], \qquad \rho_t = \frac{V(t,u)\rho V(t,u)^\dagger}{S(t \mid u,\rho)}, \tag{153}$$

$$\rho^+_{t,c} = \frac{L_c(t)\rho_t L_c(t)^\dagger}{\lambda_c(t \mid \rho_t)}. \tag{154}$$

The last formula is used only for nonzero intensity. A dark electronic event can be added as an independent Poisson mark, with unchanged material state; missed photons are distinct hidden marks with the same physical transition. Thus electronic inefficiency does not delete a physical branch. The electronic register stores the first displayed mark and is held fixed thereafter, while material evolution continues. At the deadline it is blank precisely when no displayed mark occurred.

Proposition 13.10 (Exclusive path realization of a fixed counting instrument)**.** *On every finite time interval with* $\sup_t \|C(t)\| < \infty$*, these survival functions, mark densities and updates define a unique recursive probability law on finite counting histories. The process is nonexplosive; simultaneous distinct marks have probability zero. Its complete history instrument has the density*

$$\mathcal{I}_{c_1,\dots,c_n;t_1,\dots,t_n}(X) = KXK^\dagger, \qquad K = V(T,t_n)\prod_{k=n}^{1}\left[L_{c_k}(t_k)V(t_k,t_{k-1})\right], \quad t_0 = 0,$$

The ordered product places later times to the left. Integrating over all ordered times and marks, including the no-event branch, is trace preserving. Averaging the normalized trajectories gives the specified GKSL evolution.

Proof. Differentiating the survival function gives $-\partial_t S = \sum_c f_c$ and $S(u) = 1$. This defines a first-event distribution including survival to the deadline. Repeating after every mark multiplies the conditional densities; normalization factors cancel to give $\mathrm{Tr}(K\rho K^\dagger)$. The total conditional intensity is at most $\|C(t)\|$, so the event count is dominated by a finite-rate Poisson process on each finite interval. Its ordered history expansion is consequently normalized and convergent. In an infinitesimal interval, the unnormalized no-event operation is $X - i[H,X]dt - \{C,X\}dt/2$ and the event operations sum to $\sum_c L_c X L_c^\dagger dt$. Their sum is the stated generator. Appending an arbitrary reference system leaves the proof unchanged. □

This is a standard quantum-jump realization, credited to its established development in quantum optics [37]. The contribution here is its explicit application to the material converter and its separation from an actual-source derivation. In the converter,

$$L_r = \sqrt{\kappa_r}\sum_j |j, D_r\rangle\langle j, C_r|, \qquad L_E = \sqrt{\beta_b}\sum_j |j, E\rangle\langle j, A|.$$

Hence the trigger is occupation of the converting or escape sector, the waste-event intensity is $\kappa_r\,\mathrm{Tr}(P_{C_r}\rho_t)$, and the update transfers its occupied amplitude to D_r. The sum over j preserves coherent memory blocks. No new detector threshold or independent registration-rate parameter is introduced. The rates κ_r, β_b and the counting arrangement still require the bath and monitoring identification already stated in the physical model. Conditioning only on displayed marks requires filtering over missed events, not treating the displayed record as the complete material trajectory.

Assumption 13.11 (Actual-history completion)**.** One history of the declared marked process is actual for each original run, with the conditional law (153) and transition (154). The fixed reader of that history supplies the run's recorded value or its retained blank/failure status; it covers all owned value-bearing record occurrences within its declared scope, with no competing actual code value for that same run.

The proposition constructs and normalizes the law. A full history may contain several physical marks; the first-display reader is the fixed single-valued operation that assigns its electronic output. A blank at a deadline is a complete declared status, and does not assert an earlier value-bearing detection. Thus existence of a stochastic path is not confused with existence of a successful count in every finite run. For the material protocol the complete reader retains electronic and material outputs jointly. Unequal entries are a recorded mismatch, not competing actual histories. REG's constant-string code concerns the successful agreement scope; the full instrument also retains the other declared statuses. Original-run exclusivity does not assert that a noisy apparatus always reports its material state correctly.

The mathematical construction does not deduce Assumption 13.11 from ensemble dynamics. The trace intensities already contain the quantum probability rule. Calling the same calculation an independent derivation of Born probabilities would conceal that dependence. The finite-carrier hypothesis is substantive: this proposition is not an existence theorem for counting paths of an arbitrary infinite oscillator system with unbounded loss operators. Applying an actual trajectory law to the full cosine model additionally requires a well-defined monitored realization and its counting instrument on that domain. A bound between complete quantum instruments transfers their tested predictions, but does not establish a missing infinite-dimensional trajectory-existence theorem.

The unravelling also depends on the monitored environment: different monitoring arrangements can give different conditional trajectories for the same unconditioned dynamics [38].

13.7 What probability-current constraints determine

An alternative makes a detector configuration actual while retaining the unconditioned quantum state. For a finite fixed orthogonal partition $\{P_q\}$ and unitary evolution $\dot{\rho} = -i[H, \rho]$, put

$$p_q = \operatorname{Tr}(P_q \rho), \qquad J_{rq} = 2 \operatorname{Im} \operatorname{Tr}(P_r H P_q \rho).$$

Then $J_{rq} = -J_{qr}$ and $\dot{p}_r = \sum_q J_{rq}$. The familiar minimal jump rates $\sigma(r \mid q) = [J_{rq}]_+ / p_q$ are due to the Bell-process construction [39]. We use them to make the remaining freedom explicit, rather than attributing them to AASC.

Proposition 13.12 (Current does not select a unique event dynamics)**.** *On a time interval where all $p_q > 0$, every nonnegative pairwise flow with antisymmetric part J has the form*

$$F_{rq} = [J_{rq}]_+ + S_{rq}, \qquad S_{rq} = S_{qr} \geq 0 \quad (r \neq q). \tag{155}$$

Rates $\sigma_S(r \mid q) = F_{rq}/p_q$ preserve the specified marginal evolution when initialized with $p_q(t_0)$, whenever the resulting process is nonexplosive and the forward equation has its unique solution. Zero excess traffic selects $S = 0$, but that is an additional selection principle.

Proof. The condition is $F_{rq} - F_{qr} = J_{rq}$. If $J_{rq} \geq 0$, let $S_{rq} = F_{qr} \geq 0$; then $F_{rq} = J_{rq} + S_{rq}$. The other sign gives the same symmetric definition. Conversely (155) has the required difference, and the forward equation gives $\dot{p}_r = \sum_q (F_{rq} - F_{qr})$. Positivity bounded away from zero and bounded flows suffice for finite-interval nonexplosion. This is a marginal statement, not equality of full counting instruments. □

Even a unique invariant measure does not remove this freedom. For two configurations, the symmetric generator with each transition rate $a > 0$ has unique invariant distribution $(1/2, 1/2)$ for every a. Its probability of having the initial label at time t, allowing intervening departures and returns, is $(1 + e^{-2at})/2$. All these dynamics share the stationary weights and differ on temporal observations. At $a = 0$ the same weights remain invariant but are no longer uniquely invariant. Thus neither invariance alone nor uniqueness of an invariant measure identifies a physical event clock.

A potentially useful preparation simplification is exact: taking q to label only the detector sectors, an initially loaded buffer has $q = A$ with probability one for every coherent memory input. This removes a nontrivial initial probability distribution over an added *detector* coordinate. It neither makes one coordinate physically occupied by theorem nor fixes later event rates. Zero-probability sectors at readiness also require the usual extension of the jump construction; the strictly positive proposition above does not prove that extension. The bounded counting-process construction does handle the ready state directly. Neither marginal construction alone establishes the full history instrument and conditional state law; the counting proposition does so under its expressly supplied stochastic physical law.

14 Sequential records and local covariant registration

A record law must remain consistent when its output controls later operations, when readable records are stored in successive intervals, and when localized apparatuses have no common preferred ordering. The complete classical-record–retained-state instrument is the object that composes in all three cases. Its existence and consistency do not add an actual-history selection law to Section 13.

