Color Realization from a Premetric Response Atlas

Invariant Networks, Global Response, and Physical Continuation

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Abstract

We construct a color realization atlas for a premetric, constraint-derived Standard Model, beginning with one color-transport carrier and its joint matter and Higgs source. Its adjoint realization is one irreducible support, while its local law determines the canonical self-interactions. Compact-group averaging constructs neutral responses; boundary-respecting graph reduction and invariant probes reconstruct classical transport, with full-path and smooth extensions. On explicitly witnessed tensor branches, elementary pairings and volume tensors generate every invariant polynomial observation, so observation completeness follows from source operations rather than being assumed polynomial by polynomial. The shared global quotient determines electric and magnetic descent, marked mutually local dyonic branches, exact screening normal forms and determining joint phase characters. Joint operator and anomaly constraints derive the minimal chiral charge ratios, with primitive normalization stated separately. Actually represented sectors and physical attachments construct configuration-to-neutral-boundary channels with exact domains, nonzero-amplitude criteria and matched continuation. Matched Yang–Mills results give spectral transport, local-vacuum neutrality and source-qualified subgap rigidity. Complete metric probes uniquely determine the color source distribution; compatible local responses glue, and the canonical local law yields its Hilbert source and balanced matter–Higgs–color exchange. The complete native response algebra then gives an exact observational quotient, whole-interface role uniqueness over each complete realization datum, and a necessary and sufficient criterion for lossless deletion of information. Each conclusion applies to its independently specified source branch. Structural neutralization, charge screening and metric closure remain distinct from unrestricted physical confinement and a universal hadron spectrum.

Keywords: premetric constraints; color transport; invariant theory; global gauge form; dyonic screening; metric response; confinement interface; role-occupancy closure.

Contents

1 Source identity and the color realization target

The color sector of the Standard Model distinguishes several objects often represented by the same particle label: non-Abelian transport, an adjoint field, a neutral observable, an extended-support sector and an asymptotic state. Understanding their relation requires explicit realization and restriction maps. Here those maps begin with the premetric carrier classification of BCRS-I and preserve its joint matter and Higgs attachments [1, 4, 5, 2].

The non-degenerate source is kernel-governed: admissibility, standing, reference and irreversibility precede metric realization. Its gravitational role governs licensed relational localization and consistent continuation. Color geometry is used within that realized description when the operations require it; geometry is not an additional premetric primitive. At the metric level, color contributes a response to geometric variation and exchanges energy and momentum with the other attached sectors. Section 11 derives this contribution from complete response probes and, on the canonical local branch, from the interaction law itself. This does not make gravity a color carrier or determine global gravitational dynamics. The common-source and metric boundary retain their established scope [1, 2, 12, 13].

1.1 The joint source and its restrictions

Write Ξ for a fully admitted joint source instance. Its color restriction comprises the group and global form, transport and representation actions, source-preserving comparisons, evidence of primitive standing and color-response incidences. The remaining restrictions retain matter, charge, weak, Higgs, anomaly, boundary and localization data. A licensed realization further specifies its geometry, bundle, operator category, domains and represented physical data. All these structures belong to one instance; agreement between selected marginal descriptions alone does not supply their joint attachment.

Definition 1.1 (Color comparison target)**.** A color comparison target T specifies the admitted source class, its fixed anchors, allowed varying inputs, observation types, continuation types and physical boundary requirements. Its actual instance domain $\mathcal{X}_T$ consists of received instances with those source attachments. In a licensed branch it also fixes the field and operator grammar, global and boundary scope, and the kind of physical authority required.

The activated realization interfaces may include a local interaction law; classical transport and joint tensor responses; a closed-support sector diagram; a represented cyclic state-and-operation interface; or independently authorized composite, analytic, persistence or asymptotic channels. A required interface with an unresolved source antecedent is recorded as unresolved, not removed from the target to obtain a positive result. An interface outside the declared target is instead inactive.

The source domain precedes both the observational quotient and the role count. Specifying a connection and gauge group alone does not supply the attachments required of a physical instance. The definition therefore asserts neither nonemptiness of every fibre nor existence of a universal physical lift.

Proposition 1.2 (Conservative joint restriction)**.** *Every actually admitted color realization restricts to its accepted BCRS-I, matter and electroweak/Higgs source instances, with all common incidences preserved. A source-authorized comparison of full instances induces comparisons of these restrictions. Any assertion depending only on one such restriction is unchanged by adding the realized color description of that same instance.*

Proof. Use the actual source restriction maps of Ξ. Their incidence squares commute before any realization is forgotten. A full comparison commutes with those maps, so restriction gives the asserted comparisons. A use depending only on a restriction factors through it and consequently has the same value and domain before and after the extra color description is retained. This constructs the conservative forgetful map on actual instances; it does not provide a section over every bare predecessor quotient. □

Mixed inputs make this distinction concrete. A comparison with the Higgs or matter input held fixed must preserve its incident maps at that fixed input. Simultaneous transport of both inputs establishes joint covariance instead. The full incidence diagram records which comparison is meant.

1.2 Source results and their scope

SM-2 identifies nonasymptotic color transport as a primitive carrier role and exhausts the source-status alternatives for additional primitive color carriers [4, Sec. 17]. SM-3 places quark descriptions at the intersection of matter, color transport and the specified confinement-compatibility structure [5]. These conclusions concern carrier identity, independently of adjoint-coordinate counting or a free-gluon spectrum.

BCRS-I includes the joint color structure and the source's fixed-local, closed-extended-support construction [1, Sec. 8]. Physical applications retain its local root, sector maps and source premises. A provenance distinction is important here. The nonlocal Yang–Mills source attributes part of its sector construction to a companion reconstruction paper. The companion version available for comparison proves the local reconstruction but does not contain the separately recalled general sector-realization map. We therefore retain the established source result and use the supplied sector attachments, without claiming an independent reconstruction of that general map [11, 10]. The dynamical GNS and spectral results below state separately the matched-instance hypotheses needed for their conclusions.

The global-form classification of SYM-9 is reproved constructively on its specified Standard Model representation family. The resulting screening quotient remains distinct from a phase-dependent confinement law [6]. Actual hadronic persistence and infrared branches supply neutral boundaries within their stated scope [8, 7]. Connecting a particular colored preparation to such a boundary additionally requires a channel between those objects.

1.3 Realization, reconstruction and classification

The argument proceeds from realization to reconstruction and then classification. Sections 2–4 develop the local interaction law, joint neutral responses and boundary-respecting transport reconstruction. Section 5 derives a complete invariant probe family from elementary admitted contractions. The global form calculation in Section 6 extends in Section 7 to magnetic duals, mutually local dyonic branches, screening normal forms and joint phase characters.

The represented-sector and physical-continuation results retain their own source authorities. Their joint maps produce a two-level colored-configuration/neutral-boundary interface, with physical attachment distinguished from neutral tensor projection. Section 11 adds the distributional metric source, local-to-global response gluing and balanced exchange among attached sectors. The complete native algebra, role-closure and minimality results then compare these whole interfaces rather than selected lists of outputs. Section 14 assembles the classification and its physical branch atlas. Appendix A derives the charge data used by the global construction.

The classification is fibred over the actual source-response image: each complete-realization interface datum has a unique lawful realization class. This does not select a numerical color theory, a global group or a universal hadron spectrum. Where the admitted observations determine the full reconstruction, its image is the terminal quotient. Otherwise the native observational quotient removes precisely the unobserved distinctions. This separation is central: reconstructing an interface, observing it and establishing its physical realization are related but different claims. The atlas makes their dependence explicit without replacing the source domain by the subset of successful observations.

2 Color geometry and the local interaction law

Non-Abelian color transport is fixed by the source before a gluon field is chosen. Its geometric realization retains the group, principal bundle, boundary frames, connection, matter representations and cross-sector incidences. The primitive color role is established by SM-2 independently of adjoint coordinate counting [4]. The construction below realizes the complete source-relative transport and representation response of BCRS-I [1, Sections 8, 16–17].

2.1 The adjoint response is one invariant support

For the Standard Model color branch let $\mathfrak{g}_c = \mathfrak{su}(3)$. Its real invariant positive pairing is

$$\kappa(X, Y) = -\operatorname{tr}(XY).$$

The full global group can be a shared quotient; its color adjoint action descends because the center acts trivially.

Theorem 2.1 (Irreducibility of the color adjoint support)**.** *The real adjoint representation on* $\mathfrak{su}(3)$ *has dimension eight and has no invariant real subspaces other than zero and the whole space. Thus its nonzero invariant-support lattice has one atom. Its invariant positive pairing and bracket are intrinsic to that support, not additional support occupants.*

Proof. A traceless anti-Hermitian 3×3 matrix has eight real parameters. If W is invariant under conjugation by $SU(3)$, differentiation gives $[\mathfrak{su}(3), W] \subset W$, so W is a real Lie ideal. The compact real Lie algebra $\mathfrak{su}(3)$ is simple. One direct verification complexifies a nonzero real ideal into an ideal of $\mathfrak{sl}_3(\mathbb{C})$. Commutation with diagonal matrices isolates root components; commuting a nonzero diagonal component with an elementary matrix produces a root component. From any elementary off-diagonal matrix, commutators with elementary matrices generate every root space and the traceless diagonal matrices. Thus $\mathfrak{sl}_3(\mathbb{C})$ is simple, and the real ideal has full dimension. Hence $W = 0$ or $W = \mathfrak{su}(3)$.

Conjugation preserves κ by cyclicity of trace and preserves the bracket by multiplication. Their arguments and outputs remain in the same specified adjoint representation, so they introduce no new invariant subspace beyond the one just classified. □

Irreducibility classifies the internal color response. Polarization, spacetime propagation and particle standing involve additional structures. The eight coordinates therefore imply neither eight primitive color carriers nor eight free asymptotic particles.

2.2 Connections, curvature and self-interaction

In a local frame write a connection as a $\mathfrak{g}_c$-valued one-form A, with curvature

$$F_A = dA + A \wedge A.$$

Matrix multiplication is used in the wedge product; equivalently the quadratic term is one half the Lie-bracket wedge. A local gauge comparison u acts by

$$A^u = uAu^{-1} - (du)u^{-1}.$$

Both the bundle transition data and any prescribed boundary values of u are retained.

Proposition 2.2 (Covariant color descent)**.** *For an actual smooth gauge comparison,*

$$F_{A^u} = uF_A u^{-1}.$$

For a field in a supplied representation, the covariant derivative $D_A = d + \rho_*(A)$ obeys

$$D_{A^u}(\rho(u)\psi) = \rho(u)D_A\psi.$$

The resulting curvature, covariant derivatives, invariant pairings and admitted multilinear contractions are well-defined on the same source-preserving gauge quotient.

Proof. Differentiate $uu^{-1} = 1$ to obtain $d(u^{-1}) = -u^{-1}(du)u^{-1}$. Substituting the formula for A^u into $dA^u + A^u \wedge A^u$ cancels its mixed and pure du terms, leaving $u(dA + A \wedge A)u^{-1}$. The derivative of $\rho(u)\psi$ is canceled by the inhomogeneous term in $\rho_*(A^u)$, giving the second identity. Invariant pairings and actual equivariant contractions remove the remaining conjugations. On overlapping frames these identities use the supplied transition maps, so the local expressions descend together. □

Fix the canonical Yang–Mills local-action branch with its kinetic normalization and metric realization. For the positive Riemannian action, or a positive spatial energy comparison with the same algebraic expansion, put

$$S_c(A) = \frac{1}{2g_c^2}\int \kappa(F_A \wedge *F_A), \qquad g_c > 0.$$

The Lorentzian functional has the same algebraic coefficient relations, without the positivity assertion. Topological terms, boundary terms and matter-current couplings are independent inputs, either fixed or separately observed. Higher operators define a different local-action target. The bare-form expansion below uses a fixed admitted trivialization over the integration region. On a general bundle, take a reference connection A_{ref} and a global adjoint-valued one-form a:

$$F_{A_{\mathrm{ref}}+ta} = F_{A_{\mathrm{ref}}} + t d_{A_{\mathrm{ref}}} a + t^2 a \wedge a.$$

The full curvature law glues between frames; the separate bare dA and $A \wedge A$ terms are not global tensors.

Theorem 2.3 (One local color law determines its self-interactions)**.** *For the canonical local branch, curvature, its first variation and the quadratic, cubic and quartic connection terms are determined by one retained connection law, κ and g_c:*

$$\delta F_A = d_A(\delta A).$$

$$S_c(A) = \frac{1}{2g_c^2}\big(\langle dA, dA\rangle + 2\langle dA, A \wedge A\rangle + \langle A \wedge A, A \wedge A\rangle\big).$$

Here the brackets denote the integrated pairing on two-forms. Every local vertex of this stipulated functional is recovered by differentiation. Deleting stored copies of those vertices or curvature expressions is lossless if the underlying action and its complete source inputs remain.

Proof. Expand $F_{A+\epsilon a}$ to first order:

$$F_{A+\epsilon a} = F_A + \epsilon(da + A \wedge a + a \wedge A) + \epsilon^2 a \wedge a.$$

The linear term is $d_A a$. Substitution of $F_A = dA + A \wedge A$ into the symmetric pairing gives the displayed three terms. The law has polynomial degree four in the connection and its first jets. Its derivatives therefore recover all of its multilinear local vertices with their exact shared coefficients. Re-evaluation from retained data recovers any deleted copy without adding information. Gauge covariance is a property of the full law; its individual cubic or quadratic expressions need not be separately gauge invariant. □

The nonzero bracket expresses self-interaction within the same color law, without adding a carrier or a physical pole. The action determines these linked responses but not independently varied matter, topological or boundary inputs.

Proposition 2.4 (A local coupling witness)**.** *Fix a source-admitted test one-form a on the positive local branch with*

$$N_a = \langle a \wedge a, a \wedge a \rangle > 0.$$

If the target observes the quartic response coefficient q_a of $S_c(A_{\mathrm{ref}} + ta)$ (or $S_c(ta)$ on the trivialized zero-reference branch), then

$$q_a = \frac{N_a}{2g_c^2}, \qquad g_c^2 = \frac{N_a}{2q_a}.$$

With every other local-action input fixed, this one response determines the entire canonical color functional. On an admitted comparison family containing two different positive values of g_c, erasing that response without another datum determining g_c is not lossless.

Proof. Take the coefficient of t^4 in Theorem 2.3; its numerator is positive and fixed. Solving gives the inverse. Different positive couplings give different quartic observations, while the connection, bracket, metric and retained independent inputs are the same. No reconstruction from those equal retained inputs can return both distinct values. □

Non-Abelian test forms with $N_a > 0$ can be constructed on a sufficiently large local positive-metric chart from two independent coordinate one-forms, two noncommuting color elements and a nonzero smooth compactly supported coefficient. Their use as observations still requires source admission. On a narrower native image where retained data already determine g_c, the separate coupling entry is redundant. The result reconstructs the local response law; it does not calculate the physical coupling or its renormalization flow.

3 Joint color invariants and neutral continuation

Color neutrality concerns the joint response: colored constituents can have a neutral composite without being individually neutral. This statement precedes any particle interpretation. Throughout this section, G is the compact color comparison group of the declared realization branch, with its supplied representations, invariant pairings and boundary ports. Averaging uses only transformations that are gauge redundancies of that target. A physical boundary symmetry is not removed merely because its Lie algebra agrees with that of the gauge group.

3.1 Invariant tensors and continuation maps

Let V be a finite-dimensional unitary representation with action ρ_V, and normalize Haar measure by $\int_G dg = 1$. Put

$$P_V = \int_G \rho_V(g)\, dg, \qquad V^G = \{v : \rho_V(g)v = v \text{ for all } g \in G\}.$$

The same notation is used for a strongly continuous unitary representation on a Hilbert space, with the integral taken strongly.

Theorem 3.1 (Joint invariant projection)**.** *The operator P_V is the orthogonal projection onto V^G. For an equivariant bounded linear map $A : V \to W$,*

$$P_W A = A P_V.$$

In finite dimensions, every equivariant linear map is obtained from the projection

$$\mathcal{R}_{V,W}(B) = \int_G \rho_W(g) B \rho_V(g)^{-1}\, dg \quad \textit{on } \operatorname{Hom}(V, W).$$

On a tensor response the relevant projector is $P_{V_1 \otimes \cdots \otimes V_r}$ for the joint action; it is not in general $P_{V_1} \otimes \cdots \otimes P_{V_r}$.

Proof. Left invariance of Haar measure gives $\rho_V(h) P_V = P_V$. Thus the image is invariant, and every invariant vector is fixed by P_V. Inversion invariance and unitarity give $P_V^* = P_V$, proving the first assertion. Integrating $A\rho_V(g) = \rho_W(g)A$ proves naturality. The same change of variable shows that $\mathcal{R}(B)$ intertwines the two actions; an intertwiner is fixed by $\mathcal{R}$, so its image is exactly the intertwiner space.

For the final assertion, take a nontrivial irreducible unitary representation V. Then $V^G = (V^*)^G = 0$, whereas the identity endomorphism is a nonzero invariant vector in $V \otimes V^* \simeq \operatorname{End}(V)$. The separate projectors annihilate this tensor; the joint projector fixes it. □

The theorem constructs the full intertwiner space on the declared representations. Its elements become observations only when the source admits the corresponding contraction or measurement. Mathematical availability of an invariant tensor is thus distinct from physical authority for its use as a probe.

Proposition 3.2 (Closed-operator neutral descent)**.** *Let $A : \operatorname{Dom} A \subset V \to W$ be a densely defined closed linear operator between Hilbert representations. Suppose*

$$\rho_V(g) \operatorname{Dom} A = \operatorname{Dom} A, \qquad A\rho_V(g)v = \rho_W(g)Av \quad (v \in \operatorname{Dom} A).$$

Then

$$P_V \operatorname{Dom} A \subset \operatorname{Dom} A, \qquad AP_V v = P_W A v.$$

The restriction from $\operatorname{Dom} A \cap V^G$ to W^G is closed. It is densely defined in V^G.