14.1 Storage uses the surviving state

For a readable band B of a finite classical pointer, let T_B be its transition generator killed at first exit and let p_B be its initial unnormalized population row. The retained mass is $S_B(t) = p_B e^{tT_B}\mathbf{1}$. This notion includes every first exit, even when a pointer returns to the band before a final readout. It is different from the endpoint-band event.

Proposition 14.1 (Successive storage intervals)**.** *For one fixed killed generator,*

$$p_B e^{t_1 T_B} e^{t_2 T_B} = p_B e^{(t_1+t_2)T_B}. \tag{156}$$

If $S_B(t_1) > 0$, the second-interval survival uses posterior population $p_B e^{t_1 T_B}/S_B(t_1)$, and

$$S_B(t_1 + t_2) = S_B(t_1) S_B(t_2 \mid \textit{survived } t_1). \tag{157}$$

Proof. The first equality is the semigroup law. Normalizing the population remaining at t_1 gives the second. A killed generator need not preserve the shape of its initial distribution, so reusing that initial population for the second factor is not justified. □

This lemma concerns a declared finite-rate storage model. The selected quiet-hold apparatus instead uses the stated conditional hazard cap and its dominating-death comparison. The same event distinction remains: retained-band survival is not preservation of every microscopic spin coherence, and an earlier successful readout is not a new preparation.

14.2 Complete sequential errors and acceptance

Theorem 14.2 (Complete adaptive instrument error)**.** *Let ideal and comparison experiments use the same finite adaptive control tree of depth at most n, padded with terminal identities. Their complete node instruments, including failure outputs and retained states, differ in diamond norm by at most δ_i at depth i, uniformly over every admitted prior history, setting and ancillary input. Suppose their initial states differ in trace norm by at most δ_0. Then the complete classical-record–retained-state outputs obey*

$$\left\|\Omega - \widetilde{\Omega}\right\|_1 \leq \Delta := \min\left\{2, \delta_0 + \sum_{i=1}^{n} \delta_i\right\}. \tag{158}$$

Every joint recorded probability law consequently differs in TV by at most $\Delta/2$.

Proof. Regard each depth as one CPTP map on the retained system and orthogonal history register. Its controlled direct sum has complete norm difference bounded by the worst admitted branch bound. Telescope the product of depth maps, changing one factor at a time. Complete trace-norm contraction of all remaining factors bounds each term by its δ_i; the initial-state term is bounded by δ_0. The norm between two states is at most two. Reading the classical register is a channel, giving the TV bound. □

The node errors must hold for the actual incoming states and uniformly across admitted past-dependent settings. The initialized-source domain can be used when the intermediate maps are known to preserve the required admission conditions. An asymptotic convergence statement without a quantitative norm estimate cannot be inserted as a numerical δ_i. The current cosine comparison supplies one such bounded stage; remaining implementation errors retain their separate status in the composed ledger.

Corollary 14.3 (Fixed-input convergence through a finite protocol)**.** *Fix a finite adaptive tree with finite label and failure alphabets, fixed copying and storage intervals, endpoint completion channels, deadlines and feedback policy. Suppose each complete node instrument $\mathfrak{E}_{h,\epsilon}$ converges in trace norm to $\mathfrak{E}_{h,0}$ on every fixed normal input state, including the retained ancillary domain. Then the complete terminal labelled quantum output converges in trace norm for each fixed initial normal state. Its full history law converges in total variation, and conditional quantum states converge on every history with positive limiting probability. In particular, this applies to the finite-pulse isometries of Theorem 3.2, followed by fixed copying, storage and completed registration channels.*

Proof. Write $X_{h,\epsilon}$ and $X_{h,0}$ for unnormalized inputs at a node. Complete instruments are trace-norm contractions on self-adjoint inputs, so

$$\|\mathfrak{E}_{h,\epsilon}(X_{h,\epsilon}) - \mathfrak{E}_{h,0}(X_{h,0})\|_1 \tag{159}$$

$$\leq \|X_{h,\epsilon} - X_{h,0}\|_1 + \|(\mathfrak{E}_{h,\epsilon} - \mathfrak{E}_{h,0})(X_{h,0})\|_1 . \tag{160}$$

The second term tends to zero on its fixed limiting positive input; homogeneity covers subnormalized states. Sum over the finite nodes of each depth and induct. Orthogonal terminal label blocks give trace-norm convergence of their direct sum. Taking the trace of each block bounds the history TV distance by half that norm. For a terminal block with limiting mass $p_h > 0$, nearby masses are positive and normalization is continuous; the normalized-state distance is at most $2\|X_{h,\epsilon} - X_{h,0}\|_1 / p_h$.

For the stated source pulse, strong convergence of the acquisition isometries persists after tensoring a fixed ancillary identity: first check finite sums of product vectors and then use density and their unit operator norms. Finite-rank approximation therefore gives trace-norm convergence on every fixed normal source–ancilla state. The fixed completed endpoint channels preserve it by contraction. □

This result retains the source evolution during copying and every fixed subsequent delay. It is stronger than convergence of a finite selection of output expectation values. It supplies no uniform rate over input energies, changing feedback policies or growing tree depth. Recorded exact times require a corresponding measurable-history extension; the finite-tree statement uses the declared finite labels and statuses. Where actual histories are asserted, their event-law premise remains that of Section 13.

Proposition 14.4 (Acceptance and conditional error)**.** *For an accepted set A in the complete record alphabet, let $p = \operatorname{Tr}\Omega_A$ and $\widetilde{p} = \operatorname{Tr}\widetilde{\Omega}_A$. Under* (158)*,*

$$|p - \widetilde{p}| \leq \Delta/2. \tag{161}$$

If $p \geq a > \Delta/2$, then $\widetilde{p} > 0$ and

$$\left\| \frac{\Omega_A}{p} - \frac{\widetilde{\Omega}_A}{\widetilde{p}} \right\|_1 \leq \min\{2, \Delta/a\}. \tag{162}$$

Proof. The binary acceptance test gives the first inequality. The accepted and rejected register blocks are orthogonal. If $d_A = \left\|\Omega_A - \widetilde{\Omega}_A\right\|_1$, the rejected block has norm at least $|p - \widetilde{p}|$, whence $d_A + |p - \widetilde{p}| \leq \Delta$. Adding and subtracting $\widetilde{\Omega}_A/p$ bounds the normalized difference by $(d_A + |p - \widetilde{p}|)/p \leq \Delta/a$. The trivial state bound is two. □

The conditional bound can be attained already by classical records. Take complete laws $(.010, 0, .990)$ and $(.009, .001, .990)$, with the first two entries accepted. Their full trace-norm distance is .002 and common acceptance is .01, while the accepted conditional distance is $.2 = \Delta/a$. With model law $(.001, 0, .999)$ and comparison law $(0, 0, 1)$, the same full distance .002 removes acceptance entirely; this is the threshold $a = \Delta/2$. These equalities follow directly by summing absolute probability differences.

Thus high conditional accuracy can coexist with poor control at low yield. Statistical confidence radii from Section 12 do not absorb systematic interface errors. A complete composed response matrix may replace the converter matrix in that inference theorem when its operator and preparation hypotheses hold.

14.3 A local field–probe construction

Changing a global detector's clock to proper time does not localize its operations. Finite-size particle-detector models can exhibit ordering defects when their internal operators fail microscopic causality [40, Secs. III–IV]. We instead specify a local field construction and carry both its source and apparatus data through the covariance statement.

14.3.1 Spacetime and field data

Let (M, g) be an oriented, time-oriented globally hyperbolic spacetime. For a real Klein–Gordon field, use a normally hyperbolic operator $P = \Box_g + m^2$ with a consistent signature convention. Smooth compactly supported real test functions are smeared with the invariant volume measure. The causal propagator $E = E^- - E^+$ fixes

$$[\Phi(f), \Phi(h)] = iE(f,h)I, \qquad W(f)W(h) = e^{-iE(f,h)/2}W(f+h). \tag{163}$$

Here $W(f) = e^{i\Phi(f)}$. In a chosen regular representation, local von Neumann algebras are generated by the Weyl operators with support in their regions. For causally disjoint supports, $E(f,h) = 0$, so their generated bounded algebras commute. Normal states and normal instruments are used in that representation. No canonical vacuum on every curved spacetime is assumed: states are part of the specified preparation and transform with it.

Take an independent probe field Ψ with Klein–Gordon operator Q. A real compactly supported coupling $\rho(x)$ gives the coupled field equation

$$T\begin{pmatrix}\phi\\ \psi\end{pmatrix} = 0, \qquad T = \begin{pmatrix}P & \rho\\ \rho & Q\end{pmatrix}. \tag{164}$$

This is a local bilinear interaction; its principal symbol remains normally hyperbolic. The exact causal Green operators determine a scattering morphism Θ of the uncoupled field algebras. For a probe smearing h supported outside the causal past of the interaction region,

$$\Theta(I \otimes W_\Psi(h)) = W_\Phi(f^-) \otimes W_\Psi(h^-), \qquad \begin{pmatrix}f^-\\ h^-\end{pmatrix} = \begin{pmatrix}0\\ h\end{pmatrix} - \begin{pmatrix}0 & \rho\\ \rho & 0\end{pmatrix} E_T^- \begin{pmatrix}0\\ h\end{pmatrix}. \tag{165}$$

This formula and the causal factorization of its scattering maps follow from the Green-operator comparison, rather than a global tensor assignment of one detector degree of freedom to several spacelike points.[2]

For probe preparation σ and a probe POVM $\{F_r\}$, the Heisenberg instrument is

$$\mathcal{I}_r(A) = (\mathrm{id} \otimes \sigma)\Theta(A \otimes F_r). \tag{166}$$

[2] [41], Section 4, Eqs. (4.1)–(4.12), and Appendix D, Proposition D.1 and Corollary D.2. The polynomial-field example there is extended to bounded observables in the regular/Weyl setting used in [42].

The normal representation and normal-extension conditions of this formula are part of the chosen model. Positivity of F_r gives complete positivity; summing yields $\sum_r \mathcal{I}_r(I) = I$. In state ω, the unnormalized selective state is $\omega \circ \mathcal{I}_r$, with probability $\omega(\mathcal{I}_r(I))$. Its normalization is used only when that probability is positive. The nonselective state changes through the actual interaction, but agrees with the initial state on observables in the causal complement of the coupling. A conditional change of correlations outside that region is not a controllable spacelike signal.[3]

Equation (166) initially supplies a measurement model with a specified probe measurement. It cannot be called a fully autonomous outcome law merely because the probe is another field. The next result supplies a genuine recursive physical construction for a useful instrument family while preserving that distinction.

14.3.2 Realizable Gaussian instruments

Put $X = \Phi(f)$ for real compactly supported f, and fix resolutions $\epsilon, \delta > 0$. Define bounded operators and a dephasing channel

$$k_x(X) = (2\pi\epsilon^2)^{-1/4} \exp[-(X-x)^2/(4\epsilon^2)], \tag{167}$$

$$\mathcal{D}_\delta(A) = \frac{1}{\sqrt{2\pi}\delta} \int_{\mathbb{R}} e^{-u^2/(2\delta^2)} e^{-iuX} A e^{iuX}\, du, \tag{168}$$

$$\mathcal{G}_x(A) = k_x(X)\mathcal{D}_\delta(A)k_x(X). \tag{169}$$

Integrals are taken in the specified normal/strong sense. The effect density is

$$E_x = \mathcal{G}_x(I) = \frac{1}{\sqrt{2\pi}\epsilon} e^{-(X-x)^2/(2\epsilon^2)}, \qquad \int E_x\, dx = I. \tag{170}$$

Spectral calculus proves normalization and the positive operator-integral representation proves complete positivity. For a Weyl observable $W(h)$, the nonselective channel is

$$\overline{\mathcal{G}}(W(h)) = \exp\left[-E(f,h)^2 \left(\frac{1}{8\epsilon^2} + \frac{\delta^2}{2}\right)\right] W(h). \tag{171}$$

To obtain it, commute functions of X through $W(h)$ using the Weyl relation, complete the product of the two Gaussian exponents, and perform the x and u Gaussian integrals. In particular, spacelike h has $E(f,h) = 0$ and is unchanged by the nonselective channel. Different δ give the same effects (170) and different backaction; probabilities alone do not identify this instrument.

Mandrysch and Navascués prove that every member of this dephased family with $\epsilon, \delta > 0$ is asymptotically realizable by local scalar-field probes of the form (164), and that the probe measurement can itself be realized by further probes for any finite chain length. Their construction uses weak couplings and controlled squeezed probe states. We import that realization theorem with its stated strong-limit topology.[4] It is stronger than a purely formal local Kraus proposal, but it neither supplies a finite fabricated detector error nor derives a single actual result. Moving the point at which a probe outcome is taken as a record preserves the statistical construction; it does not eliminate that outcome premise.

14.4 Finite registration and causal protocols

Choose a finite Borel partition $\{B_j\}$ of the readout line and define

$$\mathcal{G}_j(A) = \int_{B_j} \mathcal{G}_x(A)\, dx. \tag{172}$$

[3] [41], Section 3.3, Eqs. (3.17)–(3.26) and Theorem 3.4.

[4] [42], Theorem 1, Section 5, and Appendix C, especially Eqs. (15)–(21), (49)–(62). These established formulas are used and developed here with attribution; the source is licensed under CC BY 4.0.

This is a complete finite instrument. An undecided interval is retained as its own result. The imported pointwise strong limit also admits the following precise finite-bin consequence.

Proposition 14.5 (Fixed normal-test bin convergence)**.** *Suppose $\mathcal{I}_{n,x}$ and $\mathcal{I}_x$ are measurable positive instrument densities, normalized by integration over $\mathbb{R}$, and $\mathcal{I}_{n,x}(A) \to \mathcal{I}_x(A)$ strongly for each x and bounded A in the chosen representation. For a fixed normal state ω and bounded A,*

$$\sup_{B\subseteq\mathbb{R}\ \mathrm{Borel}} \left|\omega\left(\int_B [\mathcal{I}_{n,x}(A) - \mathcal{I}_x(A)]\,dx\right)\right| \longrightarrow 0. \tag{173}$$

Consequently finite-bin probabilities converge, and the conditional expectation of a fixed bounded future test converges whenever the limiting bin probability is positive.

Proof. Put $p_n(x) = \omega(\mathcal{I}_{n,x}(I))$ and $p(x) = \omega(\mathcal{I}_x(I))$. At each x, strong convergence gives a uniformly bounded sequence of operators, so normality of ω gives scalar convergence. These nonnegative densities have unit integrals. Thus

$$\|p_n - p\|_1 = 2 - 2\int \min(p_n, p)\,dx \longrightarrow 0,$$

by dominated convergence, since $\min(p_n, p) \le p$. Write $f_n(x) = \omega(\mathcal{I}_{n,x}(A))$, $f(x) = \omega(\mathcal{I}_x(A))$ and $c = \|A\|$. The norm of a positive functional is its value at I, hence $|f_n| \le cp_n$ and $|f| \le cp$. Define

$$h_n = (|f_n - f| - c|p_n - p|)_+ \le 2c\min(p_n, p) \le 2cp.$$

It tends pointwise to zero. Dominated convergence and the preceding L^1 limit give $\|f_n - f\|_1 \le \int h_n\,dx + c\|p_n - p\|_1 \to 0$. Every bin integral is bounded by this same quantity. Dividing its numerator by a convergent positive probability proves the conditional assertion. □

On the bounded observable and admitted normal-extension domain of [42, Theorem 1], its normalized outcome-density limit supplies these hypotheses. This bridge is uniform over bins for a fixed normal state and bounded test. It does not assert strong convergence of the full binned map, a uniform operational norm, or convergence of arbitrary adaptively varying approximate protocols. The exact causal theorem below needs none of those stronger conclusions. If a local classical reporting process has conditional probabilities $C_{rj} \ge 0$ with $\sum_r C_{rj} = 1$, put

$$\mathcal{R}_r = \sum_j C_{rj}\mathcal{G}_j. \tag{174}$$

This keeps the complete conditional state corresponding to the declared reporting process, including an electronics blank. It is a classical processing of the given field instrument. It does not equate a finite detector's quantum conditional filter with a projective field binning operation, or construct its source-to-circuit encoding.