Proof. The closed graph of A is an invariant closed linear subspace of $V \oplus W$ for the diagonal action. For $v \in \operatorname{Dom} A$, the continuous graph-valued function $g \mapsto (\rho_V(g)v, \rho_W(g)Av)$ has its Haar integral in that graph. The integral is $(P_V v, P_W Av)$, proving the first two statements. Intersection of a closed graph with the closed subspace $V^G \oplus W^G$ is closed. Finally, project an approximating sequence from the dense domain onto V^G; the first statement keeps that sequence in the restricted domain. □

For differential and continuation operators, the complete domain is essential to this conclusion. A formal differential expression or a common algebraic core alone does not meet the hypotheses.

Proposition 3.3 (Nonlinear continuation uses orbit descent)**.** *Let $D \subset X$ be an invariant domain in a G-space and let $F : D \to Y$ be equivariant. There is a unique map*

$$\overline{F} : D/G \longrightarrow Y/G, \qquad [x] \longmapsto [F(x)].$$

Pullback carries invariant functions on Y to invariant functions on D. If F is continuous, the induced map is continuous for the quotient topologies. Composable equivariant partial maps descend on their exact invariant composite domains.

Proof. Two points in the same orbit have images in the same orbit, proving well-definedness and uniqueness. For invariant f, $f(F(gx)) = f(gF(x)) = f(F(x))$. The quotient topology gives continuity. For a second map $H : E \subset Y \to Z$, its composite domain is $D \cap F^{-1}(E)$, which is invariant; both descended composites send $[x]$ to $[H(F(x))]$. □

Nonlinear maps require this orbit construction rather than the linear projection identity. If $V^G = 0$ and $F(v) = \|v\|^2$ takes values in the trivial representation, then $F(P_V v) = 0$, while $P_{\mathbb{R}}F(v) = \|v\|^2$. Nonlinear gauge-invariant response remains encoded by orbit data even when the average of each charged vector vanishes.

3.2 The fundamental, adjoint and composite color responses

Specialize to $G = SU(3)$ and its standard module $V = \mathbb{C}^3$. This choice specifies a representation branch of the common source, not the global Standard Model group. For a quotient of the full color–weak–hypercharge group, descent of a factor representation must be tested together with its other group actions.

Theorem 3.4 (Basic color-neutral response spaces)**.** *For the standard $SU(3)$ module:*

1. $V^{SU(3)} = 0$.
2. *Under $V \otimes V^* \simeq \mathrm{End}(V)$,*
$$(V \otimes V^*)^{SU(3)} = \mathbb{C}I, \qquad P(X) = \frac{\operatorname{tr} X}{3} I.$$
3. $(V^{\otimes 3})^{SU(3)}$ *is the one-dimensional alternating line generated by*
$$\varepsilon = \sum_{\sigma \in S_3} \operatorname{sgn}(\sigma) e_{\sigma(1)} \otimes e_{\sigma(2)} \otimes e_{\sigma(3)}.$$
4. *The complex adjoint module $\mathfrak{sl}_3(\mathbb{C})$ has no invariant vector. It nevertheless admits the nonzero invariant pairing $\operatorname{tr}(XY)$, the invariant Hermitian pairing $\operatorname{tr}(X^*Y)$, and the equivariant bracket $[X, Y]$.*

Proof. A central element ζI, with $\zeta = e^{2\pi i/3}$, acts by ζ on V, so no nonzero vector is fixed. The standard representation is irreducible: the group acts transitively on its unit sphere. An endomorphism commuting with every group element is scalar. For completeness, a complex eigenvalue has a nonzero eigenspace, which is invariant and therefore all of V. The average of conjugates preserves trace, giving the formula in (ii).

For (iii), invariance under $\mathrm{diag}(e^{it_1}, e^{it_2}, e^{-i(t_1+t_2)})$ forces each surviving elementary tensor to contain the three basis indices once. Differentiating invariance under $SU(3)$, and complexifying its Lie algebra, gives invariance under each elementary off-diagonal generator. Replacing an occurrence of one index by another then forces the coefficients of the two corresponding permutations to be negatives. The adjacent transpositions generate S_3, so all coefficients are fixed by one value with the alternating signs. Conversely, the alternating tensor transforms by the determinant, which is one.

An invariant traceless matrix commutes with the standard action, hence is scalar and traceless and therefore zero. Conjugation preserves both displayed trace pairings and the bracket. The bracket is nonzero, for example $[E_{12}, E_{23}] = E_{13}$. □

The fundamental–antifundamental pair and the three-fundamental tensor therefore have explicit singlet contractions. These are the color factors of conventional mesonic and baryonic channels. Physical interpretation also requires the appropriate spin, flavor, statistics, localization and boundary authority; bound-state, pole or asymptotic existence must be established for the type claimed. Mass and lifetime are not determined by the alternating color line.

Proposition 3.5 (Triality is necessary, not sufficient)**.** *Suppose the center acts on each module V_j by the character ζ^{t_j}, with $t_j \in \mathbb{Z}/3\mathbb{Z}$. Then*

$$\left(\bigotimes_j V_j\right)^{SU(3)} \neq 0 \quad \Longrightarrow \quad \sum_j t_j = 0 \pmod 3.$$

The converse is false. Also, on the full classical one-loop $SU(3)$ configuration family, adjoint transport does not determine the fundamental trace. A native target inherits this non-determination when it admits both holonomies used below and the distinguishing fundamental trace probe.

Proof. The central element acts on the tensor product by the product character. A fixed nonzero vector forces that character to be one. The adjoint module has triality zero but no fixed vector, by Theorem 3.4, disproving the converse. For the last assertion, a one-loop holonomy $U = I$ and $U' = \zeta I$ have identical adjoint action. They are not conjugate in $SU(3)$, and their admitted fundamental traces are 3 and 3ζ. Thus adjoint data cannot reconstruct that trace on the displayed classical comparison family. The same conclusion holds on an actual admitted image containing both configurations and admitting the trace test. This example lies within one fixed $SU(3)$ branch; it does not compare incompatible global-form targets or require both configurations to occur in every native image. □

Neither the adjoint index range nor the singlet tensors enlarge the primitive carrier classification. Brackets, pairings and contractions describe interactions among the retained responses; an interaction vertex or generator is not a further occupant.

3.3 Neutral observable algebras and cyclic physical spaces

A corresponding operator-algebraic construction applies when the source supplies a unital C^*-algebra $\mathfrak{F}$ with a point-norm continuous action $\alpha : G \to \mathrm{Aut}(\mathfrak{F})$. This realization is additional source data; finite-dimensional tensor notation alone does not establish it. Write

$$\mathfrak{A} = \mathfrak{F}^G, \qquad E(a) = \int_G \alpha_g(a)\, dg.$$

Theorem 3.6 (Neutral conditional expectation)**.** *The map $E : \mathfrak{F} \to \mathfrak{A}$ is a faithful unital completely positive idempotent of norm one. For $b, c \in \mathfrak{A}$,*

$$E(bac) = bE(a)c.$$

If an actual equivariant unital $*$-homomorphism $\Phi : \mathfrak{F} \to \mathfrak{F}'$ is supplied, then

$$E'\Phi = \Phi E.$$

These statements retain the supplied algebra, action and boundary scope; they do not construct an absent charged-field algebra.

Proof. The integral exists in norm. Haar invariance gives its range and idempotence. Every matrix amplification is an average of $*$-automorphisms and is positive, so E is completely positive; it is unital and contractive and hence has norm one. Fixed left and right factors pass through the integral, proving the bimodule identity.

If $E(a^*a) = 0$, every state φ satisfies $\int_G \varphi(\alpha_g(a^*a))\, dg = 0$. The integrand is continuous and nonnegative, so it vanishes at the identity. Thus every state vanishes on a^*a, implying $a = 0$. Finally, integrate the actual equivariance identity for Φ. □

Theorem 3.7 (The neutral cyclic space). *Let ω be a G-invariant state of $\mathfrak{F}$ and $(\pi_\omega, \mathcal{H}_\omega, \Omega_\omega)$ its GNS representation. The formula*

$$U_g \pi_\omega(a)\Omega_\omega = \pi_\omega(\alpha_g(a))\Omega_\omega$$

defines a strongly continuous unitary representation fixing Ω_ω. Its invariant space is exactly

$$\mathcal{H}_\omega^G = \overline{\pi_\omega(\mathfrak{A})\Omega_\omega}, \qquad P_G \pi_\omega(a)\Omega_\omega = \pi_\omega(E(a))\Omega_\omega.$$

The GNS representation of $\omega|_{\mathfrak{A}}$ is canonically unitarily identified with this cyclic subspace.

Proof. Invariance of the state preserves the squared norm of every GNS vector and every null relation. Hence the displayed maps extend to unitaries, obey the group law and fix the cyclic vector. Point-norm continuity of α gives continuity on the dense GNS domain and therefore strong continuity. Integrating on that domain gives the displayed projection formula. Its right side belongs to the neutral cyclic subspace. Conversely every neutral-algebra vector is fixed; projecting the dense GNS domain proves equality of the closed spaces. The map from the restricted-state GNS class of $b \in \mathfrak{A}$ to $\pi_\omega(b)\Omega_\omega$ preserves inner products and has dense image, proving the last assertion. □

Corollary 3.8 (No isolated nontrivial color input to a neutral target). *Let $\mathcal{H}_{\mathrm{n}}$ be a Hilbert space on which G acts trivially, and V a finite-dimensional representation with $V^G = 0$. Every color-equivariant linear map $J : V \to \mathcal{H}_{\mathrm{n}}$ is zero. A nonzero equivariant composite map can instead factor through $(V_1 \otimes \cdots \otimes V_r)^G$.*

Proof. Naturality gives $J = P_{\mathcal{H}_{\mathrm{n}}} J = J P_V = 0$. For a tensor input W, it gives $J = J P_W$, which is exact factorization through the joint invariant subspace. □

The result is a two-level selection theorem for the declared neutral output. It does not place every physical sector in that output: a larger representation may contain charged sectors whose admission depends on the physical target. Projection onto invariants therefore establishes neutral response structure, not dynamical confinement.

4 Reconstruction from complete color-transport networks

A color field carries more information than its adjoint dimension. Already two loops can have a relative transport invariant that neither loop determines separately. We reconstruct this joint information first on a finite network with fixed boundary frames, then from all finite restrictions of a transport functor. Classical reconstruction, observational detectability and quantum realization remain distinct throughout.

4.1 Boundary-respecting forest reduction

Let $Q = (V_Q, E_Q, s, t)$ be a finite directed graph and $B \subset V_Q$ its fixed boundary vertices. Edges carry $U_e \in G$, with inverse transport for a reversed edge. A vertex may carry vectors in specified finite-dimensional representations and a finite family of incident tensors. The complete tuple includes those tensors, source labels and the actual domain and status information. On this branch the gauge group is

$$K_B = \{(h_v) \in G^{V_Q} : h_b = 1 \text{ for all } b \in B\}, \qquad U_e \longmapsto h_{t(e)} U_e h_{s(e)}^{-1}.$$

It acts on every incident vector and tensor slot. The admitted configuration set X_Q is invariant. A smaller lawful source-comparison group has its own orbit problem and is not silently replaced by K_B.

Choose a spanning forest $\mathcal{T}$ such that each tree in a connected component meeting B has exactly one boundary root. In a component disjoint from B, choose one tree and one free root. Such a forest is obtained by growing disjoint trees from the prescribed roots without joining two rooted trees. Let $r(v)$ be the root of the tree containing v, and p_v the product along its unique tree path from $r(v)$ to v, with $p_r = 1$. Define

$$H_e = p_{t(e)}^{-1} U_e p_{s(e)}, \qquad \widehat{v}_v = \rho_v(p_v^{-1}) v_v.$$

Every incident tensor is transported in all of its slots by the same rule. Tree edges have $H_e = 1$. Keep the non-tree H_e, all rooted vectors and tensors, and all source records.

Theorem 4.1 (Complete forest normal form)**.** *Let R_f be the set of free roots and $K_R = G^{R_\mathrm{f}}$. On the actual admitted image, forest reduction gives a bijection*

$$X_Q/K_B \;\simeq\; \{\textit{complete rooted tuples}\}/K_R.$$

The remaining action is

$$H_e \longmapsto h_{r(t(e))} H_e h_{r(s(e))}^{-1},$$

with anchored root factors equal to one, together with the corresponding actions on every rooted tensor slot. The bijection preserves and reflects all invariant source domains, incidences and failure values.

Proof. Cancellation along a tree path gives $p_v' = h_v p_v h_{r(v)}^{-1}$. Substitution gives precisely the stated action on H_e and all rooted data. At a boundary root $h_r = 1$, so the orbit map is well defined.

Apply $n_v = p_v^{-1}$. Since each boundary vertex is its own tree root, $n_b = 1$, and $n \in K_B$. This gauges the tree edges to one and produces the rooted tuple. Conversely, that tuple together with unit tree edges is its complete tree-normal representative. A gauge transformation between two tree-normal representatives obeys $h_t = h_s$ on every tree edge, so it is constant on each tree and is one on every anchored tree. Exactly K_R remains. All source records are carried by the same actual gauge transformations, proving domain and incidence preservation in both directions. □

An edge between anchored trees retains their full relative transport. Averaging a boundary frame would instead remove a physically retained symmetry action. A spanning tree through several fixed anchors therefore cannot generally be gauged to one. The forest normal form changes the presentation without adding color information.

4.2 Separating invariants of a finite network

Suppose a faithful finite-dimensional unitary realization of the declared compact group is retained for this mathematical transport comparison. In this subsection work in one fixed fibre of the retained non-coordinate source, boundary, domain and status records, after their actual source identifications. Across different such fibres, equality of those records is an additional requirement, not a consequence of polynomial tests. Matrix entries of links and coordinates of the finite tensor family embed X_Q in a finite-dimensional real representation W of K_B. Conjugate coordinates are included when complex representations are used.

For a coordinate monomial m, set

$$\mathcal{R}m(x) = \int_{K_B} m(kx)\, dk.$$

Expanding the monomial before integration shows that this is a finite polynomial with invariant coefficient tensor. The linear span of these polynomials is exactly $\mathbb{R}[W]^{K_B}$: averaging fixes any invariant polynomial and commutes with its finite monomial expansion.

Theorem 4.2 (Complete finite-network separation)**.** *For $x, y \in X_Q$,*

$$\mathcal{R}m(x) = \mathcal{R}m(y) \text{ for every coordinate monomial } m \iff y \in K_B x.$$

The result does not require X_Q itself to be compact.

Proof. Same-orbit equality follows from Haar invariance. For the converse use an invariant Euclidean norm on W and suppose the compact orbits of x and y are disjoint. Put $\delta = \operatorname{dist}(x, K_B y) > 0$. The open set

$$N = \{k : \|x - kx\| < \delta/2\}$$

contains the identity and has positive Haar measure μ. For $a > 0$, the invariant continuous function

$$f_a(z) = \int_{K_B} e^{-a\|x-kz\|^2}\, dk$$

satisfies

$$f_a(x) \geq \mu e^{-a\delta^2/4}, \qquad f_a(y) \leq e^{-a\delta^2}.$$

Choose a large enough that the first bound is larger than the second. On the compact union of the two orbits, the exponential series converges uniformly. A sufficiently long finite truncation of $e^{-a\|x-kz\|^2}$, integrated over k, therefore still separates x and y. This truncation is an invariant polynomial: its finitely many coefficients are Haar integrals of continuous coefficient functions. It is a finite linear combination of Reynolds monomials. At least one of those monomials must separate the two points. □

Each distinct pair of finite-network orbits is thus separated by a finite invariant polynomial. The required degree may depend on the pair; no universal low-degree cutoff is asserted. Nor do separate loop spectra determine the joint orbit.

Proposition 4.3 (Joint data cannot be replaced by marginal loop classes)**.** *On the full classical configuration family of a one-vertex two-loop $SU(3)$ network, the separate conjugacy classes of the loop holonomies do not determine the complete joint invariant response. The displayed pair also gives a native non-determination witness whenever both configurations and their distinguishing mixed probe are admitted.*

Proof. Let

$$A = \operatorname{diag}(1, i, -i), \qquad B = A, \qquad B' = A^{-1}.$$

All three matrices lie in $SU(3)$, and B and B' are conjugate by a determinant-one unitary matrix interchanging the last two coordinate lines. The first loop is unchanged, and the second loop has the same conjugacy class in the two configurations. Nevertheless,

$$\operatorname{tr}(AB) = -1, \qquad \operatorname{tr}(AB') = 3.$$

This mixed loop contraction is invariant under simultaneous conjugation, so the two classical joint orbits differ. On a native image containing both configurations, admission of this probe gives the same observable distinction. Probe admission alone does not imply that both configurations belong to that image. □

Definition 4.4 (Invariant-complete network observation branch)**.** A network observation target is invariant-complete when its source independently admits the representation tensors and evaluations of all computed Reynolds coefficient tensors on the declared network family, and its lawful comparisons are the gauge comparisons used above. It also retains the actual source, boundary, continuation and failure data.

Invariant completeness is an additional condition on the observation domain, not a consequence of tensor products. Adjoint probes alone may fail to separate full-group orbits, and a shared global quotient may prohibit a bare color fundamental probe. On an invariant-complete branch, Theorem 4.2 identifies the observational quotient with the network orbit. With fewer native probes, the orbit is a sufficient reconstructive description, and its native-readout quotient is the terminal object. This distinction respects the actual probe family.