Proposition 14.6 (Localized reporting preserves consistency)**.** *The maps* (174) *form a complete normal CP instrument. Spacelike commutation and covariance of the underlying field instruments are preserved when the reporting kernels and settings are transported with their local apparatus.*

Proof. Positive finite sums preserve CP and normality, and column normalization gives completeness. For spacelike instruments, commuting bounded functional calculi of their fields and dephasing unitaries give commuting selective maps. Taking finite integrals and positive linear combinations preserves that commutation. Transporting both the field maps and their scalar reporting coefficients preserves the same algebraic identities. □

More generally the physical probe scattering construction itself gives causal factorization, hence the same selective-map commutation for causally disjoint independent probes. This fact is stronger than commuting only their final effects.[5]

[5] [41], Theorem 3.5, Eqs. (3.28)–(3.32); [42] supplies the selected Gaussian instrument realization.

Definition 14.7 (Finite causal protocol)**.** A protocol consists of finitely many compact coupling/processing regions with a specified acyclic causal precedence relation. Incomparable regions must be causally disjoint and their complete selective instruments commute. Each region's local setting and reporting kernel may depend on records in its causal past, but not on an incomparable or future record. Every local outcome, including an undecided result or timeout, belongs to its finite alphabet.

Theorem 14.8 (Ordering-independent registration)**.** *For a finite causal protocol, the unnormalized state assigned to a complete labelled record, its probability, and its normalized future predictions are independent of the chosen linear ordering compatible with the causal relation. The complete record probabilities sum to one. Under an isometric transport of the spacetime, source state, couplings, probe states and local settings, they have the same values.*

Proof. Fix a complete labelled record. It fixes every adaptive setting from that region's causal-past labels. Any two compatible linear orders differ by a finite sequence of swaps of adjacent incomparable elements. The two corresponding selective maps commute, and neither setting depends on the swapped incomparable label, so each swap preserves the composed unnormalized functional. This proves order independence without normalizing intermediate zero-probability branches. Summing outcomes backwards through a compatible order successively uses each instrument's complete channel and preserves normalization at the identity. For isometric transport, the local field-algebra morphism intertwines the transported Green operators, couplings and instruments. Applying the transported state cancels that morphism and preserves the probability and all transported future expectations. Normalize only a nonzero final branch. ☐

No preferred spacelike ordering appears in this finite joint record law. An actual-event extension may use it as the distribution of one record history, provided its separate existence/exclusivity premise is retained. The theorem proves consistency of that distribution and conditional operations. It does not prove that one member, rather than a coherent quantum description, is the only physically actual history.

14.5 A definite obstruction to clock relabelling

Let one qubit be spuriously assigned to two spacelike detector regions, with selective projectors P_{z+} and P_{x+}. For initial state $\rho = P_{z+}$,

$$\mathrm{Tr}(P_{x+}P_{z+}\rho P_{z+}P_{x+}) = \tfrac{1}{2}, \qquad \mathrm{Tr}(P_{z+}P_{x+}\rho P_{x+}P_{z+}) = \tfrac{1}{4}. \tag{175}$$

The same two labelled events now have different joint probabilities in opposite orders. Changing each clock to a proper time does not change this algebra. The defect is assigning noncommuting operations to causally incomparable regions. It also produces a signaling defect: the nonselective x operation changes the $z+$ probability from one to one half.

By contrast, operations on independent local factors, or on causally disjoint field-probe algebras satisfying the preceding construction, commute at the level required by Theorem 14.8. This distinction is directly relevant to a stored global Q_j: an abstract record projection cannot be copied into several spacetime regions merely by reusing its symbol. Actual local carriers, interactions and copying operations have to be supplied.

For a simple geometry, take coupling supports inside two boxes centred at $(0, 0, 0, 0)$ and $(0, 3, 0, 0)$, each with temporal and spatial halfwidth $1/4$ in units $c = 1$. Every cross-box pair satisfies $(\Delta x^1)^2 - (\Delta t)^2 \geq 6 > 0$. Boosts with velocities $\pm 3/5$ assign the second centre times $\mp 9/4$, reversing its coordinate-time order relative to the first. The field construction has identical probabilities in both descriptions; the falsely localized common-qubit example lacks that property. These exact rational geometry and matrix checks verify the illustration, not a discretized relativistic field simulation.

14.6 A finite-energy field preparation and its readout

Take a regular Gaussian scalar-field preparation for which $X = \Phi(f)$ has variance v and mean μ. Equations (170) and Gaussian convolution give

$$p(x) = \frac{1}{\sqrt{2\pi(v+\epsilon^2)}} e^{-(x-\mu)^2/[2(v+\epsilon^2)]}. \tag{176}$$

For this example specialize to four-dimensional Minkowski spacetime, the massive real Klein–Gordon field with $m = 1$ in the chosen units, and its standard Fock vacuum. This fixes the vacuum and energy used only in this preparation example. Take

$$f_0(x) = \prod_{\alpha=0}^{3} b(4x^\alpha), \qquad b(u) = \begin{cases} e^{-1/(1-u^2)}, & |u| < 1, \\ 0, & |u| \geq 1. \end{cases}$$

The vacuum variance is the positive mass-shell integral of $|\widehat{f_0}|^2$ with the usual free-field measure. It is finite because f_0 is smooth and compactly supported. It is strictly positive: at the mass-shell point $(1,0,0,0)$ the spatial Fourier factors are positive integrals, while the temporal factor is $\int b(4t)\cos t\, dt > 0$ since $|t| < 1/4$; continuity gives a nonzero mass-shell neighborhood. Rescale f_0 to obtain the desired variance of $X = \Phi(f)$.

The resulting nonzero solution Ef has a nonzero patch before the support of f, by uniqueness of Cauchy evolution. Choose a nonnegative compact cutoff χ on such a patch and a real source smearing $h = \chi E f$. The symplectic response $E(f,h)$ is, up to convention, $\int \chi(Ef)^2$, and is nonzero. A Weyl displacement of amplitude proportional to $\mu/E(f,h)$ sets the mean to μ without changing the variance. Its energy is finite because h is smooth and compactly supported in the massive Minkowski theory. These are specified calibration preparations of a field observable; choosing their means does not choose an unknown measurement result in advance.

Use $v = 1/4$, $\epsilon = .4$, $\delta = .3$ and means $\mu = \pm 2$. Define positive and negative readouts by $x > a$ and $x < -a$, with $a = .5$, and retain $[-a, a]$ as undecided. Writing N for the standard normal CDF and $s_0 = \sqrt{v+\epsilon^2}$,

$$p_+(\mu) = \mathsf{N}((\mu - a)/s_0), \quad p_-(\mu) = \mathsf{N}((-a-\mu)/s_0), \quad p_u = 1 - p_+ - p_-. \tag{177}$$

The two opposite preparations interchange the conclusive probabilities. For these specified field preparations the conclusive correct, incorrect and undecided probabilities are approximately 0.99042521, 0.00004724 and 0.00952755. These are finite Gaussian-model predictions; no physical circuit is identified with this field preparation.

14.7 From normal tests to a finite causal experiment

Proposition 14.5 proves finite-bin convergence against each fixed normal input state and bounded future test. This does not imply a uniform error on all states. It does suffice for a finite recorded experiment with finitely many requested final tests, if its physical approximants can be chosen independently at the finitely many instrument nodes within their declared regions.

Theorem 14.9 (Finite-test approximation of a finite adaptive experiment)**.** *Consider a finite instrument tree with a fixed normal initial state and a finite set of bounded final tests. At each node, suppose complete approximating normal instruments obey, for every fixed normal functional ω, bounded A and recorded outcome z,*

$$\omega(\mathcal{I}_z^{(m)}(A)) \longrightarrow \omega(\mathcal{I}_z(A)). \tag{178}$$

For every $\epsilon > 0$, one can choose a finite approximation index at each node such that every requested complete-experiment expectation differs by less than ϵ. The choices need not share one index or give a rate, an energy-uniform error or a diamond bound.

Proof. Process the finitely many nodes in reverse chronological order. When a node is changed, all its earlier ancestors still use their exact instruments. Its subnormalized input normal functional is therefore fixed. All later operations already chosen define finitely many fixed bounded future-test effects, pulled back from the requested final tests along the finite outcome tree. Equation (178) lets us choose the node's index so that the sum of the finitely many errors for every requested test is less than ϵ/N_*, where N_* is the number of nodes. CP normality and boundedness preserve the needed test and state domains. Telescoping these finitely many replacements gives total error less than ϵ. Zero-weight prefixes contribute zero. The argument is existential and uses independent choices; no uniform convergence on the varying states of a forward approximation was assumed. □

For a causal spacetime protocol, the approximation index may depend only on admitted causal-past information. Different tree occurrences of the same physical instrument occurrence and causal-past setting must use one common index, even if a chosen linear ordering gives them different incomparable prefixes. At the reverse step, impose the finite union of their fixed-input/future-test requirements and choose one index satisfying all of them. Such occurrences are not ancestors of one another, so their grouped replacement preserves the proof. This prevents an approximation choice from introducing a forbidden dependence on a spacelike record.

For the field construction this also requires the approximants to preserve the admitted causal supports and any resource restrictions. Their probe number or coupling accuracy is not supplied by a theorem with no convergence rate. Spacelike commuting selective maps then give order-independent joint laws by Theorem 14.8. A scalar detector reporting matrix is still only local classical processing. Connecting a quantum field measurement to the actual circuit and latch requires its own quantum adapter with energy, timing, bath and error specifications.

A local stochastic reporting kernel can now act on the positive, negative and undecided bins by (174), retaining an electronics blank if present. This classical operation is not a quantum field-to-spin or field-to-circuit adapter. The dephasing parameter provides a simple counterexample to identification by probabilities: at $E(f, h) = 1$ the nonselective multiplier in (171) is about 0.4377, although changing δ leaves the outcome density unchanged. Spacelike h has multiplier one.

14.8 The source-transfer boundary

A local scalar-field construction and the finite physical detector are two specified targets. The source results established earlier include a direct common Standard Model–Einstein realization independently of removing the regulator of the interacting quantum source. Their positive content is not withdrawn by the additional comparison needed here. Conversely, a regulated Hamiltonian with a chosen time coordinate does not by itself construct a continuum Lorentz-covariant local net, and a global copied-memory projection cannot be assigned to several spacelike regions by reusing its symbol.

The local measurement construction is imported with its own normal representation, probe preparation and strong-limit conditions. A uniform diamond estimate on arbitrary high-energy field inputs does not follow from strong convergence. To identify it with the material apparatus, the comparison must preserve the actual source preparation, localized interactions, admitted operations and outcome alphabet, clock and bath structure, retained readers and stress-energy exchange. A useful finite resource or approximation bound must be proved in the norm or fixed-test class required by that claim. The normal-test and finite-tree theorems above make a positive comparison available without pretending that they already provide a rate or that all inputs share one uniform operational error.

Necessary AASC governance of determinate carriers and incidences remains in force throughout. It does not supply omitted operations or identify distinct physical sources. Local covariance establishes causal consistency of the assigned joint distribution and selective future predictions; assigning one such history as exclusively actual remains a separate physical commitment.

Accounted stage	Duration or resource
Pointer setup: sixteen gates and two ramps	0.000191831 ms
Frozen electronic detection interval	35.5870791 ms
Two receiver transfers, if independent parallel control is realized	0.1666667 ms
1024 finite coherent amplification collisions, including declared diagonal ramps	21.147475 ms
Quiet hold	100 ms
Accounted sequential duration with parallel receivers	156.901413 ms
Fresh bath/selector registers plus purifiers	$28 \times 1024 = 28672$ qubits
Reusable work register	30 qubits
Pointer; complete cold preparation resource	32; 28734 qubits
Amplifier primitive gates	2111488
Pointer preparation gates; total including setup	16; 2111504

Table 5: A shared finite clock and finite preparation resource. Initial bank preparation is a specified resource law. The duration subtotal excludes initial loading, quadratic routing and phase controls, final seed detachment and reader operations unless separately included by their implementations. Serial receiver control adds another 0.1666667 ms.

15 A common operating point and the complete physical error ledger

All channel norms in this composition refer to the admitted binary source input, with the stated apparatus initialization and an arbitrary idle reference. They are not unrestricted bounds on arbitrary bosonic inputs. The receiver and finite amplifier nevertheless act on every state in their declared Hilbert spaces. The spin apparatus has a bounded seed even when the preceding bosonic converter leaks. The detector comparison ends in the exact bare twelve-state code embedding in its forced harmonic frame; its two endpoint dressing errors have already been paid in the detector bound. Known number phases are retained. The physical pump-undressing and routing interface must still map that reference output to laboratory flag occupations $0, 1$ and independently prepared auxiliary vacua, with spectator states and phases retained. Its actual residual belongs to R below. This permits contractive channel composition: an upstream error bound on the intended source code remains valid after arbitrary downstream CPTP stages. A hypothetical perfect leakage herald is not needed for channel existence.

It is useful to distinguish three sorts of number. First, the finite Gibbs-chain probabilities are rigorously enclosed properties of an exact specified channel. Second, the receiver and monitored-detector bounds compare explicitly given physical models to simpler instruments. Third, actual apparatus control tolerances are requirements to be established independently. Combining their numerical columns as though each were an observed device error would conceal the principal remaining work.

For example, use the validated complete monitored-detector bound below 0.063241, and suppose the declared receiver assumptions hold with combined transfer and preparation bound below 0.002381. The constructed correct-record amplifier channel has bound below 0.017491. If each of the 2111504 physical primitives including pointer setup meets the required operator error 10^{-10}, the control channel allowance is below 0.000423; the initial clean-bank contribution is below 10^{-9}. These entries sum to less than 0.083536001. This is a conditional subtotal, with all values rounded upwards. It is not a certified total device error.

Let R denote the sum of the still-required complete channel bounds for initial loading and undressing, residual nonlinearities during routing, spatial and polarization mismatch, mode cross-coupling, actual confinement corrections and orbital preparation error, receiver loss/dephasing, physical seed coupling, imperfect ramps, ambient amplification errors, pointer/work/bank idle errors over the preceding detector and receiver intervals and all waiting within the collision schedule, and reader operations. Terms already included in a proved receiver or detector bound must not be counted again. If each bound holds on the actual incoming states and all stages have the stated CPTP realization, telescoping gives

$$\|\mathcal{I}_{\text{apparatus}} - \mathcal{I}_{\text{comparison}}\|_{\diamond} < 0.083536001 + R \tag{179}$$

through the amplifier endpoint. The comparison uses the dressed detector and the specified polar-normalized amplifier isometry, which agrees with actual success-conditioning on fixed-seed inputs. No numerical value for R is inferred from an empty entry.

If the comparator instead retains the actual finite amplifier, whose full original-run outcome table was just calculated, its 0.017491 normalization cost is absent: the corresponding channel bound is below $0.066045001 + R$. This distinction keeps a certified finite apparatus channel separate from an ideal correct-record comparison.

For classical final observations, each endpoint total-variation discrepancy is at most half its respective channel bound. For the normalized correct-record comparator and its $0.083536001 + R$ endpoint bound, the separate quiet-hold hazard condition permits comparison with ideal persistent bands by adding a probability allowance smaller than 2.056×10^{-8}, plus the unarmed false-band bound. This last addition compares the normalized correct-record comparator with persistent readable classical records. It does not bound every subsequent band change of the actual finite amplifier, whose initially wrong or unreadable outputs are also present. The separately stated original-run correct-display, correct-band and hold lower bounds remain valid. It is not a diamond-norm bound preserving every microscopic spin coherence for 100 ms.

The declared 10^{-10} gate-operator tolerance across more than two million gates is especially demanding. The present finite synthesis proves a physical control specification and its exact ideal channel, rather than an experimental feasibility claim at that precision. A dedicated analog energy-resolving collision could reduce this burden, but it would require its own Hamiltonian and residual proof. The independent Gibbs preparation law similarly prevents hidden dependence on a source outcome, while leaving its physical equilibration or calibration as an apparatus input.