Section 5 gives a constructive sufficient condition for invariant completeness: elementary witnessed contractions, their admitted composition rules and complete boundary readouts generate the required observations. This supplies a source-level route to the condition without attributing observation status to tensor products alone.

4.3 Reconstruction from all finite restrictions

Let $\mathcal{P}$ be a fixed small path groupoid, with object set M, and let $F, F' : \mathcal{P} \to G$ be transport functors. Fix a set $B \subset M$ of boundary frames. Finite-dimensional matter and tensor data may be attached at the objects with their actual incidences. The path domain and all nonnumerical source records are fixed or compared by their separately supplied source maps.

Theorem 4.5 (Full pointwise transport reconstruction)**.** *Suppose every finite family of arrows and attached tensor data has equal complete invariant-polynomial evaluations under the finite-vertex gauge group fixing B. Then there is a single family $g_x \in G$, with $g_b = 1$ for $b \in B$, such that*

$$F'(\gamma) = g_{t(\gamma)} F(\gamma) g_{s(\gamma)}^{-1} \quad (\gamma \in \mathcal{P}),$$

and the same family transports all attached tensor data. Conversely, such a family preserves every finite invariant evaluation.

Proof. The product $K = \prod_{x \in M \setminus B} G$, with boundary values fixed to one, is compact. Each arrow equation defines a closed cylinder subset of K. Each finite-dimensional tensor equation is likewise closed. A finite selection of these equations uses only a finite network; by Theorem 4.2, its complete invariant values provide a simultaneous vertex-gauge solution. Extend that finite solution arbitrarily outside the network. The closed constraint sets therefore have the finite intersection property. Compactness gives one family satisfying every equation. The converse follows by restriction to each finite network and invariance. □

Compactness produces one comparison satisfying all finite constraints simultaneously, rather than a separate gauge choice for each network. For nontrivial principal bundles, replace the factor G at each object by the compact torsor of equivariant identifications between its two fibres; use the fixed boundary identifications there. The same closed-constraint argument applies without choosing a global bundle trivialization.

At this stage the conclusion is pointwise gauge equivalence. Smoothness requires the connection hypotheses below; a quantum measure, operator net or spectrum requires further realization data. Restricting the comparison class, or imposing an extended-sector condition not known to be closed in the pointwise topology, likewise requires an additional argument.

Theorem 4.6 (Smooth reconstruction on a connection branch)**.** *Suppose F, F' in Theorem 4.5 are the actual parallel transports of smooth connections on a fixed smooth principal bundle. Assume their path domain contains, around every x_0, a smoothly varying family of short paths $\gamma_x : x_0 \to x$, with the corresponding smooth parallel transport. Then the pointwise comparison is a smooth gauge automorphism with the prescribed boundary values. In local frames and the convention $\dot{U} = -A(\dot{\gamma})U$, it satisfies*

$$A' = gAg^{-1} - (dg)g^{-1}.$$

Proof. In a local trivialization, the transport intertwining equation forces

$$g_x = F'(\gamma_x) g_{x_0} F(\gamma_x)^{-1}.$$

The right side is smooth in x. These local expressions agree on overlaps because they describe the already existing global pointwise family. Thus they give a smooth bundle automorphism with the same boundary values. Differentiating the transport identity on a short path starting at a fixed point gives the displayed connection transformation law. □

Complete finite-network observations therefore classify the smooth connection on this branch, beyond its values on any chosen graph. This classical reconstruction is one part of color realization. Local and extended quantum responses additionally require states, operators and their continuation data.

5 Generating complete color observations from witnessed contractions

Complete network observations need not be supplied one polynomial at a time. On a tensor-realized color branch, a small family of actual contractions generates them. This gives a sufficient source-level criterion for Definition 4.4, rather than assuming that every mathematically available invariant is already an observation.

The distinction matters for representation formation. PSM-3 requires an actual comparison with the response carrier and preservation of its boundary, quotient and transport witnesses. Its tensor and dual criteria require, respectively, a joint composite witness and a witnessed pairing or reversal; they do not grant arbitrary tensor products or duals [21, Definitions 4.1–4.3 and Theorems 8.4, 9.2, 13.1]. SR-3 further retains the same product-sector attachment, charge, anomaly, residual, vacuum and deformation data [22, Theorems 14.3, 15.2 and 24.1]. We use these source conditions for the generating operations themselves. Their consequences for all composite observations are then proved below.

5.1 The elementary color contractions

For the mathematical color calculation put $V = \mathbb{C}^3$, with its standard unitary $SU(3)$-action. Write

$$I = \sum_{a=1}^{3} e_a \otimes e^a, \qquad \varepsilon = \sum_{\pi \in S_3} \mathrm{sgn}(\pi) e_{\pi(1)} \otimes e_{\pi(2)} \otimes e_{\pi(3)}, \qquad \varepsilon^\vee = \sum_{\pi \in S_3} \mathrm{sgn}(\pi) e^{\pi(1)} \otimes e^{\pi(2)} \otimes e^{\pi(3)}.$$

Evaluation is $e^a(e_b) = \delta^a_b$; in particular $\varepsilon^\vee(\varepsilon) = 6$. A contraction diagram is a tensor expression made from these tensors, evaluation, permutations, and tensor composition. The displayed basis specifies a calculation, not a physical color frame.

Lemma 5.1 (Mixed-tensor invariant generation)**.** *For every $r, s \geq 0$, the space*

$$\left(V^{\otimes r} \otimes (V^*)^{\otimes s}\right)^{SU(3)}$$

is spanned by contraction diagrams in $I, \varepsilon, \varepsilon^\vee$. Consequently every intertwiner between two mixed tensor words is a finite linear combination of these diagrams, after turning input factors into dual output factors by evaluation and coevaluation.

Proof. Apply Schrijver's mixed-tensor generation criterion [24, Corollary 1.2] to the singleton consisting of ε. Its stabilizer in $U(3)$ is precisely $SU(3)$, because its transformation factor is the determinant. The criterion identifies the invariant algebra with the contraction-closed graded

$*$-algebra generated by this tensor and I. Its adjoint supplies $\varepsilon^\vee$, up to the harmless sign from reversing the factor order. The canonical identification $\mathrm{Hom}(A, B) = B \otimes A^*$ gives the intertwiner assertion. □

Proposition 5.2 (Finite calculation of the averaging coefficients)**.** *For each mixed tensor word, its Haar projector has an expression using finitely many contraction diagrams and rational coefficients in the displayed tensor bases. Its computation requires no new invariant tensor as input. The same conclusion, with coefficients in $\mathbb{Q}(i)$, holds after taking real and imaginary coordinates.*

Proof. Enumerate the finite contraction expressions with the required output type. Each has integral coordinates: its entries are obtained by finite sums and products of the entries of $I, \varepsilon, \varepsilon^\vee$. Retain linearly independent columns until they span the invariant space. There is a finite stopping test independent of the desired conclusion: compute the common kernel of the infinitesimal action of eight unnormalized matrix generators of $\mathfrak{su}(3)$. Their mixed-word matrices have entries in $\mathbb{Q}(i)$. Since $SU(3)$ is connected, this kernel is exactly its fixed space. Lemma 5.1 therefore guarantees termination when the retained columns reach that computed dimension.

Let D be the matrix of those independent diagram columns. If the fixed space is nonzero, its orthogonal projector is

$$P = D(D^*D)^{-1}D^*.$$

The Gram matrix is integral and invertible, so its inverse is rational. The unitary Haar projector is the orthogonal projector onto the same fixed space; hence it equals P. If the fixed space is zero, the projector is zero. The coordinate changes $\mathrm{Re}\, z = (z + \overline{z})/2$ and $\mathrm{Im}\, z = (z - \overline{z})/(2i)$ only introduce coefficients in $\mathbb{Q}(i)$. Thus each specified coefficient calculation is finite, even though no uniform bound on tensor degree is asserted. □

5.2 From generating operations to admitted observations

Work first on a source branch with an actual $SU(3)$ color realization and its retained boundary frames. Fix the common source, domain, continuation, failure and non-color records. A finite network has link matrices and finitely many attached color tensors as in Section 4. In the following definition a generator is an actual source operation, not merely its symbolic tensor.

Definition 5.3 (Contraction-generated observation branch)**.** A source has contraction-generated color observations on a supplied nonempty network image when the following data and closure laws hold.

1. Each finite coordinate family is an actual equivariant realization in a finite direct sum of mixed color tensor words, with its conjugate coordinates and all incident inputs retained. The component inclusions and projections are actual admitted source maps on the common domains, or the corresponding component inputs are independently supplied with their actual incidence maps. A bare direct-sum identification does not supply these operations. Any subrepresentation used instead has a supplied equivariant inclusion and retraction. Lawful color comparisons are exactly the vertex gauge comparisons fixing the retained boundary frames.
2. The source supplies evaluation, coevaluation, the two volume tensors above, identity and permutation maps on their actual domains. They obey the displayed normalizations and the duality identities. The finite-dimensional algebraic maps are total on their declared tensor fibres. Any additional native-domain restrictions are fixed beforehand on a common source-admitted domain on which these typed operations and the closure laws below are defined; this domain coverage is supplied, not inferred from a failure label. For every occurrence their full input and output types, including non-color attachments, are specified. Their source comparisons preserve the PSM-3 and SR-3 witness data just described.

3. Finite tensor products and composition of these actual maps, repeated use of the same retained input, conjugation, and finite $\mathbb{Q}(i)$-linear combinations of parallel maps and scalar readouts are admitted. Real and imaginary parts are readouts. Each composite keeps the common input incidences and is defined on the actual intersection of its input domains; its failure values are not replaced by numerical zero.
4. At a vertex not acted on by the gauge group, the actual retained boundary or spectator readouts include the linear evaluations of a retained dual frame for every uncontracted coordinate slot. Those readouts and their source domains are transported by the same comparison as the network inputs.

These hypotheses concern a fixed list of generator types and their composition laws. They do not posit the admission of all invariant polynomials or all Reynolds coefficient tensors.

Theorem 5.4 (Source generation of complete finite-network observations)**.** *On a contraction-generated observation branch, every Reynolds coordinate monomial of Section 4 is an admitted finite composite readout. The resulting diagram readouts separate precisely the boundary-fixed gauge orbits on each fixed fibre of the retained non-coordinate records. Thus this branch satisfies Definition 4.4 by construction.*

Proof. Choose one real coordinate monomial. Express it using complex and conjugate coordinates, with coefficients in $\mathbb{Q}(i)$. Each resulting term is a coordinate functional on a finite tensor power of the network input representation. Repeated occurrences refer to the same actual input, as required in Definition 5.3; they are not independently varied copies. Dualizing coordinate functionals, regroup their color indices at the network vertices.

The internal gauge group is a product of the vertex copies of $SU(3)$. Its averaging projector is the product of the individual vertex projectors. On the regrouped tensor word, Proposition 5.2 expresses each such projector by finitely many elementary contractions and rational coefficients. At a boundary vertex there is no averaging: the retained determining readouts evaluate its open slots. Tensoring the vertex expressions, restoring the original slot order, and taking real and imaginary parts therefore gives exactly the chosen Reynolds monomial. If an input is in a supplied subrepresentation, include it in its tensor word before this calculation and restrict back by its supplied maps.

Admission follows by induction on this finite expression. Its leaves are the actual network inputs, coefficient constants and generating source maps. Each internal vertex uses one of the stated closure laws. The same induction preserves the entire tuple of incident inputs and the source comparison squares: tensoring combines their squares and composition composes them. Intersections of the actual input domains are retained at every step. Consequently this is an admitted readout with the correct failure status, not a formal invariant assigned an observation after the fact.

Every resulting scalar diagram is gauge invariant. Conversely, equal diagram values imply equal Reynolds monomials by the finite expressions just constructed. Theorem 4.2 then gives one boundary-fixed gauge transformation of the complete finite tuple. Records outside that numerical fibre are still compared separately; polynomial equality is not used to reconstruct them. □

Corollary 5.5 (Adjoint and full-transport consequences)**.** *The theorem includes an actual complexified adjoint input in $\mathfrak{sl}_3(\mathbb{C})$, the trace-free part of $V \otimes V^*$, and an actual real adjoint input in $\mathfrak{su}(3)$, its anti-Hermitian trace-free real subspace, with the respective supplied inclusions and retractions. The latter has eight real dimensions; its complexification has eight complex dimensions. If the hypotheses hold on every finite restriction of the fixed path groupoid, equality of the generated observations gives the single pointwise transport comparison of Theorem 4.5. Under the connection hypotheses of Theorem 4.6, that comparison is smooth.*

Proof. The complex-linear retraction onto the complexified adjoint is

$$X \longmapsto X - \frac{\operatorname{tr} X}{3} I.$$

The real-linear retraction onto the real adjoint is

$$X \longmapsto \frac{X - X^\dagger}{2} - \frac{\operatorname{tr}(X - X^\dagger)}{6} I.$$

Its output is anti-Hermitian and trace-free, and it fixes every $X \in \mathfrak{su}(3)$. Both maps intertwine conjugation by $SU(3)$: unitarity gives $(gXg^{-1})^\dagger = gX^\dagger g^{-1}$, and trace is invariant. They use evaluation, coevaluation and rational linear combinations; the real retraction additionally uses the admitted conjugation and permutation of the two matrix slots. Hence no additional invariant generator is needed. Products of either kind of adjoint input use their respective inclusion and retraction in each slot; complexifying the real representation is a calculation device, not an increase in the number of real color directions. Theorem 5.4 supplies the complete invariant-polynomial equality on every finite restriction. The two cited reconstruction theorems then apply with the same path, boundary and attached-tensor data. □

The adjoint assertion does not say that adjoint measurements alone distinguish fundamental transport. Completeness is for the actual input representation and actual determining readouts. To reconstruct full color transport rather than a center-blind quotient, that input realization must retain a faithful color action as in Section 4.

5.3 Joint global descent and fixed non-color inputs

The preceding color calculation has a precise global domain. For the shared groups G_n of Section 6, a bare fundamental has labels $(t, s, q) = (1, 0, 0)$ and descends only for $n = 1, 2$. For $n = 3, 6$, the symbol V in a local contraction calculation cannot by itself denote a global associated bundle. Full Standard-Model tensor factors and their weak and hypercharge attachments must be used.

Proposition 5.6 (Joint interpretation of the generated contractions)**.** *Fix an admitted principal G_n-bundle and its common electroweak and Higgs target. A local color contraction constructed above defines a global source operation when its inputs are genuine joint associated bundles and its completed map intertwines their full transition functions into its specified joint output. If the generating maps, their closure operations and determining spectator readouts satisfy these conditions, every generated composite does also. The separation theorem consequently applies on the resulting fixed non-color fibre, provided its actual residual comparisons and input realization meet Definition 5.3.*

Proof. The elementary map in this proposition is the completed full intertwiner, not its color component alone. A color contraction with the remaining slots left open has their inherited weak and hypercharge action on its output. Conversely, a volume insertion into charged joint tensor factors requires a matching input representation; it is not automatically an operation from the scalar unit. A further contraction of the remaining slots must itself be an admitted intertwiner; color invariance does not supply it. For a color-singlet output the inherited labels obey $3s + q = 0 \pmod{n}$, by Proposition 6.5. They need not be individually zero.

Write the full transition intertwining square for each generating map. Tensor products, composition and linear combinations of parallel maps preserve that square. Induction therefore makes the composite independent of the chosen local presentation and glues it to its actual associated-bundle map. The same induction applies to the retained source domains and joint comparison squares. Bare local factor indices have been contracted inside full transition-covariant expressions; no separate fundamental bundle has been introduced.

The common non-color input is fixed throughout the separation argument. Its comparison is the identity, or a separately supplied identification with that fixed input; it is not merely a simultaneous transport to a different input. With the stated residual group and determining readouts, the finite-network proof now applies to that actual fibre. Without those hypotheses, the result established here is the gluing of the generated operations, not completeness of its observational scope. □

Elementary witnessed operations thus determine the invariant observations needed for classical color reconstruction. They neither select a global-group branch nor create a new standing carrier for each tensor expression. SR-3's irreducible composite criterion still distinguishes a new joint role from equivalent tensor bookkeeping [22, Proposition 14.4 and Corollary 14.5]. Likewise, contraction generation does not create a state, a stable pole or an asymptotic hadron. Those are the independently realized objects and continuation requirements of the physical color analysis.

6 Global form, screening, and the shared color target

Local adjoint data leave a global ambiguity. Color, weak isospin and hypercharge can share a quotient even though their Lie algebras split. The global group and bundle must therefore be the same throughout the color, matter and Higgs realization. We use the global-form analysis of SYM-9 [6], while retaining the primitive color and matter roles established by SM-2 and SM-3 [4, 5].

6.1 The common representation kernel

Fix the admitted minimal chiral matter and one-doublet Higgs branch. Use integer hypercharge $q = 6Y$, color triality $t \in \mathbb{Z}/3\mathbb{Z}$, and weak center parity $s \in \mathbb{Z}/2\mathbb{Z}$. One generation and the Higgs have center labels

	Q	u^c	d^c	L	e^c	H
t	1	-1	-1	0	0	0
s	1	0	0	1	0	1
q	1	-4	2	-3	6	3

where u^c, d^c are left-handed color anti-fundamentals. Repeating the representation action across generations leaves the common kernel unchanged. The calculation in this section takes this source-admitted table as input. Appendix A derives its charge ratios from joint operator and anomaly constraints on the minimal chiral branch and identifies the separate primitive-lattice condition required for this normalization. Let

$$G_0 = SU(3)_c \times SU(2)_L \times U(1)_q, \qquad \xi = (\omega_3, -1, e^{i\pi/3}), \qquad \omega_3 = e^{2\pi i/3}.$$

Theorem 6.1 (Common-kernel and global-family calculation)**.** *The common kernel of the displayed matter and Higgs action is $K_{\mathrm{mat}} = \langle \xi \rangle \simeq \mathbb{Z}/6\mathbb{Z}$. Within the source class of compact connected groups obtained by quotienting G_0 by a central subgroup invisible to this matter action, the complete family is*

$$G_n = G_0/\Gamma_n, \qquad \Gamma_n = \langle \xi^{6/n} \rangle, \qquad n \in \{1, 2, 3, 6\}.$$

All four groups have the same local Lie-algebra action on the admitted matter and Higgs fields.