16 Distinguishable predictions and comparison of measurement accounts

The relevant empirical object is a complete experiment: a preparation family, material apparatus, controls, clock origin, output arrangement, failure alphabet and retained state for later tests. The construction above makes these data explicit and provides a useful complete detector bound. It does not follow that each ingredient is available only within AASC, or that a successful test of a shared detector law identifies one account of actuality.

A matched comparison also needs an explicit outcome criterion. One can ask for one globally exclusive original-run history, or for an account of the consistent definite records that observers encounter. A theory that posits branch-relative records answers the latter question and declines the former. The comparison below makes that difference visible without converting it into an undisclosed empirical score.

16.1 Equality of complete instruments is operational parity

Theorem 16.1 (No discrimination on a common protocol class)**.** *Suppose two accounts assign the same initial state and the same complete outcome-and-retained-state instrument $\mathcal{I}_{z|h,u}$ to every admitted control u and prior recorded history h. Every finite adaptive protocol formed from those operations has the same joint record probabilities and the same conditional future-test statistics in both accounts. Consequently every decision statistic has the same distribution; its optimal equal-prior discrimination error is* $1/2$*.*

Proof. For a fixed recorded history $(z_1, \ldots, z_n)$, the unnormalized conditional state is

$$\mathcal{I}_{z_n|h_{n-1},u_n} \circ \cdots \circ \mathcal{I}_{z_1|u_1}(\rho_0). \tag{180}$$

Equal initial states and maps give the same expression by induction, including history-dependent controls. Its trace is the joint record probability; applying any common bounded final effect gives the same joint future-test probability. Common randomized controls, recorded stopping rules, finite time bins and coarse-graining sum the same histories and preserve equality. A decision statistic is another such processing. □

The hypothesis concerns complete controlled instruments. Equal final Born histograms do not establish equal sequential experiments. Equal unconditional Lindblad generators likewise do not determine a physical monitoring arrangement: resolved ports, mixed output fields and different probe measurements can have different selective maps. The comparison keeps the actual apparatus output, not a convenient unravelling selected after the calculation.

Where equilibrium Bohmian apparatus theory or a Born-weight Everettian account realizes these same operations, Theorem 16.1 applies. It is not itself a construction of a microscopic realization of every present component in those accounts. General Bohmian apparatus theory already derives experiments, generalized measurements and conditional instruments under its equilibrium and apparatus assumptions [43, Secs. 3–4, especially Sec. 4.3]. The present record bookkeeping is therefore not an exclusive capability established by its algebra alone.

16.2 What a same-device test can establish

The analytic detector kernel predicts a cubic count onset:

$$F_r(t) = \frac{\kappa_r |g_r|^2}{3} t^3 + O(t^4), \tag{181}$$

because the converted amplitude starts as $-ig_r t + O(t^2)$. A slow weak-conversion population approximation instead has a linear initial count law. This is a difference between specified apparatus models on their shared Markov-time domain. The expansion is not asserted below a bath's correlation time, and the two-state closed mean latency is not substituted for the current three-active-state detector mean. Holding calibrated material coefficients fixed while testing additional pump, detuning and timing settings can therefore reject an inaccurate rate approximation. Fitting a fresh free rate to every histogram would remove much of that test's content.

Retaining timestamps can reveal transient differences that a terminal label distribution conceals. A current quantitative comparison must use the same fixed dressed Hamiltonian, output ports, physical clock and complete outcome alphabet in both models. It must also extend the record-routing comparison to the time bins being tested. The present full-deadline detector certificate cannot, merely by renaming its output, become a numerical separation already calculated for a different quartic target or different time bins.

Proposition 16.2 (Separation with model uncertainty)**.** *If two actual record laws lie within total-variation errors δ_p, δ_q of calculated laws p, q, their guaranteed separation is at least*

$$\max\{0, \mathrm{TV}(p, q) - \delta_p - \delta_q\}. \tag{182}$$

Proof. Apply the triangle inequality for total variation to the two calculated and two actual laws. □

The complete current converter bound contributes at most $0.063241/2$ to any terminal-event probability comparison on its admitted prepared domain. Its value is a conservative bound, not a measured discrepancy. Loading, material interfaces, parameters and controls can consume further proposed separation. The final apparatus ledger keeps these errors explicit. Confirmation of the resulting shared quantum instrument would support its physical dynamics; it would not by itself select AASC's actual-history commitment over an account satisfying Theorem 16.1.

16.3 A material collapse comparison requires its actual operators

For definiteness, one mass-proportional white-noise CSL convention uses

$$\begin{aligned} h_{r_C}(x) &= (\pi r_C^2)^{-3/4} e^{-|x|^2/(2r_C^2)}, \\ L_x &= \frac{\sqrt{\lambda}}{m_0} \sum_a m_a h_{r_C}(x - \widehat{x}_a), \\ \mathcal{C}(\rho) &= \int d^3x \left(L_x \rho L_x^\dagger - \tfrac{1}{2}\{L_x^\dagger L_x, \rho\} \right). \end{aligned} \tag{183}$$

For one particle this convention damps position coherence at rate $\lambda(m/m_0)^2[1 - e^{-|q-q'|^2/(4r_C^2)}]$. In the convention of Bassi and Ghirardi, $r_C = 1/\sqrt{\alpha}$ and $\lambda = \gamma(\alpha/(4\pi))^{3/2}$ [44, Sec. 8.1, Eqs. (8.12)–(8.16); Sec. 8.6]. This specifies one published dynamical model, not every collapse variant.

Josephson participation coefficients and microwave number states do not supply the positions and mass-density modes in (183). The ionic and electronic geometry and microscopic-to-circuit embedding are additional physical inputs. Replacing a circuit excitation by $\hbar\omega/c^2$ is not a derivation of that nonrelativistic mass-density operator.

Proposition 16.3 (Projection retains leakage)**.** *Let P be a finite material-code projection, $\rho = P\rho P$, $A_x = PL_xP$ and $B_x = (I-P)L_xP$, on the common operator domains. With $\mathcal{D}[L](\rho) = L\rho L^\dagger - \{L^\dagger L, \rho\}/2$,*

$$P\mathcal{D}[L_x](\rho)P = A_x\rho A_x^\dagger - \tfrac{1}{2}\{A_x^\dagger A_x + B_x^\dagger B_x, \rho\}. \tag{184}$$

The initial code-trace loss is $\mathrm{Tr}(B_x^\dagger B_x \rho)$. In particular, scalar PL_xP does not prove insensitivity.

Proof. Insert $P + (I-P)$ between $L_x^\dagger$ and L_x to obtain $PL_x^\dagger L_x P = A_x^\dagger A_x + B_x^\dagger B_x$. The gain and anticommutator terms involving A_x cancel in trace; the remaining trace derivative is negative with the stated magnitude. ☐

A finite collapse comparison must therefore prove code invariance or retain the resulting leakage and heating channels with controlled errors. If its derived operators preserve the code and are diagonal in actual material configurations $|j, \alpha\rangle$, their pairwise coherence rates are

$$\Gamma_{j\alpha,k\beta} = \tfrac{1}{2} \int d^3x \, |\ell_{j\alpha}(x) - \ell_{k\beta}(x)|^2. \tag{185}$$

These rates share common-profile Gram constraints; they cannot be invented independently. Differences between active A and C_r configurations can change conversion and timing. A different projection that acts only on the stored memory has the more restricted effect below.

Once a material generator perturbation is specified, a record effect E_h in the same augmented register has response

$$\partial_\theta p_h|_0 = \mathrm{Tr}\left[E_h \int_0^T \mathcal{T}_0(T, s)\mathcal{C}_{\mathrm{aug}}\mathcal{T}_0(s, 0)(\rho_0)\, ds \right]. \tag{186}$$

Duhamel proves this expression for bounded generators, or on the appropriately controlled common domains. It includes the declared readout and time routing. The collapse noise is not automatically an electronic count, so its unravelling cannot simply replace the physical output mechanism. No circuit-specific collapse rate or sensitivity limit is assigned in the absence of these material operators.

16.4 An exact restricted invisibility and coherence result

Consider the finite target with its preserved memory projections and $Z_m = Q_0 - Q_1$. Add the generator

$$\mathcal{C}_\gamma = \frac{\gamma}{2}\mathcal{D}[Z_m]. \tag{187}$$

Here γ is a declared coherence-decay parameter, not an inferred microscopic CSL parameter.

Theorem 16.4 (Memory dephasing and recorded statistics)**.** *If all target Hamiltonians, material jumps and record routes preserve memory sectors, and the target generator commutes with $\mathcal{C}_\gamma$, the added term leaves every label, timestamp and blank probability unchanged for every input. At the common deadline it multiplies each cross-memory block of every retained conditional map by $e^{-\gamma T}$ and leaves the diagonal blocks unchanged.*

Proof. On $Q_j \rho Q_k$ the added generator is zero for $j = k$ and equals $-\gamma$ times that block otherwise. Commutation factors the augmented propagator into the original propagation and this dephasing map, which also commutes with reading the classical register. Cross-memory blocks have trace zero. Therefore none of the recorded probabilities changes. □

This applies to the specified dressed comparator, whose memory blocks are invariant. It is not an exact symmetry asserted for all unprojected cosine corrections, nor a statement that every CSL operator is invisible. It demonstrates that even complete sector-calibration and count-timing data need not detect a restricted change of conditional quantum state.

For a retained logical qubit, choose $|0\rangle, |1\rangle$ in the two sectors and factor out known common storage. Suppose the retained instrument acts as

$$\mathcal{I}_h(|0\rangle\langle 1|) = c_h |0\rangle\langle 1| \tag{188}$$

for each recorded bin/label or blank h. This scalar-action hypothesis specifies the retained output; it is not assumed for an arbitrary material handoff.

Proposition 16.5 (Conditional coherent sensitivity)**.** *On input $(|0\rangle + |1\rangle)/\sqrt{2}$, the optimal total variation between the original instrument and* (187)*, using the joint record and a subsequent retained-memory measurement, is*

$$\mathrm{TV}_{\mathrm{coh}}(\gamma) = \frac{1 - e^{-\gamma T}}{2} \sum_h |c_h|. \tag{189}$$

Proof. Each record block has the same diagonal in the two outputs and an off-diagonal difference $(1 - e^{-\gamma T})c_h/2$. The difference's two eigenvalues are opposite and have that absolute magnitude. Summing its trace norm over the orthogonal record blocks and dividing by two gives the formula. A record-conditioned equatorial logical measurement with phase matched to c_h attains the Helstrom optimum. □

The coefficients depend on terminal phases, time bins, retained modes and the full common deadline. A coefficient for a different quartic detector cannot be inserted into this formula for the present dressed instrument. The optimum also requires physically available coherent access to the retained memory. In an exact projective-output limit all c_h vanish, and this added memory dephasing is invisible even to that retained-memory test.

16.5 Developed alternatives and their actual commitments

The Curie–Weiss programme provides a microscopic spin, metastable interacting magnet and bath, with trigger, registration, erroneous registration and erasure analyses. Its macroscopic apparatus calculation is a developed comparison, not an unexplained projection instruction

[45, 46]. The subensemble programme supplies additional relaxation analysis and principles governing physical subsets [47, Secs. 5.2–6.2]. Its explicit 2024 formulation states Postulate P: an individual prepared apparatus reaches one equilibrium alternative and the relevant outcome-defined subsets admit density operators. Frequencies and selective states are obtained using this premise [48, Secs. 5.3–5.4]. Thus it should neither be credited with premise-free exclusive actualization nor dismissed as nothing beyond diagonalizing an ensemble.

Bohmian mechanics supplies an actual configuration and a guiding quantum state. Equilibrium/typicality and an apparatus mapping to readable configuration regions yield outcome probabilities and conditional instruments [49, 43]. Equivariance preserves a stated equilibrium law; it does not force every initial configuration measure to equal it. Bell-type quantum field constructions with explicit regularization and creation events exist within their stated domains [39]. It would be inaccurate to contrast the present interacting AASC source with a claim that Bohmian theory can contain no field interactions. Neither the ordinary apparatus theorem nor that field construction alone implements the exact device built here.

Objective-collapse programmes add a stochastic dynamical law and a physical-space ontology. Concrete binary-pointer calculations address localization, duration and persistence; the GRW flash formalism includes POVMs, CP conditional maps and suitable random-duration experiments [50, 51]. Spatial tails, stochastic parameters and the domain of the physical ontology remain material qualifications. The timing of spatial localization is not the same observable as this detector's first emitted or displayed event. XENONnT constrains specified Markovian white-noise collapse laws and parameter choices; such bounds are not a detection of collapse, an exclusion of every variant, or positive evidence for AASC's independent actual-history premise [52].

Everettian accounts retain unitary dynamics and interpret decohering quasiclassical records as branch structure. Their definiteness is relative to a branch rather than one globally selected successor [53]. Probability arguments do additional work: Wallace's decision-theoretic theorem invokes assumptions on available acts and preferences, while Sebens and Carroll use an epistemic separability principle and self-location [54, 55]. These are substantive conditional arguments, not probabilities obtained from the Schrödinger equation alone. Decoherence and redundant record access can support several ontological accounts; the present copying and retention theorems do not uniquely select one of them.

Operational or Copenhagen accounts foreground recorded experimental conditions and the quantum probability rule; Bohr's discussion includes classical communication and amplification rather than a requirement that consciousness produces collapse [56]. Relational accounts make events relative to physical systems; cross-perspective record links are an additional developed proposal [57, 58]. QBism gives the Born rule a normative role among an agent's probabilities and treats outcomes as experiences [59]. Consistent histories uses compatible history families and their probability rules, while transactional accounts add an emitter–absorber transaction and a selection account [60, 61]. These are contextual comparisons of explanatory targets, not matched microscopic implementations of the present device. A demand for absolute original-run exclusivity must be stated before using it to assess approaches that formulate the outcome question differently.

The current AASC physical construction now includes a useful complete full-interval detector bound and a finite material receiver/amplification schedule. That is a stronger positive result than a merely proposed record-controlled kernel. Its source content also exceeds the deliberately small input of several canonical pointer examples. Source breadth, apparatus accuracy and occurrence-law scope are nevertheless distinct comparisons. The new local scalar-field setting does not by itself show that every added material operation is realized at the full AASC common physical target.

A relative advantage must be demonstrated for a declared common measurement with matched resources, output definitions and error obligations, or by a justified implication between substantive premise sets. Counting named axioms conceals how much physics a single premise

Account	Constructed measurement content	Additional outcome/probability bridge
AASC extension	Occupied interacting source imports, informative acquisition, complete detector, finite receiver/amplifier and controlled record bounds.	Original-run actuality; affine/calibration premises or a specified history law; the explicit source, bath and implementation conditions.
Curie–Weiss	Metastable spin magnet, bath-driven registration and finite-size record analysis.	Macroscopic exclusivity and physical-subset/frequency principles, plus apparatus and approximation conditions.
Bohmian	Configuration-based apparatus dynamics and conditional measurement formalism.	Configuration ontology, guidance, equilibrium/typicality and the physical configuration-to-reading map.
Objective collapse	Modified dynamics, concrete binary pointers and GRWf measurement formalism.	Stochastic law and parameters, primitive ontology and outcome mapping, with model-specific tails and domain.
Everett	Unitary apparatus/environment dynamics and persistent branch records.	Branch ontology and a decision or self-location probability account; global exclusive selection is declined.

Table 6: A qualitative task comparison. These entries state capabilities under their premises; they are not empirical confidence scores, a count of equally weighted axioms, or a matched hardware-performance ranking.

can contain. Granting AASC its realization commitment while charging competitors for their ontologies would likewise predetermine the comparison. The established result is a connected and quantitatively controlled conditional construction, with explicit routes for testing its physical dynamics and explicit boundaries on the actual-event claim.