Proof. The action of ξ on a label (t, s, q) is $\exp(i\pi(2t + 3s + q)/3)$. Each table entry gives the identity. Conversely, let $(g_3, g_2, e^{i\alpha})$ lie in the common kernel. Its action on e^c gives $e^{6i\alpha} = 1$, so $\alpha = k\pi/3$ modulo 2π. Its action on L gives $g_2 = e^{3i\alpha} I_2 = (-1)^k I_2$. Its action on d^c gives $\overline{g_3} = e^{-2i\alpha} I_3$, hence $g_3 = \omega_3^k I_3$. Thus the element is ξ^k and the kernel is exactly the cyclic group of order six. That group has precisely one subgroup of each order dividing six, and every such subgroup acts trivially on the table. The quotient maps are local isomorphisms and the matter representations descend with unchanged differentials. This is the explicit content of SYM9-Q02 and SYM9-Q03 on their declared quotient family [6]. □

Remark 6.2. The theorem does not choose one member by examining the local adjoint response. It does not enlarge the family to disconnected or noncompact groups, change the Abelian period, or introduce spin-charge refinements. The source global fingerprint also retains the principal-bundle class, operator category, magnetic policy, line branch, admitted backgrounds and their action, Yukawa and phase data, and geometric scope. Their omission cannot be repaired by the name $SU(3)$ alone.

6.2 Electric descent and screening

Write the center-charge group as

$$L = (\mathbb{Z}/3\mathbb{Z}) \oplus (\mathbb{Z}/2\mathbb{Z}) \oplus \mathbb{Z}, \qquad \kappa(t, s, q) = 2t + 3s + q \pmod 6.$$

The coefficients make κ well-defined on the residue classes of t and s. Let L_{dyn} be the subgroup generated by the center charges of the admitted dynamical matter and Higgs fields, with antiparticles represented by additive inverses. A heavy probe is not added to this subgroup merely because its line is admitted.

Theorem 6.3 (Electric descent and exact screening)**.** *A center-charge label descends to G_n precisely when it belongs to*

$$L_n = \{\ell \in L : \kappa(\ell) = 0 \pmod n\}.$$

Moreover

$$L_{\mathrm{dyn}} = \ker \kappa, \qquad L_n/L_{\mathrm{dyn}} \simeq n(\mathbb{Z}/6\mathbb{Z}) \simeq \mathbb{Z}/(6/n)\mathbb{Z}.$$

The electric one-form action on these screened charge classes is the dual finite group, equivalently $K_{\mathrm{mat}}/\Gamma_n$.

Proof. The generator $\xi^{6/n}$ acts by $\exp(2\pi i\kappa(\ell)/n)$. Its action is trivial exactly on L_n, which proves descent. Every dynamical table entry lies in $\ker \kappa$. For the reverse inclusion choose integer representatives t, s and write

$$(t, s, q) = -t(-1, 0, 2) + s(0, 1, 3) + \frac{q + 2t - 3s}{6}(0, 0, 6).$$

If $\kappa(t, s, q) = 0$, the last coefficient is integral: its numerator differs from $q + 2t + 3s$ by $6s$. The three displayed generators are d^c, H, e^c. Hence every element of $\ker \kappa$ belongs to L_{dyn}, proving equality without assuming that an unseen screening class is trivial.

The restriction $\kappa : L_n \longrightarrow n(\mathbb{Z}/6\mathbb{Z})$ is surjective because $(0, 0, nj)$ realizes every multiple of n. Its kernel is L_{dyn}, so the first isomorphism theorem gives the stated quotient. The character pairing with $K_{\mathrm{mat}}/\Gamma_n$ is nondegenerate: a class represented by $\kappa(\ell) = nj$ acquires the phase $\exp(2\pi ij/(6/n))$ under the residual generator. This proves the screened charge and electric one-form assertions of SYM9-Q04 and SYM9-Q05 at their charge-level scope [6]. □

The four explicit descent rules and residual groups are

n	electric descent	L_n/L_{dyn}
1	no congruence	$\mathbb{Z}/6\mathbb{Z}$
2	$q = s \pmod 2$	$\mathbb{Z}/3\mathbb{Z}$
3	$q = t \pmod 3$	$\mathbb{Z}/2\mathbb{Z}$
6	$q = 3s - 2t \pmod 6$	0.

The table classifies center charges, not complete representations. Equal center labels may accompany different highest weights and multiplicities, so a color network still requires its representation and junction data.

Proposition 6.4 (A global-form deletion witness)**.** *Across the computed global-group family, the local adjoint and matter differentials do not determine the electric line-admission structure. For $n \mid m$ with $n < m$, the pure hypercharge probe $(0, 0, n)$ belongs to L_n but not to L_m. The $n = 2$ and $n = 3$ branches are separated by $(0, 0, 2)$ and $(0, 0, 3)$. These pairs give native non-determination witnesses when the target permits both compared global-form choices and admits the corresponding line-admission tests.*

Proof. The local agreement is Theorem 6.1. The probe admission tests are direct applications of Theorem 6.3. For $n \mid m$ the induced inclusion $L_m/L_{\mathrm{dyn}} \subset L_n/L_{\mathrm{dyn}}$ has orders $6/m < 6/n$, so it is not an isomorphism. At a target observing only local operators these line tests are unavailable; their absence is a coarser observational scope, not a selection of a unique global group. The examples prove failure of reconstruction from the shared local differentials. They do not make the separate storage of n essential when a fixed target anchor or other retained data already determine it. □

6.3 Associated bundles and open color ports

Fix a principal G_n-bundle P on the licensed base. The adjoint color representation always descends through Γ_n, since central elements act trivially on $\mathfrak{su}(3)$. In contrast, an isolated fundamental color factor has label $(1, 0, 0)$; it descends only when n divides two. For $n = 3, 6$ it is therefore not an associated representation of the full quotient group.

Proposition 6.5 (Joint bundle descent and neutral outputs)**.** *The color adjoint bundle is defined from P for every admitted n. Every full quark, lepton and Higgs representation in the table also defines an associated bundle for every n. Open color-factor descriptions on local frames are compatible with a general shared bundle only when their joint weak/hypercharge transition data are retained, or a separate lawful lift or reduction is supplied. A color-singlet contraction of admitted joint tensor factors has zero triality but retains its remaining weak and hypercharge output; that output obeys*

$$3s + q = 0 \pmod{n}.$$

Color-neutralization therefore neither erases the electroweak attachment nor proves electric neutrality.

Proof. The adjoint assertion follows from trivial central action. For the full matter and Higgs representations, $\kappa = 0 \pmod{6}$, hence also modulo n. An associated bundle requires an honest representation of G_n; the pure fundamental test above exhibits its failure on the two stated branches. Local color factors may still be used as part of a full transition-function presentation, but no independent global factor bundle follows without extra lift or reduction data.

The weak and hypercharge actions commute with the color action. They therefore preserve the color-invariant subspace and commute with its Haar projection. This projection is a full intertwiner onto that subspace with its inherited electroweak action. A particular contraction to a specified output is either a color-index contraction tensored with the identity on the other slots, or must separately intertwine their actions; color equivariance alone does not establish this property on multiplicity spaces. For these admitted full intertwiners the tensor product has total center character trivial on Γ_n. The singlet color output has $t = 0$, so its remaining character is exactly $3s + q$ modulo n. Equivariance passes that trivial character to the image. Nothing in the argument sets the individual weak or hypercharge labels to zero. □

6.4 Transport of line and phase information

The global description also includes the admitted line branch, electric and magnetic lattices, Dirac–Schwinger–Zwanziger pairing, dynamical screening subgroups, theta monodromy and observed background or junction responses. SYM9-Q07 reconstructs their *charge-level* line normal form at a fixed branch; it does not supply all line dynamics or a full higher category. SYM9-Q08 further distinguishes the no-dynamical-monopole branch from branches with an admitted monopole lattice. These differences are not gauge-coordinate changes [6].

Proposition 6.6 (Shared-target screening transport)**.** *Let a source-authorized shared-target redescription induce an isomorphism of electric charge groups $f : L_n \to L'_n$ carrying the actual dynamical subgroup onto L'_{dyn}. Then*

$$\overline{f} : L_n/L_{\mathrm{dyn}} \longrightarrow L'_n/L'_{\mathrm{dyn}}, \qquad [\ell] \longmapsto [f(\ell)]$$

is a unique induced isomorphism. It transports the dual electric one-form action contragrediently. If the same redescription intertwines the source tensor incidences and transports the complete admitted magnetic, phase and boundary data, these maps form one jointly transported global color packet. A marginal color isomorphism alone does not establish the mixed-input conclusion.

Proof. If $\ell - \ell' \in L_{\mathrm{dyn}}$, then $f(\ell) - f(\ell') \in L'_{\mathrm{dyn}}$, so the displayed map is well-defined. The inverse of f gives its inverse, and surjectivity of the quotient map gives uniqueness. A character χ' is transported by composition $\chi' \circ \overline{f}$, preserving its evaluation on every transported charge. The source incidence equations commute by the joint transport hypothesis; tensor composition preserves their commutativity. The admitted phase, magnetic and boundary transports therefore attach to the same quotient map. This is the explicit screening part of the joint global-completion covariance of SYM9-Q13. If only the color marginal is supplied, none of these mixed incidence equations is implied, so that weaker datum cannot license the last claim. □

A fixed Higgs or matter input requires identity transport on that input, or an independent comparison identifying its transported value with the fixed one. Joint covariance with the input varied does not establish this stronger comparison. An anomalous fermion rephasing similarly transports theta and Yukawa/mass phases together: the gauge-topological theta torus is only part of the physical phase structure.

Remark 6.7 (Screening is not a phase law)*.* The screened quotient records which line charges can end on the admitted dynamical matter. It does not evaluate a Wilson-loop expectation, establish an area or perimeter law, or construct an asymptotic hadron. SYM-9 separates these questions explicitly in section 9.4. Consequently the continuation and confinement-response information constructed or received elsewhere in this paper remains a distinct part of the full color signature. A common screened quotient permits its transport when the actual dynamics and observations are intertwined; it cannot replace those data.

7 Magnetic, dyonic and joint phase response

The shared global group constrains both sides of a line comparison. Electric descent fixes its magnetic dual, while dynamical endpoints determine which combinations of electric and magnetic charge remain distinguishable. Theta transport then acts on this joint structure, not on an independent list of angles. We make these consequences constructive on the primitive, positively oriented charge branch of Section 6. The underlying line and topological realization is the spin, fixed-branch interface of SYM-9 [6, Secs. 10–13]. Highest weights, junction multiplicities, background responses and represented dynamics remain attached wherever the target observes them; center-charge data alone do not reconstruct those objects.

7.1 The magnetic lattice as an electric dual

Retain the electric group L, descent subgroup L_n and dynamical subgroup L_{dyn} already computed. In the magnetic coordinates

$$(b, a, g) \in \mathcal{M} = (\mathbb{Z}/3\mathbb{Z}) \oplus (\mathbb{Z}/2\mathbb{Z}) \oplus \tfrac{1}{6}\mathbb{Z},$$

b is the color magnetic residue, a the weak residue, and g the hypercharge magnetic flux. The pairing is

$$\beta\big((t, s, q), (b, a, g)\big) = \frac{tb}{3} + \frac{sa}{2} - qg \quad \text{in } \mathbb{R}/\mathbb{Z}.$$

This is SYM-9's Dirac convention, divided by six and reordered to match the color-first electric coordinates. In particular, the minus sign belongs to the Abelian term.

Theorem 7.1 (Exact electric–magnetic duality)**.** *For $n \in \{1, 2, 3, 6\}$, the annihilator of L_n is*

$$M_n = L_n^{\perp} = \mathbb{Z}m_n, \qquad m_n = \left(-\frac{6}{n} \bmod 3, \frac{6}{n} \bmod 2, \frac{1}{n}\right).$$

Conversely $M_n^{\perp} = L_n$. The magnetic charge group is infinite cyclic; its normalized charge is $r = ng$. Its generators in the four cases are

n	$m_n = (b, a, g)$
1	$(0, 0, 1)$
2	$(0, 1, \frac{1}{2})$
3	$(1, 0, \frac{1}{3})$
6	$(2, 1, \frac{1}{6})$.

These are center/cocharacter congruences, with the non-Abelian coroot data suppressed, not a classification of complete monopole representations.

Proof. The map $\kappa \bmod n : L \to \mathbb{Z}/n\mathbb{Z}$ is surjective, with kernel L_n. If a magnetic label m annihilates L_n, its character $\beta(-, m)$ factors through this cyclic quotient. It therefore has the form $j\kappa/n$ modulo one. Comparison on the three generators of L gives

$$\frac{b}{3} = \frac{2j}{n}, \qquad \frac{a}{2} = \frac{3j}{n}, \qquad -g = \frac{j}{n} \pmod 1.$$

Thus $r = ng$ is an integer, $j = -r \bmod n$, $b = -6r/n \bmod 3$, and $a = 6r/n \bmod 2$. The last sign may be reversed modulo two, but the color sign cannot be reversed without changing the other conventions. Hence $m = rm_n$. Conversely direct substitution gives

$$\beta(e, rm_n) = -\frac{r\kappa(e)}{n} \pmod 1.$$

It vanishes for all $e \in L_n$. The same identity with $r = 1$ proves the double-annihilator assertion. The map $rm_n \mapsto r$ is an isomorphism because its last component is r/n. □

For $n = 6$, the source's two displayed generators $(0, 1, \frac{1}{2})$ and $(1, 0, \frac{1}{3})$, now in color-first order, are $3m_6$ and $2m_6$. Their difference is m_6. The two displays therefore describe the same cyclic magnetic group, not two independent magnetic species [6, Thm. 10.1; App. C.3].

7.2 Mutually local branches and joint screening

For charges $(e,m),(e',m') \in L \oplus \mathcal{M}$, put

$$\Omega((e,m),(e',m')) = \beta(e,m') - \beta(e',m).$$

A charge subgroup is mutually local when Ω vanishes on every pair. Fixing the electric intersection and full magnetic projection permits an explicit algebraic classification.

Theorem 7.2 (Marked dyonic charge branches)**.** *Let $e_0 \in L$. The subgroup*

$$\Lambda_{n,e_0} = \{(e + re_0, rm_n) : e \in L_n,\ r \in \mathbb{Z}\}$$

is maximal mutually local in $L \oplus \mathcal{M}$. Every mutually local subgroup whose pure-electric intersection is L_n and whose magnetic projection is all of M_n has this form. Two such marked subgroups are equal precisely when $e_0 - e_0' \in L_n$; their algebraic labels therefore form $L/L_n \simeq \mathbb{Z}/n\mathbb{Z}$. The source's reference charge branch is $e_0 = 0$.

Proof. For two displayed elements, the terms involving $rr'e_0$ cancel in the antisymmetric pairing. The remaining terms vanish by Theorem 7.1. If a charge (v,m) is mutually local with this entire subgroup, comparison with $(L_n, 0)$ first gives $m = rm_n$. Comparison with (e_0, m_n) then gives $\beta(v - re_0, m_n) = 0$, hence $v - re_0 \in L_n$. No additional mutually local charge can be adjoined.

For the converse choose any lift (e_0, m_n) of the magnetic generator. Subtracting its r-fold multiple from an element of magnetic charge r leaves a pure electric element of L_n. All such elements are present by the assumed electric intersection, proving the formula. Finally the magnetic-generator lifts in two equal subgroups differ by a pure electric element of L_n. The converse is immediate. ☐

This theorem counts marked charge subgroups, not inequivalent quantum field theories. Physical admission, Weyl/highest-weight refinement, spin data, and source-authorized theta or branch identifications must still be applied. In particular, it does not assert a physical realization of every algebraic label or select one branch from the local gauge algebra. For any actually supplied pairing-preserving Witten isomorphism W, the image $W\Lambda_{n,e_0}$ is again maximal mutually local: its inverse would pull any proposed enlargement back to an enlargement of Λ_{n,e_0}. The same transport must carry the dynamical endpoint subgroup, not just the displayed probe charges.

Theorem 7.3 (Joint charge-screening quotient and one-form annihilator)**.** *Write $d = 6/n$, and choose a lift e_0 for an admitted branch. Screening by the admitted electric matter alone gives the explicit isomorphism*

$$\Lambda_{n,e_0}/(L_{\mathrm{dyn}}, 0) \simeq (\mathbb{Z}/d\mathbb{Z}) \oplus \mathbb{Z}, \qquad [(e + re_0, rm_n)] \longmapsto \left(\frac{\kappa(e)}{n} \bmod d, r\right).$$

If additional actual dynamical endpoints have images (a_j, r_j), let S be the subgroup they generate. The full screened charge group and its character action are

$$Q = ((\mathbb{Z}/d\mathbb{Z}) \oplus \mathbb{Z})/S, \qquad \widehat{Q} = \{(z,u) \in \mu_d \times U(1) : z^{a_j} u^{r_j} = 1 \text{ for every } j\}.$$

Here $\mu_d = \{z \in U(1) : z^d = 1\}$. For purely magnetic endpoint images $(0, r_j)$, put $p = \gcd\{r_j\}$, with $p = 0$ if all vanish. Then the surviving action is $\mu_d \times \mu_p$ for $p > 0$, and $\mu_d \times U(1)$ for $p = 0$.

Proof. The first map is well defined by the electric screening theorem. It is onto: $(0,0,n) \in L_n$ maps to the electric generator, and (e_0, m_n) maps to the magnetic generator. Its kernel is exactly $(L_{\mathrm{dyn}}, 0)$. Quotienting by the further endpoint subgroup gives Q.

Every character of $(\mathbb{Z}/d\mathbb{Z}) \oplus \mathbb{Z}$ is uniquely $(a,r) \mapsto z^a u^r$, with $z^d = 1$. It descends through S exactly when it is one on each generator, which proves the displayed annihilator. In the purely magnetic case these conditions are $u^{r_j} = 1$. The integer subgroup generated by the r_j is $p\mathbb{Z}$; Bezout's identity therefore gives $u^p = 1$ when $p > 0$, and no condition when $p = 0$. ☐

Proposition 7.4 (Constructive endpoint-subgroup normal form)**.** *For any subgroup $S \subset (\mathbb{Z}/d\mathbb{Z}) \oplus \mathbb{Z}$, there is a unique divisor $a \mid d$ with $S \cap ((\mathbb{Z}/d\mathbb{Z}) \oplus 0) = \langle (a, 0)\rangle$. Its magnetic projection is $p\mathbb{Z}$, for a unique $p \geq 0$. If $p = 0$, then $S = \langle (a, 0)\rangle$. If $p > 0$, there is a unique $u \bmod a$ such that*

$$S = \langle (a, 0), (u, p)\rangle.$$

Consequently an endpoint difference (c, r) belongs to S exactly when

$$\begin{cases} r = 0 \text{ and } a \mid c, & p = 0, \\ p \mid r \text{ and } a \mid (c - ur/p), & p > 0. \end{cases}$$

For $p > 0$, the screened group has order ap and

$$Q \simeq \mathbb{Z}/h\mathbb{Z} \oplus \mathbb{Z}/(ap/h)\mathbb{Z}, \qquad h = \gcd(a, u, p).$$

For $p = 0$, $Q \simeq \mathbb{Z}/a\mathbb{Z} \oplus \mathbb{Z}$.

Proof. Subgroups of the cyclic electric factor have precisely the stated divisor form, and subgroups of $\mathbb{Z}$ have the form $p\mathbb{Z}$. If $p > 0$, choose an element of S with magnetic coordinate p. Any two such elements differ by its pure electric subgroup, giving the unique residue $u \bmod a$. Subtracting r/p copies of this element from an arbitrary member proves the generation and membership assertions. Quotienting first by $(a, 0)$ leaves the relation (u, p). The integer relation matrix has determinant ap; integer row and column Euclidean reduction gives diagonal entries h and ap/h, because the first invariant factor is the greatest common divisor of its entries. These operations are invertible changes of generators and relations and therefore preserve the presented group. The case $p = 0$ is immediate. □

The no-monopole case recovers SYM9-Q08 and its unit magnetic current $J^{(2)}_n = n \star da_Y/(2\pi)$. A unit magnetic endpoint removes the continuous magnetic action. Mixed dyonic endpoints instead impose coupled character equations. For example, at $d = 2$, quotienting by $\langle (0, 2)\rangle$ gives $(\mathbb{Z}/2)^2$, whereas quotienting by $\langle (1, 2)\rangle$ gives $\mathbb{Z}/4$: in the latter quotient the electric generator equals minus twice the magnetic generator. Both choices have the same magnetic projection $2\mathbb{Z}$ and no additional purely electric endpoint, yet their joint screened groups differ. This is an algebraic separator; it becomes a native physical separator only on a comparison family admitting both endpoint patterns and the requisite line-action tests.

Changing the lift e_0 changes these product coordinates by a shear and transports S with them. Thus Q, its evaluation pairing and the actual endpoint maps are intrinsic, whereas a separate assignment of an electric coordinate to a dyon need not be. Any admitted pairing-preserving branch isomorphism carrying the full endpoint subgroup induces the quotient isomorphism and contragredient character action. Background fractionalization, anomalies and junction responses remain additional structures; equality of the abstract one-form group does not determine them.

7.3 Coupled theta periods and their determining characters

Write the dimensionless theta coordinates as

$$x = (x_2, x_3, x_Y) = \frac{1}{2\pi}(\theta_2, \theta_3, \widetilde{\theta}).$$

On the source reference line branch, at fixed matter-phase convention, the period lattice has basis

$$v_2 = (1, 0, A_n), \qquad v_3 = (0, 1, B_n), \qquad v_Y = (0, 0, N_n),$$

n	A_n	B_n	N_n
1	0	0	1
2	2	0	4
3	0	3	9
6	18	12	36.

The coupled-shift convention is important. For $n = 6$, the primary line calculation identifies $(\theta_3 = 2\pi, \widetilde{\theta} = u)$ with $(\theta_3 = 0, \widetilde{\theta} = u + 48\pi)$. Their difference is $(0, 2\pi, -48\pi)$, equivalent to $(0, 2\pi, 24\pi)$ after a 72π Abelian period. This fixes $B_6 = 12$. A positive 48π *difference vector* would give a different lattice; it is not the same convention. We use the primary equivalence in this orientation [25, Sec. 3.1], together with the other fixed-branch periods of SYM9-Q09 [6, Sec. 13.1; App. C.5]. No modification of a theta coordinate is performed without its actual line monodromy.

Theorem 7.5 (Theta reconstruction and anomalous-direction reduction)**.** *Let* $\Pi_n = \mathbb{Z}v_2 + \mathbb{Z}v_3 + \mathbb{Z}v_Y$*. The three characters*

$$\begin{aligned}\chi_2(x) &= e^{2\pi i x_2},\\ \chi_3(x) &= e^{2\pi i x_3},\\ \chi_Y(x) &= e^{2\pi i (x_Y - A_n x_2 - B_n x_3)/N_n}\end{aligned}$$

give an isomorphism $\mathbb{R}^3/\Pi_n \simeq U(1)^3$*. The relative covolume is* $N_n = n^2$*, and the smallest pure hypercharge period is* $2\pi n^2$*.*

If the target admits the baryon/lepton anomalous rephasing direction

$$x \longmapsto x + \alpha(-1, 0, 18), \qquad \alpha \in \mathbb{R},$$

with its actual source action, its gauge-theta quotient is

$$\frac{\mathbb{R}^2}{\mathbb{Z}(1, B_n) + \mathbb{Z}(0, N_n)} \quad \textit{in coordinates} \quad (x_3, \eta), \qquad \eta = x_Y + 18x_2.$$

It is determined by the two characters

$$e^{2\pi i x_3}, \qquad e^{2\pi i (\eta - B_n x_3)/N_n}.$$

This is the quotient by this admitted anomalous direction, not yet the full matter-phase quotient.

Proof. The linear map

$$x \longmapsto \left(x_2, x_3, \frac{x_Y - A_n x_2 - B_n x_3}{N_n}\right)$$

takes the three period generators to the standard basis of $\mathbb{Z}^3$. Exponentiation therefore gives a surjective homomorphism with kernel exactly Π_n. Its inverse reconstructs the theta class. The determinant of the triangular period matrix is N_n. A lattice vector with its first two coordinates zero is an integer multiple of v_Y, proving the pure-period statement.

The linear map $p(x) = (x_3, x_Y + 18x_2)$ has kernel $\mathbb{R}(-1, 0, 18)$. Its image lattice is generated by

$$(0, A_n + 18), \quad (1, B_n), \quad (0, N_n).$$

The four displayed rows satisfy $\gcd(A_n + 18, N_n) = N_n$, so this lattice is precisely $\mathbb{Z}(1, B_n) + \mathbb{Z}(0, N_n)$. If two images differ by this lattice, subtract a corresponding vector of Π_n; the remaining difference lies in $\ker p$. Conversely both period shifts and the anomalous direction have the stated images. This proves the quotient identity, including its exact fibres. The two characters now follow by the same triangular coordinate inversion. □

The rephasing premise is the renormalizable source's baryon/lepton action; new violating operators must carry their own transformed coefficients or change that phase target [6, Sec. 13.2]. The invariant electroweak combination is $2\pi\eta = \widetilde{\theta} + 18\theta_2$. Its relation to color remains visible in the second determining character when $B_n \neq 0$.

For the gauge-topological torus alone, inversion fixes exactly the eight classes $\frac{1}{2}\Pi_n/\Pi_n$: multiplying by two identifies this set with $\Pi_n/2\Pi_n$. This is a lattice statement, not a criterion for CP invariance of the full Yukawa and matter theory. With $n = 6$, its representatives at $(\theta_2, \theta_3) = (0, \pi)$ have $\widetilde{\theta} = 12\pi, 48\pi$, consistently with the coupled-shift convention above.

7.4 Joint matter phases and fixed-input comparisons

The complete source phase object remains

$$\Phi_{\text{phys}} = ((\mathbb{R}^3/\Pi_n) \times \mathcal{Y}_{\text{ph}})/\mathcal{R}_{\text{anom}},$$

where $\mathcal{R}_{\text{anom}}$ acts on theta and the retained mass/Yukawa data together [6, Sec. 13.2]. A general Yukawa orbit need not be a single phase torus. On a source-admitted torus stratum, however, a determining physical phase family can be computed explicitly.

Theorem 7.6 (Constructive joint phase quotient)**.** *Fix a source stratum whose retained Abelian phase coordinates form V/Π, with $V = \mathbb{R}^m$ and full-rank period lattice $\Pi = P\mathbb{Z}^m$. All other matter, Higgs and background invariants are retained on their specified common fibre. Suppose the actual admitted rephasings on this stratum are the torus action*

$$z \longmapsto z + A\alpha, \qquad \alpha \in \mathbb{R}^r/\mathbb{Z}^r, \qquad W = P^{-1}A \in \mathbb{Z}^{m\times r}.$$

Let $v_1, \ldots, v_k$ be an integral basis of

$$J = \{v \in \mathbb{Z}^m : W^{\mathsf{T}} v = 0\}.$$

Then the characters

$$\psi_j(z) = \exp\bigl(2\pi i\, v_j^{\mathsf{T}} P^{-1} z\bigr)$$

give a complete quotient coordinate

$$(V/\Pi)/\operatorname{im}(A) \simeq U(1)^k.$$

In particular, equality of these characters constructs precisely an allowed simultaneous phase rephasing modulo periods.

Proof. In coordinates $w = P^{-1}z \bmod \mathbb{Z}^m$, a character is specified by an integer vector v. It is constant on every rephasing orbit exactly when $v^{\mathsf{T}} W\alpha$ vanishes modulo integers for all real α, which is equivalent to $W^{\mathsf{T}} v = 0$.

The sublattice J is saturated: if a nonzero integer multiple of v lies in J, then $W^{\mathsf{T}} v = 0$ already. Integral row and column reduction, or equivalently the Euclidean algorithm for integer matrices, extends a basis of this saturated kernel to a basis of $\mathbb{Z}^m$. Use those basis vectors as rows of a unimodular matrix U, with the first k rows equal to v_j^{T}. The first k rows of UW vanish. The remaining block has full row rank $m - k = \operatorname{rank} W$, so its real linear image is all of $\mathbb{R}^{m-k}$. Consequently the torus image of UW is exactly the last $m - k$ torus coordinates. Its quotient consists of the first k coordinates, which are the displayed characters. This also constructs a rephasing whenever those characters agree. □

The torus action is an actual premise, not a replacement for an unconstructed Yukawa quotient. Singular-mass strata, non-Abelian flavor orbits, discrete identifications and additional line or background monodromy retain their own source actions. On an invariant subfamily the theorem restricts to its actual quotient image; it does not add phase points or rephasings absent from that family.

Proposition 7.7 (Fixed siblings require the stabilizer action)**.** *For an admitted simultaneous action on a joint phase and sibling record (z, y), an orbit comparison from (z, y) to (z', y) with the sibling record held fixed uses a group element in the stabilizer of that full record, or an independently admitted comparison restoring it. A comparison that only sends (z, y) to (z', y') supplies no such conclusion.*

On an admitted nonsingular quark-mass phase stratum, use the convention

$$\phi = \arg\det(M_u M_d), \qquad \overline{\theta} = \theta_3 + \phi.$$

A source rephasing whose strong-sector components are

$$\theta_3 \mapsto \theta_3 - \alpha, \qquad \phi \mapsto \phi + \alpha$$

preserves $e^{i\overline{\theta}}$*. Changing only* θ_3 *by* $-\alpha$ *while holding* ϕ *fixed changes this character whenever* $\alpha \notin 2\pi\mathbb{Z}$*.*

Proof. Equality of the final sibling record with its initial value is precisely the stabilizer condition. For the strong-phase example,

$$(\theta_3 - \alpha) + (\phi + \alpha) = \theta_3 + \phi, \qquad (\theta_3 - \alpha) + \phi = \overline{\theta} - \alpha.$$

Exponentiation proves both claims. All other anomaly-induced theta shifts and all transformed Yukawa entries accompany the actual joint rephasing; none is being held fixed by omission. Starting with $\theta_3 = \phi = 0$, the lawful joint pair $(-\alpha, \alpha)$ has character one, while the fictitious fixed-mass-phase replacement $(-\alpha, 0)$ has character $e^{-i\alpha}$. □

The strong-phase convention is the mass-determinant interface retained in SYM-9, not a prediction of its numerical value [6, Sec. 13.2]. When the source family actually admits both compared points and this phase readout, the example is a native fixed-sibling separator. A fixed Higgs input likewise includes its tensor incidences and transition data; fixing a displayed vacuum symbol alone does not fix that whole input.

7.5 Native reconstruction and exact information loss

On an admitted global realization retain the electric embedding, chosen dyonic branch, actual endpoint subgroup, theta monodromy, phase action and the mixed matter/Higgs and background maps. The preceding proofs supply explicit determining coordinates for the charge and phase components:

$$m_n, \quad [e_0] \in L/L_n, \quad (a, p, u), \quad (\chi_2, \chi_3, \chi_Y), \quad (\psi_1, \ldots, \psi_k),$$

with u absent for $p = 0$, and the joint phase characters used only on their proved torus strata. The displayed theta characters apply to the reference branch $e_0 = 0$, or after an actual source identification transports that branch and its complete period lattice. Without such an identification, an alternative marked branch retains its independently supplied theta lattice and monodromy; the four-row period table is not assigned to it by its charge label alone. These entries are compatible coordinates, not independently adjustable data. The theta characters refer to the fixed matter convention; their joint physical quotient is imposed by the actual rephasing action. The non-Abelian representation and junction information not determined by these coordinates remains in the source record.

Theorem 7.8 (Constructive global-use factorization)**.** *On the actual admitted charge family, with theta and phase components restricted as above, the following operations factor through the displayed coordinates and the retained joint maps: electric and magnetic admission, dyonic addition and pairwise locality, membership in the subgroup generated by dynamical endpoint charges, one-form character evaluation, theta-period comparison, and physical phase comparison on the stated rephasing strata. Their finite compositions with the received Witten transport and matter/Higgs incidences preserve exact domains and source failure values under a lawful joint comparison. A fixed-sibling comparison uses the corresponding stabilizer or actual fixed-sibling incident map.*

Proof. Electric admission is the congruence defining L_n. Magnetic admission and its inverse coordinates are Theorem 7.1. The marked dyonic subgroup is reconstructed by Theorem 7.2; addition, inversion and Ω are then evaluated on its reconstructed charges. Subtracting two charge vectors and applying Proposition 7.4 decides their exact charge-screening equivalence. Theorem 7.3 gives the character evaluation $z^c v^r$, with (z, v) the group parameters subject to its endpoint equations.

Theta equality is equality of the three determining characters in Theorem 7.5; the theorem also reconstructs the admitted anomalous-direction quotient. On a joint torus stratum, Theorem 7.6 reconstructs the entire allowed rephasing orbit. The supplied Witten maps and mixed incidences are evaluated on these same objects, not independently chosen isomorphic copies. Actual physical endpoint and junction maps are retained with their own exact domains; subgroup membership does not construct such a map for every representative of a charge class.

These are determining formulas for every new charge or phase vertex. Replacing the vertices in a received finite diagram by these formulas does not change its incident maps or their exact pullback domains. Induction along the diagram, as in Theorem 12.3, therefore preserves its value or first recorded failure. The fixed-sibling case uses Proposition 7.7; joint transport with a sibling varied is not substituted for it. □

The coordinate formulas are algebraically defined whenever their stated premises hold. Their use as *observations* still requires the corresponding line, endpoint, background or phase probes. Unresolved physical realization is not turned into a successful charge test merely by solving a congruence.

Corollary 7.9 (Determining probes and no-repair witnesses)**.** *For this enlarged global response family, retain the actual image of the joint source reconstruction. If its target admits the determining membership, endpoint-charge, character and phase readouts above, together with determining readouts for all remaining retained global maps, equality of these readouts reconstructs the whole global response up to the actual joint equivalence. Without those probe admissions, the native quotient is their actual response image, possibly coarser.*

A proposed deletion is lossless precisely when its kernel is contained in the kernel of that native response on the actual admitted family. In particular, a dyonic endpoint pattern or a joint mass-phase attachment cannot be discarded when an admitted pair has the same retained data but is separated by its endpoint or joint phase character.

Proof. The determining formulas reconstruct the specified components, and the assumed remaining readouts recover their actual joint maps. Factorization proves the reverse implication. The exact image and deletion claims are Theorems 12.4 and 13.4 applied to this received extension, not to a larger set of formal charge assignments.