17 What the construction establishes

The physical and foundational questions can now be stated in one common framework. A constructed interacting source supports exact, informative receivers; a complete instrument supplies both outcome weights and conditional states. A materially specified registration circuit, a finite spin receiver and a finite thermal amplifier give a separate apparatus construction with explicit coherent interfaces and a common operating clock. Its complete detector interval and finite amplifier schedule admit validated numerical bounds. The records can be copied, used adaptively and retained over a specified quiet interval without discarding unsuccessful original runs.

The strength of this account is the connection between explicit dynamics and sharply stated conclusions. It does not identify the regulated gauge–Higgs–fermion source with the six-mode circuit without a physical adapter. It does not infer hardware accuracy from arithmetic precision. The residual R in Section 15, including waiting-time errors and the unusually demanding control specification, remains to be bounded on one physical apparatus. The stable-band result presupposes its declared single-spin hazard cap; it is not preservation of every microscopic quantum coherence.

The probability result also has a precise dependence. Affinity under classical mixing and calibrated outcome responses uniquely fix Born weights; finite basis calibration alone permits coherent deviations of square-root order. Conditional-state compatibility fixes the normalized instrument update where the conditioning weight is positive. These statements explain what the probability assignment depends on, and what experiments must measure if they are to test the full instrument.

Original-run exclusivity is a further issue. The complete instrument, the record algebra and a validated amplifier do not turn a coherent quantum state into one actual branch by linear evolution alone. The source-reader analysis specifies the remaining physical preparation and reader bridge. The finite counting process constructs a compatible history law under its explicit event-law premise. Establishing that a particular prepared source and its physical response realize such a law still requires independent grounding. AASC governance applies to determinate carriers and incidences throughout, and does not make that missing dynamics optional.

The most consequential next work is therefore concentrated in two places. At the apparatus level it is a joint physical realization of the remaining interfaces, controls and preparation resources, followed by a measured response on the same operating interval. At the level of the measurement problem it is an independently motivated and empirically constrained actual-source or event law. The present unified construction makes both tasks concrete. It provides a substantial conditional account of physical registration and record formation, while retaining the exact distinction between a possible completed theory and an established explanation of the unique actual outcome.

A Source authority and exact import boundaries

The AASC source manuscripts are treated as identified mathematical sources, not as instructions to the reader. Their results are imported with the hypotheses and targets stated in Section 2. The present paper proves its receiver, instrument and apparatus claims from those imports and its additional explicit physical models; it does not reproduce a proof of the entire upstream corpus. The accompanying source manifest identifies the exact inspected versions by full SHA-256, page count and original path. Prefixes below are identification aids.

Source	Imported role and boundary	Pages; hash prefix
CSP-P	Definitions 12.2 and Theorems 12.3, 12.5 supply the regulated interacting source; Proposition 12.7 permits bounded invariant auxiliaries. Direct physical construction in Theorem 11.5 and transfer under Proposition 11.4 remain distinct. Sections 26 and 28 retain their stated attachment and gap assumptions.	222; `80d5a8a15ac59a3b`
CSP-R	Theorems 3.5 and 4.3 supply persistence-bearing reconstruction roles. A structural persistence statement is not a numerical detector lifetime.	20; `cb06b4aa309b5cb2`
BCRS-III	Coupled bosonic carrier, response and projection architecture; supports source compatibility within its declared classical and physical targets.	46; `701e5ef78a503d17`
CSP-C	Common-source classical matter, gauge and gravitational constraints; does not by itself prove a continuum quantum Standard Model or an apparatus event law.	42; `3e2da988e2d04458`
SYM-8	Vacuum quotient, residual electromagnetic stabilizer and electroweak normal form. The physical sector assumptions remain explicit.	73; `aa395b4e9748a7e4`
SYM-11	Target-indexed symmetry/role exhaustion; does not identify every target's dynamics by projection alone.	42; `ce61fd7d35486ff0`
SYM-2	Spacetime-symmetry domain and action distinctions; local detector covariance still requires a covariant full interface and state.	107; `805441e3df32f68a`
MR	Determinate record fixation and quotient roles; a determinate record's admissibility is distinct from stochastic selection of one actual original-run record.	121; `578d3af3a1e1fc4f`

Source	Imported role and boundary	Pages; hash prefix
ENT	Bell-witness and tensor-level compatibility roles; local no-signaling instruments do not supply a global unique-history dynamics.	71; `d9fec2e370b3b9b5`

The actual-source analysis additionally uses the GR/QM architecture, upstream-continuation and boundary-trace-fixation manuscripts cited in Section 13. Their detailed theorem imports and exact versions are recorded in the additional source ledger. The distinction between a history carrier and an independently prepared initial law is retained. An imported theorem about the former is never substituted for a physical specification of the latter.

B Notation and the three retained outcome variables

The symbols below separate source observables, material occupations and reported outcomes. Local dummy indices in proofs do not change these physical roles.

Symbol	Meaning
H_{src}, N_F	Regulated interacting source Hamiltonian and bounded total fermion number.
m, b, f_r, w_r	Memory, buffer, material flag and resolved waste circuit modes.
β_b, β	Buffer loss rate and inverse temperature, respectively; local conventions are stated before use.
$e \in \{\varnothing, 0, 1\}$	First electronic display, frozen after its first observed label while material dynamics continues.
D_0, D_1, E	Terminal material carrier states: flag 0, flag 1 and no flag. An electronic dark display need not create a material flag.
$q = n_{s_0} - n_{s_1}$	Conserved bounded receiver-spin seed; both unoccupied and both occupied receiver sectors have $q = 0$.
$a = q^2$	Coherent arming projector; the controlled collision acts as identity on the entire unarmed sector.
$k, M = 2k - 32$	Number of up pointer spins and pointer magnetization.
B	Amplifier endpoint band (lower, ready or upper) after 1024 collisions. Ready includes the unreadable intermediate band.
$\mathcal{I}_e, \mathcal{A}$	Complete detector operations and finite amplifier channel, including their quantum conditional states.
R	Sum of independently required physical channel-error bounds not already included in the proved detector/receiver/finite-control ledger. It is not assigned a numerical value here.

A complete run retains its electronic display, material carrier and amplifier band together. Exclusivity refers to one tuple in this complete alphabet and does not impose agreement among its components. This is essential when counting dark displays, missed detections and wrong bands.

C Reproducibility and the numerical certificate

The accompanying project contains one main article, its editable LaTeX sections, the numerical models and proof certificates used for the current operating point, a source-coverage map and independent review records. The source-coverage map accounts for all thirteen supplied documents. Repeated introductions and intermediate research scores are not mathematical premises. Superseded amplifier designs and short-time diagnostics are identified in provenance; their numerical values are not mixed with the current detector's response.

The current detector data include the exact-number operating point, the losslessly archived finite Fourier trial, its comparison matrices, displacement and Bessel-factor bounds, infinite-tail estimates and the complete interval ledger. The trial is a fixed certificate object: it does not depend on reproducing a floating eigensolver's search path. The fast replay checks file integrity and recomputes the arithmetic enclosures, comparator and final bound. The full replay additionally recomputes the finite cosine action. Its largest data object is about 79 MB and full evaluation requires several gigabytes of working memory. The proof's full oscillator propagator and common-core hypotheses remain mathematical assumptions; numerical replay does not prove them.

Finite amplification and quiet hold are evaluated with outward exact rational intervals. Separate checks cover the reversible collision, work-register erasure, coherent unarmed sectors, finite preparation, resource counts, receiver formulas and coherent channel comparisons. The finite detector target is evaluated by rigorous 256-bit Arb matrix exponentials and independently reduced occupation integrals. Its complete electronic/material table is then composed with the finite amplifier using exact rational intervals. Every original-run outcome is retained.

NumPy, SciPy and python-flint are the numerical runtime dependencies. The project README gives a clean build and replay sequence and records the versions used for this release. Ordinary script tests and interval certificates have different roles: numerical agreement is a diagnostic, whereas the outward bounds and analytic residual theorem support the stated certification. Neither form of computation is an experimental measurement of an unspecified device parameter.

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