For a concrete endpoint test in the $d = 2$ example, $(0, 2)$ belongs to $\langle(0, 2)\rangle$ but not to $\langle(1, 2)\rangle$. For the phase example, $e^{i(\theta_3+\phi)}$ distinguishes the fixed-mass replacement from the lawful joint one. Each yields a deletion witness only if both inputs and that readout belong to the actual comparison family. Conversely coordinates already determined by the retained source anchor or other observations may be deleted without loss. □

The construction supplies magnetic and joint phase work for the same color interface rather than a new carrier. Its explicit coordinates resolve the charge and phase information within the full line and phase interface. Neither a one-form group nor its breaking by endpoints gives a confinement law: Wilson-loop asymptotics, represented sectors, channel existence and asymptotic completeness retain their distinct physical authorities.

8 Extended-support sectors and their transport

The local color law and the nonlocal support-sector diagram describe different, incident structures. Within its stated theorem scope, the Yang–Mills source organizes extended-support sectors over an unchanged local root and supplies comparisons that distinguish the admitted global-form data. BCRS-I retains this result [1, Sec. 8]. The primitive color response remains the one source role identified by SM-2 [4]; sector labels do not add primitive color carriers.

We study the transport of the supplied represented diagram. Its objects remain the source's closed supports. Neither a representation for every formal label nor a quantum realization of arbitrary networks follows from the construction.

8.1 The closed-support diagram

For the fixed source group G, the Yang–Mills comparison class is

$$\mathcal{E}(G) = \mathcal{E}_{\mathrm{line}}(G) \sqcup \mathcal{E}_{\mathrm{surf}}(G).$$

A line object is (γ, λ), with γ a compact oriented closed piecewise smooth curve and $\lambda \in \Lambda_{\mathrm{line}}(G)$. A surface object is (Σ, μ), with Σ a compact oriented closed piecewise smooth surface without boundary and $\mu \in \Lambda_{\mathrm{surf}}(G)$. The label sets are the source-admitted role labels, not arbitrary external decorations. Source equivalence uses the prescribed orientation-preserving support reparametrizations and label-preserving compactly supported ambient isotopies [11, Defs. IV.1–IV.12].

The sector theorem and global-form recovery are source results [11, Thms. VI.10–VI.14, VII.5–VII.8]. The statements below concern their instantiated data, rather than an independent proof of universal sector realization.

Definition 8.1 (Received support-sector datum)**.** A received support-sector datum is an actual source instance

$$z_\xi = (\xi, \pi_\xi : \mathcal{A}_0 \to \mathcal{B}(\mathcal{H}_\xi))$$

over the same fixed local algebra $\mathcal{A}_0$, with its witnessed incidence to the BCRS-I color packet. It retains the source support and label, the represented sector, and the specified local vacuum agreement: for every bounded region O spacelike separated from $\operatorname{supp}\xi$,

$$\pi_\xi|_{\mathcal{A}_0(O)} \simeq \pi_0|_{\mathcal{A}_0(O)}$$

in the source's sense. A received deformation comparison from z_ξ to z_η consists of a source-admitted support comparison $d : \xi \to \eta$ and its actual unitary intertwiner

$$T_d : \mathcal{H}_\xi \to \mathcal{H}_\eta, \qquad T_d\pi_\xi(a) = \pi_\eta(a)T_d \quad (a \in \mathcal{A}_0).$$

The datum includes any source restrictions on boundary values and allowed comparisons.

This is the source's represented sector and deformation structure [11, Defs. VI.1–VI.7]. A vacuum state for an enlarged algebra, a Hamiltonian on $\mathcal{H}_\xi$, and an asymptotic-particle map are additional attachments, not part of Definition 8.1.

Theorem 8.2 (Composition of received sector comparisons)**.** *For a composable finite sequence of received deformation comparisons*

$$z_{\xi_0} \xrightarrow{d_1} z_{\xi_1} \xrightarrow{d_2} \cdots \xrightarrow{d_n} z_{\xi_n},$$

the composite

$$T_{d_n} \cdots T_{d_1}$$

is a unitary intertwiner of the endpoint representations over the same local algebra. Its support comparison is the concatenated source deformation chain. Identity comparisons and inverses give identity unitaries and adjoint unitaries. Thus the received source arrows and their admitted compositions form a groupoid over the fixed local root.

On its object isomorphism classes, this construction descends to the received sector classes. It identifies neither different global-form fibres nor source-distinct role labels. Two deformation chains with the same endpoints need not have equal intertwiners.

Proof. For two successive comparisons,

$$T_{d_2}T_{d_1}\pi_{\xi_0}(a) = T_{d_2}\pi_{\xi_1}(a)T_{d_1} = \pi_{\xi_2}(a)T_{d_2}T_{d_1}.$$

The product is unitary, and induction proves the formula for a finite chain. The source permits the corresponding concatenation and inverse comparisons [11, Props. VI.6, VI.8]. The identity and inverse intertwining relations are immediate. The local algebra is unchanged throughout, so this is a groupoid over that fixed root.

Passing to object isomorphism classes forgets only those actual comparison arrows. Their support components preserve the source's global-form fibre and admitted labels. A comparison across either would require a different supplied source map. Finally, intertwining fixes neither a unique arrow nor all representation automorphisms; equality of two paths is imposed only when the source proves or supplies that equality. □

The groupoid retains automorphisms and path-dependent transport information that the quotient of sector classes may forget. A path used by a mixed response must therefore remain attached to that response.

Proposition 8.3 (Exact preservation of the assigned subrecord)**.** *Each received deformation comparison preserves, up to its actual intertwiner, the following assigned subrecord:*

$$(\textit{fixed local root},\ \textit{support comparison},\ \textit{admitted label},\ \pi_\xi).$$

In particular,

$$T_d\pi_\xi(a_1)\cdots\pi_\xi(a_r)T_d^* = \pi_\eta(a_1)\cdots\pi_\eta(a_r)$$

for every finite word in the fixed local algebra, with the same conclusion for adjoints. Any matrix-element comparison must transport its state vectors as well:

$$\langle T_d v, \pi_\eta(a)T_d w\rangle = \langle v, \pi_\xi(a)w\rangle.$$

The structural restriction received by BCRS-I preserves the corresponding support–sector incidence and composition identities while omitting the represented Hilbert data.

Proof. The root, support and label statements are the source comparison conditions. Multiplying the intertwining equations proves the finite-word identity; taking adjoints preserves it. Unitarity gives the matrix-element formula. The target-relative restriction used by BCRS-I is applied to the same witnessed diagram, so it retains these incidence and comparison identities in its structural codomain [1, Sec. 8]. It does not turn a Hilbert space or a unitary into a premetric object. □

Spectral transport additionally requires the Hamiltonian, or its time-translation group, to be intertwined. The dynamical local comparison in Section 9 gives the corresponding hypotheses and conclusion.

8.2 Mixed inputs and fixed-sibling comparisons

Intertwining a sector representation need not intertwine an incident interaction. Consider a supplied bounded response

$$M_\xi : \mathcal{H}_\xi \otimes V \longrightarrow W.$$

If the comparison holds the sibling input V and output W fixed, the required equality is

$$M_\eta(T_d \otimes I_V) = M_\xi. \tag{MI}$$

If V or W is also transported, its actual transport replaces the corresponding identity in (MI). Equivariance of a simultaneous action does not authorize replacing those transports by identities.

Proposition 8.4 (Sector intertwining does not imply mixed-port preservation)**.** *The representation-intertwining equation in Definition 8.1 does not by itself imply* (MI) *for an independently supplied mixed response.*

Proof. Take $\mathcal{A}_0 = \mathbb{C}$, its scalar representation on $\mathcal{H}_\xi = \mathcal{H}_\eta = \mathbb{C}$, and $T_d = -I$. This unitary intertwines the representations. Let $V = W = \mathbb{C}$ and let both mixed maps be multiplication: $M_\xi(z \otimes v) = M_\eta(z \otimes v) = zv$. Then

$$M_\eta(T_d z \otimes v) = -zv,$$

which is not $M_\xi(z \otimes v)$ in general. The example concerns the logical implication between the two equations; it is not asserted to be an additional Yang–Mills sector. It shows why the actual mixed incidence must be checked. □

When the generator comparisons do preserve the mixed incidences, that preservation extends to their composites as follows.

Definition 8.5 (A jointly transported response diagram)**.** Consider an actual typed diagram of source-supplied response spaces X_i and primitive partial maps

$$f : D_f \subset X_i \longrightarrow X_j.$$

A joint transport to a second diagram consists of actual bijections $\theta_i : X_i \to X_i'$ such that, for every primitive map,

$$\theta_i(D_f) = D_{f'}, \qquad f'\theta_i = \theta_j f \quad \text{on } D_f.$$

Every admitted test, branch predicate, or failure readout used in the diagram is itself included as a primitive map or subset with its corresponding preservation equation. Products use the actual product of the component transports. A sibling held fixed has transport I, not an independently chosen replacement.

These equations test the supplied primitive operations and their exact domains. They do not assert completeness for an enlarged physical observation class or supply an operation whose existence has not been established.

Theorem 8.6 (Constructive transport of composite sector responses)**.** *A jointly transported response diagram preserves every finite typed response expression formed from its primitive maps by composition, products, and restriction to its admitted branch domains. It preserves both successful values and the exact domain/failure alternatives recorded by those operations.*

Consequently a received sector deformation preserves all native mixed responses generated by such a diagram when its intertwiner is the sector component of the actual joint transport. Preservation of the assigned subrecord alone, as in Proposition 8.3, is insufficient for this conclusion.

Proof. For a primitive operation this is Definition 8.5. Suppose it holds for $f : X \supset D_f \to Y$ and $g : Y \supset D_g \to Z$. The composite domain is

$$D_{g\circ f} = \{x \in D_f : f(x) \in D_g\}.$$

Exact domain transport for f, followed by that for g, gives $\theta_X(D_{g\circ f}) = D_{g'\circ f'}$. On this domain,

$$g'f'\theta_X = g'\theta_Y f = \theta_Z gf.$$

Products satisfy the same identity componentwise, with product domains. Restriction to an admitted branch is preserved because its defining subset or predicate is among the transported primitives. Structural induction on the finite expression proves the claim. The same induction preserves each recorded failed-domain or branch alternative, rather than merging all failures into a common untyped symbol.

The sector case uses the received unitary at its corresponding sort. Equation (MI), or its actual multiport version, is then one of the primitive equations in the induction. The preceding proposition shows that a sector intertwiner alone supplies no such equation for an independent mixed port. ☐

Source-proved equalities between response expressions are preserved under this transport; no additional identities between expressions are imposed. For unbounded operations the primitive data include the exact domain and graph. Agreement of formal expressions or agreement on an unspecified core is insufficient.

8.3 Global information and the relation to networks

Corollary 8.7 (A received global-form distinction cannot be deleted locally)**.** *Suppose the received source diagram contains two role-distinct closed-support instances with the same local shadow whose sector distinction is supplied by the Yang–Mills global-form theorem. A color signature retaining that admitted distinction cannot be reconstructed from the common local shadow and fixed local root alone.*

Proof. The source supplies the same local shadow but distinct sector-role data [11, Thms. VII.5–VII.8]. A function of the common shadow and local root has the same value on both instances, so it cannot reconstruct the two distinct retained values. The statement requires the actual source pair; it does not infer that every global-form fibre contains such a pair or that every external label is physically distinct. ☐

The finite-network theorem in Section 4 reconstructs a different object: classical group-valued edge transport, rooted tensors and invariant observations on the admitted graph family. Vertices and open edges belong to that classical target. A junction web, an open Wilson line with endpoint fields, or a surface with boundary does not thereby become an object of the closed-support class $\mathcal{E}(G)$. Quantum realization of such a network requires its own represented source data and comparison maps.

Closed-support sectors and classical networks thus supply distinct parts of the color description. Their simultaneous use requires common-source maps and the joint incidence equations of Theorem 8.6. Neither the number of network edges nor the number of sector labels changes the primitive color role [4, 1].

9 Physical color realization over a fixed local theory

A premetric color-response class specifies structural incidence; a represented Yang–Mills theory additionally supplies a state and dynamics. The BCRS-I color packet retains the fixed local root and extended-support incidences, but not a vacuum vector, Hamiltonian, or spectral projections [1, Sec. 8]. A physical realization therefore requires a matched lift of this diagram. The results below determine its operator-algebraic and spectral consequences while preserving the premetric classification.

9.1 The realized local object and its attachment

Fix a compact connected simple color group G, including its global form, and a supplied local Yang–Mills theory

$$\mathcal{B}_0(G) = (\mathcal{A}_0(\cdot; G), \pi_0, \mathcal{H}_0, \Omega_0, H_0, \tau^0).$$

Here $\mathcal{A}_0$ is the quasi-local algebra of the bounded-region gauge-invariant net, $(\pi_0, \mathcal{H}_0, \Omega_0)$ is its vacuum GNS representation, and

$$U_0(t) = e^{-itH_0}, \qquad U_0(t)\pi_0(a)U_0(t)^* = \pi_0(\tau_t^0(a)), \qquad U_0(t)\Omega_0 = \Omega_0.$$

The local source supplies both this represented theory and its spectral conclusion [9, 10]. On its gapped branch, fix a source-proved $m_* > 0$ for which

$$E_{H_0}([0, m_*))\mathcal{H}_0 = \mathbb{C}\Omega_0. \tag{PL}$$

The number m_* is a lower spectral threshold established by the source, not necessarily the first nonzero spectral value or a measured hadron mass.

Definition 9.1 (Matched physical lift)**.** A matched physical lift of a color packet consists of the actual object $\mathcal{B}_0(G)$, its witnessed incidence to that packet, and the realization maps used by the target. The lift preserves the packet's global-form fibre, local-root identity, boundary data, and retained support–sector comparisons. Any additional matter, Higgs, or external ports belong to the same source diagram, with their actual incidence maps and continuation domains. Equality of the premetric restrictions alone does not identify two vacuum states, represented dynamics, or physical port maps.

A matched lift preserves the incidence between the represented theory and its premetric restriction. Its operator-algebraic conclusions then apply at the represented level; they are not used to establish the standing of the underlying premetric source.

Definition 9.2 (Scope-faithful represented extension)**.** A scope-faithful represented extension of $\mathcal{B}_0(G)$ consists of a unital C^*-algebra $\mathcal{A}$, an injective unital $*$-homomorphism $i : \mathcal{A}_0 \to \mathcal{A}$, a state ω, and an ω-preserving automorphism group α_t, such that:

1. i is the actual source attachment of the fixed local net, including its bounded-region inclusions;
2. $\omega \circ i = \omega_0$, where $\omega_0(a) = \langle \Omega_0, \pi_0(a)\Omega_0 \rangle$;
3. $\alpha_t \circ i = i \circ \tau_t^0$;
4. in the GNS representation $(\pi, \mathcal{H}, \Omega)$ of ω, the implementation $U(t) = e^{-itH}$ is strongly continuous, has $H \geq 0$, fixes Ω, and satisfies $U(t)\pi(b)U(t)^* = \pi(\alpha_t(b))$.

Write

$$\pi_{\mathrm{loc}}(a) = \pi(i(a)), \qquad \mathcal{H}_{\mathrm{loc}} = \overline{\{\pi_{\mathrm{loc}}(a)\Omega : a \in \mathcal{A}_0\}}.$$

These conditions specify the algebra, state, and dynamics to be compared. They are the scope-faithful part of the nonlocal Yang–Mills extension interface [11, Def. VIII.22]. They do not, by themselves, exclude additional spectral content in $\mathcal{H}_{\mathrm{loc}}^{\perp}$.

9.2 Dynamical GNS comparison and spectral transport

Theorem 9.3 (Canonical dynamical local equivalence)**.** *For a scope-faithful represented extension there is a unique unitary*

$$W_{\mathrm{loc}} : \mathcal{H}_{\mathrm{loc}} \longrightarrow \mathcal{H}_0$$

satisfying

$$W_{\mathrm{loc}}\Omega = \Omega_0, \qquad W_{\mathrm{loc}}\pi_{\mathrm{loc}}(a)|_{\mathcal{H}_{\mathrm{loc}}} W_{\mathrm{loc}}^* = \pi_0(a).$$

The subspace $\mathcal{H}_{\mathrm{loc}}$ reduces H, and

$$W_{\mathrm{loc}} U(t)|_{\mathcal{H}_{\mathrm{loc}}} W_{\mathrm{loc}}^* = U_0(t).$$

Consequently, with equality of self-adjoint operators including their domains,

$$W_{\mathrm{loc}} H|_{\mathcal{H}_{\mathrm{loc}}} W_{\mathrm{loc}}^* = H_0, \qquad W_{\mathrm{loc}} E_H(\Delta)|_{\mathcal{H}_{\mathrm{loc}}} W_{\mathrm{loc}}^* = E_{H_0}(\Delta)$$

for every Borel set $\Delta \subset \mathbb{R}$.

Proof. On the dense local cyclic subspace define

$$W_{\mathrm{loc}}(\pi_{\mathrm{loc}}(a)\Omega) = \pi_0(a)\Omega_0.$$

For $a, b \in \mathcal{A}_0$, state restriction gives

$$\langle \pi_{\mathrm{loc}}(a)\Omega, \pi_{\mathrm{loc}}(b)\Omega\rangle = \omega(i(a^*b)) = \omega_0(a^*b) = \langle \pi_0(a)\Omega_0, \pi_0(b)\Omega_0\rangle.$$

The map is therefore well defined and isometric, including on finite linear combinations. Its range is dense because Ω_0 is cyclic. It extends uniquely to a unitary. Multiplication in $\mathcal{A}_0$ gives the representation intertwining relation on the dense cyclic subspace and then everywhere by boundedness. Any other unitary with the stated properties agrees there, proving uniqueness.

Covariance and vacuum invariance give

$$U(t)\pi_{\mathrm{loc}}(a)\Omega = \pi_{\mathrm{loc}}(\tau_t^0(a))\Omega.$$

Thus $U(t)\mathcal{H}_{\mathrm{loc}} = \mathcal{H}_{\mathrm{loc}}$ for all real t. The orthogonal projection onto this subspace commutes with the entire unitary group; hence the subspace reduces its self-adjoint generator. On cyclic vectors,

$$W_{\mathrm{loc}} U(t)\pi_{\mathrm{loc}}(a)\Omega = \pi_0(\tau_t^0(a))\Omega_0 = U_0(t) W_{\mathrm{loc}}\pi_{\mathrm{loc}}(a)\Omega.$$

Density proves the asserted group equivalence. Uniqueness of the self-adjoint generator of a strongly continuous unitary group gives operator equality with its full domain. The spectral-projection identity then follows from the spectral calculus under unitary equivalence. □

The theorem expresses the dynamical content of the source GNS comparison [11, Lem. VIII.27]. Agreement of the represented local algebra alone would not justify the Hamiltonian conclusion: agreement of the state and implemented dynamics was used.

Corollary 9.4 (Local subgap rigidity)**.** *If the fixed local object satisfies* (PL)*, then every scope-faithful represented extension satisfies*

$$E_H([0, m_*))\mathcal{H}_{\mathrm{loc}} = \mathbb{C}\Omega.$$

Proof. Apply Theorem 9.3 with $\Delta = [0, m_*)$ and conjugate (PL) by W_{loc}. □

Proposition 9.5 (Continuation of the physical local comparison)**.** *Let two matched represented lifts be related by an actual unitary $T : \mathcal{H} \to \mathcal{H}'$ which maps Ω to Ω', intertwines their attached local representations, and satisfies $TU(t) = U'(t)T$. Then $T\mathcal{H}_{\mathrm{loc}} = \mathcal{H}'_{\mathrm{loc}}$,*

$$TE_H(\Delta) = E_{H'}(\Delta)T,$$

and the canonical local GNS comparisons commute with T and the corresponding source local equivalence. If a bounded port F and its transport F' satisfy $F'T = SF$ for the actual codomain transport S, their restrictions to any transported local spectral subspace satisfy the same relation.

Proof. The local intertwining and vacuum equations map the cyclic generators of $\mathcal{H}_{\mathrm{loc}}$ onto those of $\mathcal{H}'_{\mathrm{loc}}$. Taking closures proves the subspace statement. Intertwining the unitary groups intertwines their spectral measures. The two routes from a local cyclic vector to the source vacuum representation agree; uniqueness in Theorem 9.3 gives commutation of the local comparison diagram. Restricting the supplied port equation gives the final claim. No port-intertwining relation is inferred from marginal Hilbert-space equivalence. □

9.3 Admitted completions and the full subgap space

The nonlocal source distinguishes a representation over a fixed local algebra from an admissible completion of that theory. Completion admissibility additionally requires its actual role and scope assignment, vacuum-preserving state extension, implemented positive-energy dynamics, and kernel-faithful same-domain continuation [11, Defs. VIII.31, VIII.40; Thms. VIII.41–VIII.45]. These are not automatic properties of every sector representation.

The following lemma isolates the precise source exclusion used to pass from the local cyclic space to the full completion.

Lemma 9.6 (Source completion exclusion)**.** *Let the fixed local object satisfy* (PL)*, and let a scope-faithful represented extension be an admissible same-domain completion in the sense of the Yang–Mills source, with its completion conditions discharged on the matched object $\mathcal{B}_0(G)$. For a unit vector $\eta \in E_H([0, m_*))\mathcal{H}$, neither of the following can occur:*

1. *the local state $a \mapsto \langle \eta, \pi_{\mathrm{loc}}(a)\eta \rangle$ differs from ω_0;*
2. *$\eta \perp \Omega$ and that local state equals ω_0.*

Proof. These are the two operator-algebraic exclusions of Theorem VIII.32 of the source, applied to the source-admitted same-domain completion; its Corollary VIII.48 records their completion consequence [11]. The second alternative is an orthogonal subgap carrier of the same fixed local vacuum state. The exclusions are imported conclusions of that completion theorem, not defining clauses of a scope-faithful represented extension. □

Theorem 9.7 (Complete subgap rigidity on the matched completion)**.** *Under the hypotheses of Lemma 9.6,*

$$E_H([0, m_*))\mathcal{H} = \mathbb{C}\Omega.$$

In particular,

$$E_H((0, m_*)) = 0, \qquad \ker H = \mathbb{C}\Omega.$$

Proof. Let ψ lie in the subgap spectral subspace. By Theorem 9.3, $\mathcal{H}_{\mathrm{loc}}$ and its orthogonal complement reduce H. The orthogonal decomposition

$$\psi = \psi_{\mathrm{loc}} + \psi_\perp$$

therefore has both terms in the same subgap spectral subspace. If $\psi_\perp \neq 0$, normalize it to a unit vector η. Its local vector state either differs from ω_0, contradicting the first source exclusion, or equals ω_0. In the latter case $\eta \perp \mathcal{H}_{\mathrm{loc}}$, hence $\eta \perp \Omega$, contradicting the second source exclusion. Thus $\psi_\perp = 0$. Corollary 9.4 now places ψ in $\mathbb{C}\Omega$. Conversely, vacuum invariance puts Ω in the zero-energy spectral subspace. This proves equality of the entire subgap space with the vacuum ray and the two spectral consequences. □

This direct argument uses reduction of the canonical local cyclic subspace and the source's exact completion exclusion. It does not require a multiplicity decomposition for arbitrary equal vector states. The result is the matched-instance form of the source's complete-theory spectral endpoint [11, Thms. VIII.57–VIII.58].

Proposition 9.8 (Scope-faithfulness alone does not extend the gap)**.** *The conditions of Definition 9.2, without the source completion conditions, do not imply the conclusion of Theorem 9.7.*

Proof. Choose $0 < \varepsilon < m_*$ and put

$$\mathcal{A} = \mathcal{A}_0 \otimes M_2(\mathbb{C}), \qquad i(a) = a \otimes I, \qquad \omega = \omega_0 \otimes \langle e_0, (\,\cdot\,)e_0 \rangle.$$

The GNS realization is $\mathcal{H}_0 \otimes \mathbb{C}^2$ with vacuum $\Omega_0 \otimes e_0$. Give the two-dimensional factor the Hamiltonian $h_2 = \mathrm{diag}(0, \varepsilon)$ and use the product dynamics. Then

$$H = H_0 \otimes I + I \otimes h_2 \geq 0.$$

All the scope-faithful conditions hold, but $\Omega_0 \otimes e_1$ is an orthogonal eigenvector of energy ε with the same restricted local state as the vacuum. Thus the full subgap range is not the vacuum ray. This extension fails the second source completion exclusion; it is not a counterexample within the admitted same-domain completion class. □

9.4 Neutrality of observable-generated physical states

Theorem 9.9 (Color neutrality of the local vacuum sector)**.** *Suppose the represented lift carries an actual unitary color action $V : G \to \mathcal{U}(\mathcal{H})$ such that*

$$V(g)\Omega = \Omega, \qquad V(g)\pi_{\mathrm{loc}}(a)V(g)^* = \pi_{\mathrm{loc}}(a) \quad (a \in \mathcal{A}_0,\ g \in G).$$

Then

$$V(g)|_{\mathcal{H}_{\mathrm{loc}}} = I \quad (g \in G).$$

Consequently every state vector generated from the vacuum by the fixed gauge-invariant local algebra is color neutral.

Proof. For every local cyclic vector,

$$V(g)\pi_{\mathrm{loc}}(a)\Omega = \pi_{\mathrm{loc}}(a)V(g)\Omega = \pi_{\mathrm{loc}}(a)\Omega.$$

Such vectors are dense in $\mathcal{H}_{\mathrm{loc}}$. Boundedness of $V(g)$ extends the equality to that entire subspace. □

Corollary 9.10 (No colored channel into the local vacuum sector)**.** *Let R be a strongly continuous unitary representation of the compact group G on $\mathcal{K}$, with $\mathcal{K}^G = 0$. Under Theorem 9.9, every bounded equivariant map*

$$J : \mathcal{K} \longrightarrow \mathcal{H}_{\mathrm{loc}}, \qquad JR(g) = V(g)J,$$

vanishes. In particular, there is no nonzero isometric equivariant realization of a nontrivially colored irreducible one-particle channel inside this local vacuum sector.

Proof. The theorem gives $JR(g) = J$. Integrating against normalized Haar measure yields $JP_{\mathcal{K}^G} = J$. Since the invariant subspace is zero, its orthogonal projection is zero and $J = 0$. □

The color action fixes both the admitted observable algebra and its vacuum. Boundary transformations acting nontrivially on either are outside the theorem's hypotheses. Its conclusion concerns the local vacuum sector, not every representation of the observable algebra; nonlocal and charged representations retain their own source attachments.

Corollary 9.11 (Neutrality under matched continuation)**.** *If the unitary T of Proposition 9.5 also intertwines the actual color actions, it maps the neutral local vacuum sector onto the corresponding neutral sector of the second lift. Every supplied equivariant physical port into these sectors retains the vanishing conclusion of Corollary 9.10 under that continuation.*

Proof. The proposition identifies the local cyclic subspaces; color intertwining preserves their invariant vectors. Conjugating the supplied port equation preserves both equivariance and a zero map. □

9.5 Massless channels and the meaning of the gap

Theorem 9.12 (Spectral obstruction to a massless one-particle embedding)**.** *Let $\mathcal{H}_{\mathrm{ph}}$ be either the local cyclic space of a scope-faithful lift satisfying* (PL)*, or the whole Hilbert space of a matched admissible completion as in Theorem 9.7. Denote its Hamiltonian by H_{ph}. Suppose a candidate one-particle space $\mathcal{K}$ has a strongly continuous positive-energy time-translation group e^{-ith}, with*

$$E_h((0, m_*))\mathcal{K} \neq 0.$$

There is no isometric map $J : \mathcal{K} \to \mathcal{H}_{\mathrm{ph}}$ satisfying

$$Je^{-ith} = e^{-itH_{\mathrm{ph}}}J \quad (t \in \mathbb{R}).$$

In particular, this excludes a standard nontrivial massless one-particle representation from either stated physical realization.

Proof. A bounded intertwiner of the strongly continuous unitary groups intertwines their spectral projections. Thus

$$JE_h((0, m_*)) = E_{H_{\mathrm{ph}}}((0, m_*))J = 0,$$

where the final equality is Corollary 9.4 or Theorem 9.7. The assumed nonzero low-energy spectral subspace contains a nonzero vector annihilated by J, contradicting isometry.

For a standard massless one-particle representation, the momentum-space energy is $p^0 = |\mathbf{p}|$ on the nonzero future light cone. Every region $0 < |\mathbf{p}| < m_*$ has nonzero one-particle spectral subspace. Hence its time translations satisfy the hypothesis. □

On the matched gapped branch, a massless physical embedding of the stated kind is impossible. This spectral obstruction is distinct from confinement: a gap alone neither excludes massive colored sectors above its threshold nor constructs hadronic scattering states or a Wilson-loop area law. Color neutrality and spectral rigidity therefore give complementary, rather than interchangeable, restrictions.

9.6 Nonlocal sectors, matter coupling, and physical taxonomy

The BCRS-I packet retains the source-stated closed-support sector layer over the unchanged local root [1, Sec. 8]. In a represented lift, any supplied member of that layer retains its actual support, admissible label, representation, local vacuum agreement away from the support, and source-authorized deformation transport. These are the incident data of the source's sector construction and global-form comparison [11, Secs. IV–VII]. They are not additional primitive color response carriers. The results here apply to supplied represented sectors; they do not independently establish a realization for every formal support label. Nontrivial sector content requires a corresponding representation, not a relabelling of the vacuum representation.

A sector representation, a filtered same-domain completion, and a different matter-coupled theory remain distinct cases. The first is a representation over the local algebra. The second supplies the stronger state, dynamics and completion conditions used above. The third may change the local algebra, vacuum restriction, or dynamics and therefore requires its own common-source comparison. Choosing $G = SU(3)$ does not by itself identify pure Yang–Mills theory with a theory containing dynamical quark and Higgs couplings. A simultaneous color–matter–Higgs use must retain the actual mixed source diagram; separately valid marginal source theorems do not supply it [1, 11].

Likewise, the following physical uses have different mathematical codomains:

1. a confined or gauge-presented color field is a field-level description on its licensed domain;
2. a gauge-invariant composite response is an admitted joint observable or invariant contraction;
3. a composite asymptotic state requires an actual physical representation and the relevant spectral or scattering map;
4. a partonic approximation retains its factorization, resolution and kinematic domain;
5. a jet or detector reconstruction is a process-dependent observation map, not a new primitive color carrier.

A pole or resonance assigned to one of these uses requires the actual corresponding channel, continuation domain, and pole authority. Neither the local vacuum gap nor the neutral tensor projection supplies those maps by itself.

The resulting physical classification comprises dynamical equivalence and spectral transport on the matched local theory, full subgap rigidity on its admitted same-domain completions, neutrality of its observable-generated vacuum sector, and exclusion of massless one-particle embeddings on the specified gapped branches. Each result retains its own realization domain and leaves the premetric carrier classification unchanged.

10 Composite channels and two-level continuation

An invariant color tensor specifies a contraction of joint inputs. A physical color-neutral channel additionally requires a represented operator or response, an independently admitted boundary, and the maps connecting them. These data define a two-level configuration-to-boundary interface whose variance agrees with the aggregate-first interface of FCGS-II [3, Sec. 4.2]. Neither an isolated quark nor a gluon component is assigned a preferred hadron.

10.1 Independently admitted neutral boundaries

For the realized source branch under discussion, let $\mathcal{C}^{\mathrm{pre}}_{\mathrm{col}}$ denote the category of source-admitted joint colored configurations with the response realizations used below, before attachment to a physical boundary. Its records retain all constituent and support incidences, including weak, charge, Higgs, global-form, anomaly, and source data. The internal continuation arrows are those actually licensed by the source. Let $\mathcal{B}^0_{\mathrm{col}}$ denote the category of independently admitted color-neutral boundary records. An object specifies its boundary type and its authority; distinct types are not identified by sharing a color-singlet tensor.

A source of objects in this second category is the hadronic persistence construction [8, Defs. 5.1–5.3; Lem. 5.4; Thm. 5.8]. Its admission datum has the form

$$\mathrm{HadAdm}(B) = (D_{\mathrm{phys}}, G_{\mathrm{SM}}, Q_{\mathrm{lat}}, M_{\mathrm{slot}}, \partial_B, \mathcal{H}^{\mathrm{hist}}_B, \rho_B).$$

Here $\mathcal{H}^{\mathrm{hist}}_B$ is a family of licensed histories, not a Hilbert space; ρ_B types the object as downstream composite persistence. The source's hadronic admission also certifies localized color-neutral status and reidentification through the specified histories. This persistence datum does not by itself determine a hadron spectrum.

Proposition 10.1 (Supplied composite-persistence boundary)**.** *On the realized physical and recovered Standard-Model target of the hadronic source, an actual* $\mathrm{HadAdm}(B)$ *supplies a color-neutral composite-persistence boundary and its licensed identity and history comparisons. This is a downstream boundary over the existing matter and color carriers, not a new fundamental carrier. Admission alone does not supply an asymptotic Hilbert-space embedding, an isolated spectral mass, or a colored-configuration channel into* B.

Proof. The source's hadronic persistence admission lemma applies to the actual localized composite and its supplied datum. Its persistence certificate then identifies the same boundary object under the admitted histories, with charge, inertial-response and role records retained. These are precisely the claimed boundary and comparison data. None of the cited conclusions constructs a spectral projection, scattering operator or configuration-to-boundary attachment, so none is inserted into this boundary by its admission. The matter/color primitive typing remains the intersection supplied by SM-2 and SM-3 [4, 5]. □

For a stable asymptotic boundary, retain an actual positive-energy representation and its admitted incoming or outgoing attachment. For a bound-state boundary, retain the specified physical spectral or persistence authority. For a resonance boundary, retain the gauge-invariant channel, continuation domain or sheet, pole and residue authority. A resonance need not be an invariant Hilbert subspace. These are the same type distinctions used for electroweak and Higgs physical response [2]. A jet, detector output or partonic reconstruction instead belongs to an observation or inference category and must be distinguished from the physical boundary it describes.

10.2 Actual source maps and their composite domains

The realization diagram contains the admitted configuration responses, neutral tensor spaces, composite-operator domains, represented state or response spaces, and physical boundary ports. Every generating edge is an actual source-authorized operation, with its domain and boundary type. Examples include an admitted invariant contraction, an operator acting on its declared common domain, an actually supplied time evolution, or a licensed physical port attachment. An absent source operation is an absent edge, not a zero operation.

Definition 10.2 (Source-witness composition category)**.** Let $\mathcal{R}$ be the category generated by this realized source diagram and the realized arrows of $\mathcal{C}^{\mathrm{pre}}_{\mathrm{col}}$ and $\mathcal{B}^0_{\mathrm{col}}$. Work with small source categories in one fixed universe. Their objects are typed source interfaces; equality of underlying response spaces alone does not identify two interface objects. A path carries the composition of its actual operations, their source warrants, all fixed sibling incidences, the exact composite domain, and the induced failure record. Paths are identified only by the source-licensed equalities, with the resulting congruence closed under composition. Every generating equality preserves the actual operation, its exact domain and failure behavior, retained physical authority and joint incidence data. The relations include the identity and composition laws of both supplied source categories. Denote the induced functors by

$$A : \mathcal{C}_{\mathrm{col}}^{\mathrm{pre}} \longrightarrow \mathcal{R}, \qquad B : \mathcal{B}_{\mathrm{col}}^{0} \longrightarrow \mathcal{R}.$$

The generators and their physical authority are specified before paths and their quotient are formed. The resulting category therefore organizes the supplied operations and their composites, rather than all formal Hilbert-space or set-theoretic maps. Coverage of a larger, independently characterized configuration domain is a separate theorem; it is not implied by the existence of this realized category.

Lemma 10.3 (Exact-domain source composition)**.** *For actual partial operations $f : D_f \subset X \to Y$ and $g : D_g \subset Y \to Z$, the composite domain is*

$$D_{gf} = \{x \in D_f : f(x) \in D_g\}.$$

For three composable operations, both parenthesizations have the domain

$$\{x \in D_f : f(x) \in D_g,\ g(f(x)) \in D_h\}$$

and the same value. Recording the first failed operation and its transported source witness makes failure propagation associative as well. Consequently $\mathcal{R}$ is a category of witnessed actual operations, even when some composite domains are empty or some linear composites vanish.

Proof. The formula is the definition of where $g(f(x))$ can be evaluated. Expanding it twice gives the displayed three-operation domain for both parenthesizations. On that domain the values agree by composition of functions. Outside it, the ordered path first fails at f, then at g, or then at h, independently of parentheses. Identities have full domains and change no failure record. Finally the stipulated source equalities generate a composition congruence, so the quotient preserves these category laws. □

This lemma does not assert that the composition of closed unbounded operators is closed. If a physical operation requires a closed graph, its actual domain and closedness warrant are retained separately. The category law is the law of the witnessed partial operations.

10.3 The two-level channel profunctor

Theorem 10.4 (Constructed two-level source interface)**.** *The realized source diagram determines a Set-valued profunctor*

$$\mathsf{K}_{\mathrm{col}} : (\mathcal{C}_{\mathrm{col}}^{\mathrm{pre}})^{\mathrm{op}} \times \mathcal{B}_{\mathrm{col}}^{0} \longrightarrow \mathbf{Set}, \qquad \mathsf{K}_{\mathrm{col}}(C, D) = \mathrm{Hom}_{\mathcal{R}}(A(C), B(D)).$$

For $u : C' \to C$, $v : D \to D'$ and a channel k, its action is

$$\mathsf{K}_{\mathrm{col}}(u, v)(k) = B(v)kA(u).$$

It retains the exact composition domains, inherited authority and mixed incidences of the generating source paths. The construction asserts no nonzero coupling for an arbitrary pair (C, D) and no preferred single-valued boundary image of C.

Proof. The displayed composite has domain $A(C')$ and codomain $B(D')$ in $\mathcal{R}$. The category laws and functoriality of A, B prove identity, composition and commutation of the left and right actions. Lemma 10.3 supplies their actual domain and failure interpretation. Every path ends in an independently admitted boundary through actual source edges, so a missing boundary authority has not been manufactured by path formation. A Hom set may be empty, or may contain only operations that vanish on a selected preparation; the theorem makes neither an existence nor a nonzero assertion beyond the actual source paths. □

A well-typed channel need not have a nonzero response. Where response spaces are linear, a nonzero witness consists of an admitted input x in the full composite domain and a certified nonzero response $k(x)$. For a persistence-record operation it includes an actual admitted history and the indicated boundary record. Such witnesses are not preserved under every postcomposition: a later projection can annihilate a nonzero response. They are preserved by a source-authorized invertible continuation with its domain transported bijectively.

Proposition 10.5 (Boundary gluing and module composition)**.** *For a second actual source interface $\mathsf{L} : (\mathcal{B}^0_{\mathrm{col}})^{\mathrm{op}} \times \mathcal{D} \to \mathbf{Set}$ realized in the same small diagram, let $F_{\mathcal{D}} : \mathcal{D} \to \mathcal{R}$ be its boundary functor. Each $l \in \mathsf{L}(D, E)$ is an actual source-witness arrow $B(D) \to F_{\mathcal{D}}(E)$, with its left and right actions given by composition in $\mathcal{R}$. The balanced channel construction is*

$$(\mathsf{L} \otimes \mathsf{K}_{\mathrm{col}})(C, E) = \left(\coprod_D \mathsf{K}_{\mathrm{col}}(C, D) \times \mathsf{L}(D, E) \right) / \sim,$$

where $(vk, l) \sim (k, lv)$ for each admitted intermediate boundary arrow v. It has a well-defined realized composition map $[k, l] \mapsto lk$. Its image is exactly the channels lk with $k \in \mathsf{K}_{\mathrm{col}}(C, D)$ and $l \in \mathsf{L}(D, E)$ for an admitted intermediate boundary D. It need not contain a factorization whose second leg is outside the supplied interface L. Different such factorizations may still become equal under further source relations in $\mathcal{R}$.

Proof. Both representatives of a generating balanced relation evaluate to lvk, including the exact domain and failure data by Lemma 10.3. Hence composition is well-defined on the quotient. Every image has the stated factorization, and every factorization using the specified $\mathsf{K}_{\mathrm{col}}$ and L legs supplies a pair in the coproduct. Associativity is inherited from $\mathcal{R}$. The balanced relation imposes only its stated identifications; additional actual source equalities can identify its images, so no injectivity beyond the proved statement is asserted. ☐

10.4 Represented composite-to-boundary channels

Explicit composite-to-boundary maps provide generating channels for the preceding category. Fix a supplied joint unitary color representation V_C and its joint invariant projector P_C. Suppose the source admits a linear composite-operator realization $\mathcal{O}_C$ on a domain containing the actual vacuum Ω, and the map

$$j_C : V_C^G \longrightarrow \mathcal{H}, \qquad j_C(t) = \mathcal{O}_C(t)\Omega$$

is bounded and preserves the specified mixed source incidences. For finite-dimensional V_C^G, boundedness follows once the linear map is everywhere defined; it does not establish that the operator realization exists. Assume the realized vectors are color neutral, as holds for source-admitted gauge-invariant operators acting on an invariant vacuum.

Independently let a neutral physical boundary have a supplied Hilbert space $\mathcal{H}_D$, an actual isometric attachment $a_D : \mathcal{H}_D \to \mathcal{H}$, and the source authorization for the corresponding boundary-amplitude readout. Where dynamics is used, let $U(t)$ be the actual unitary evolution at this same target, preserving the required neutral sector and source domain.

Theorem 10.6 (Joint composite-to-boundary amplitude)**.** *These data construct the bounded channel*

$$J_{D,C}(t) = a_D^* U(t) j_C P_C : V_C \longrightarrow \mathcal{H}_D.$$

Its nonzero use is witnessed precisely by an admitted x for which

$$\langle U(t) j_C P_C x, a_D a_D^* U(t) j_C P_C x \rangle > 0.$$

Let source-authorized unitaries S_C, T, R_D transport the configuration, represented source and boundary, respectively, with

$$S_C P_C = P'_C S_C, \quad T j_C = j'_C S_C|_{V_C^G}, \quad TU(t) = U'(t)T, \quad T a_D = a'_D R_D.$$

Then the entire channel, not merely its separate marginal spaces, obeys

$$R_D J_{D,C}(t) = J'_{D,C}(t) S_C.$$

Its nonzero witnesses and their exact physical-authority type are preserved by this matched continuation.

Proof. The displayed operator is a composition of the actual bounded maps. Because a_D is isometric, $a_D a_D^*$ is the orthogonal projection onto its range and the displayed nonnegative number equals $\|a_D^* U(t) j_C P_C x\|^2$. Strict positivity is therefore equivalent to a nonzero response on this input; it is not inferred from the existence of the tensor $P_C x$.

The last transport equation and unitarity give $R_D a_D^* = a_D'^* T$. Substitution, followed by the other three equations, gives

$$R_D a_D^* U(t) j_C P_C = a_D'^* U'(t) T j_C P_C = a_D'^* U'(t) j'_C P'_C S_C.$$

This proves joint covariance. Unitarity preserves norms, and the source continuation preserves the boundary and authority data by hypothesis, proving the final assertion. □

The map a_D requires independent spectral, persistence or asymptotic authority for its boundary; an interpolating operator alone supplies no such attachment. The operation P_C is the kinematic neutral projection, whereas physical evolution, when present, is supplied by $U(t)$. These operations have distinct roles in the channel.

When both incoming and outgoing ports are used, retain their actual attachments in this same diagram. If $T a_{\text{in}} = a'_{\text{in}} R_{\text{in}}$ and $T a_{\text{out}} = a'_{\text{out}} R_{\text{out}}$, their joint channel satisfies

$$R_{\text{out}} a_{\text{out}}^* U(t) a_{\text{in}} = a_{\text{out}}'^* U'(t) a'_{\text{in}} R_{\text{in}}.$$

The same substitution used in the theorem proves this equation. Separate identifications of the two marginal port spaces do not imply it; the common source attachment is essential.

Proposition 10.7 (A nonempty neutral-response seed)**.** *On a unital source-admitted gauge-invariant vacuum representation, the scalar unit response supplies a nonzero neutral channel. In the joint tensor model $V \otimes V^* \simeq \mathrm{End}(V)$, for $d = \dim V > 0$, take*

$$t_0 = I_V/\sqrt{d}, \qquad \lambda(t) = \mathrm{Tr}(t)/\sqrt{d}, \qquad j(t) = \lambda(t)\Omega.$$

If this contraction is an admitted source operation, then $j(t_0) = \Omega$ and the vacuum-ray attachment gives a unit nonzero boundary amplitude. This populates a neutral response channel, not a massive composite or hadronic scattering channel.

Proof. The identity is invariant under conjugation, and trace is an invariant linear contraction. Thus $P_C t_0 = t_0$ and $\lambda(t_0) = 1$. The unit of the represented observable algebra acts as the identity on the nonzero vacuum. Choosing the actual vacuum-ray inclusion therefore gives the scalar amplitude one. No additional spectral or composite claim appears in this construction. □

10.5 Resonances, nonlinear histories, and mixed inputs

For a resonance, the relevant response is a gauge-invariant continued channel on its admitted domain or sheet, with incoming and outgoing ports retained jointly. The generating map is this analytic channel composed with the admitted configuration preparation and boundary readout. Its source intertwining equations compose exactly as above. Equality of the resulting continued channel preserves its poles and Laurent coefficients on that domain; a putative pole with zero coupling to the retained ports need not appear in that channel. Thus neither a formal propagator denominator nor a nonzero composite operator establishes a physical resonance. This channel-relative interpretation of a pole agrees with that in BCRS-2EW/H [2].

Nonlinear histories are represented by their actual equivariant partial maps and descend on orbit spaces, on the exact composite domains. The proof is orbit equivariance, not an identity asserting that a nonlinear map commutes with Haar averaging. In particular, projecting a colored input vector to zero can discard a nonzero invariant polynomial response; the full joint operation must be retained.

For a fixed-sibling comparison, every source equation involving the Higgs, matter, boundary or preparation sibling must use that same fixed input. If a source redescription changes it, an independent lawful identification with the fixed sibling is required before the comparison is used. Simultaneously transporting color and Higgs data proves covariance of the joint datum; it does not prove a comparison that holds the Higgs input fixed. The construction of $\mathcal{R}$ retains these incidence equations, so no marginal isomorphism supplies them silently.

10.6 Requirements for a complete confinement interface

The complete confinement claim requires both configuration coverage and physical realization. The following definition specifies the evidence for each part without assuming the existence of the required objects.

Definition 10.8 (Complete target-relative confinement acquisition)**.** A complete confinement and asymptotic-sector acquisition at a declared target requires the following source theorems and witnesses.

1. An independently characterized, nonempty, role-complete upstream domain of configurations, with the actual common color, matter, Higgs, global-form, boundary and vacuum attachments.
2. A proved comparison of that domain with the realized source configuration operations, including every terminally relevant configuration class; restricting attention to already successful channels is not a coverage proof.
3. Actual color-neutral physical boundaries and channel witnesses for every positively claimed realization, or exact source-proved impossibility or inapplicability on the corresponding branch. A missing attachment or unproved existence is not such an impossibility theorem.
4. The source's phase-appropriate confinement statement, with its exact color-sector and asymptotic meaning. If the conclusion excludes freely propagating colored sectors, that exclusion must be proved for the stated sectors, not inferred from a singlet projection or from a gap alone.
5. Independent spectral, pole, stability, persistence and scattering authority for each physical boundary type invoked. A claim of asymptotic completeness additionally requires coverage of all physical states in its stated scattering domain.
6. Actual continuation and gluing theorems preserving or lawfully transporting all these data, including mixed inputs, domains, failure cases and physical boundary type.
7. A source-complete outcome classification on the declared domain, with no unresolved required authority hidden as a zero channel, and a common-apex comparison with the retained matter and Higgs realization.

These data specify a complete physical confinement interface. Each narrower claim activates only the subset of these obligations that its stated conclusion actually uses.

The source-witness profunctor gives the realization and composition interface for its supplied generators. Forming its Hom sets establishes neither configuration coverage nor physical existence beyond those generators. A particular physical branch can nevertheless be established without an unrestricted continuum classification.

Proposition 10.9 (Positive branch and acquisition separation)**.** *The following conclusions have different exact support.*

1. *The invariant and unit-response constructions populate a structural and represented neutral-response branch where their source operations are admitted.*
2. *An actual hadronic admission datum populates its composite persistence boundary and licensed history comparisons by Proposition 10.1.*
3. *An actual joint operator realization and independently supplied physical attachment construct the channel of Theorem 10.6; a nonzero overlap is a separately tested productive witness.*
4. *The registered infrared QCD and heavy-hadron branches of SYM-6 retain their specific vacuum, matching, lifetime and domain premises. Their source conclusions apply to those branches, not to every color configuration.*

None of these individual results supplies unrestricted continuum confinement or all-sector asymptotic coverage. Those stronger claims remain governed by Definition 10.8.

Proof. The first three statements are the cited constructions and their explicit nonzero tests. For the fourth, SYM6-V01 retains the renormalized QCD vacuum witness as a registered input; V02–V03 use its specified target and physical hypotheses. SYM6-H06 additionally uses actual heavy-hadron occupancy, including the lifetime/hadronization condition, and SYM6-R01 transports the corresponding admitted ancestry paths [7]. In particular the formal heavy-mass regime does not by itself admit a long-lived top-hadron boundary. These source theorems therefore provide genuine target-specific boundary and response information, but their quantifiers do not cover the unrestricted domain or scattering completeness in Definition 10.8. □

The two-level construction determines channel operations, their variance and composition, exact nonzero tests, and matched continuation. Its physical conclusions apply to the source branches on which those data are supplied. Hadronization, where established, is a dynamical channel on such a branch, distinct from a static color contraction.

11 Color response in the licensed metric source

Color neutrality and gravitational source response answer different questions. A gauge-invariant color field can have nonzero metric response without becoming an asymptotic particle. Conversely, a formal particle label does not determine its metric source. The source construction of Module A starts with the response of an already admitted object and represents that response by a tensor distribution [23, Secs. 3–8, 10–12]. We specialize that construction to the color interface and its actual joint matter and Higgs couplings.

11.1 Attachment and complete metric probes

Fix a licensed smooth Lorentzian metric envelope (M, g), with signature $(-+++)$, and use compactly supported interior variations

$$\mathcal{V}_g = \Gamma_c(S^2T^*M).$$

The topology is the usual test-section LF topology. Boundary responses, when required, are additional declared inputs. For an actual color instance x, let $\mathcal{M}_x \subset \mathcal{V}_g \times \mathbb{R}$ be its metric first-response relation. This relation is determined by the admitted deformation acts, their metric tangents and their protocol outputs before a tensor representative is sought. Not every color target activates these protocols.

Definition 11.1 (Complete color metric response)**.** The metric-response branch is complete when every test variation has an incident admitted protocol, all protocols at that variation give the same output, and for every pair of variations and every real scalar the source admits sum and rescaling protocols whose outputs are respectively the sum and scalar multiple of the original outputs. The resulting function must be LF-continuous. The full relation, its exact domain, and its boundary and transport data are retained; no admitted metric protocol is omitted.

Theorem 11.2 (Attached color source and its observable quotient)**.** *On the complete metric-response branch, $\mathcal{M}_x$ is the graph of a unique continuous linear functional W_x, and there is a unique symmetric tensor-density distribution T_x such that*

$$W_x(h) = \frac{1}{2}\langle \mathsf{T}_x, h\rangle, \qquad \mathsf{T}_x \in \mathcal{D}'(S^2TM \otimes |\Lambda|M).$$

On one fixed metric and boundary envelope, equality of all these metric responses is equivalent to equality of the distributions. Lawful joint transport by a source-authorized diffeomorphism ϕ, with $g' = \phi_* g$, carries this equivalence to other metric presentations. The support of the distribution is the response support; no stronger regularity is implied by continuity alone.

Proof. Totality and single-valuedness give the function with graph $\mathcal{M}_x$. The sum and rescaling identities give linearity, and the continuity hypothesis places it in $\mathcal{V}_g'$. The dual bundle of S^2T^*M is S^2TM; by the definition of distributional sections its continuous functional is represented by exactly one density-valued distribution. Multiplication by two gives the displayed convention. Equality against every test section is distributional equality. If a lawful comparison ϕ carries the protocol relation to its counterpart, then $W_{x'}(\phi_* h) = W_x(h)$. Duality gives $\mathsf{T}_{x'} = \phi_* \mathsf{T}_x$. Finally, both supports are the complement of the union of open sets on which every supported probe has zero response. Delta distributions show why continuity does not establish smoothness. □

This is an equivalence of the metric-response quotient, not injectivity on the entire color source domain. Distinct transport or phase structures can share a metric first response. A zero source attaches precisely when the complete incident protocol family has zero output; an absent metric protocol is not a zero response. These distinctions preserve the domain of Module A's attachment theorem rather than assigning a tensor to every upstream response class.

11.2 Local-to-global response gluing

Theorem 11.3 (Local-to-global metric-response reconstruction)**.** *Let a locally finite open cover of the paracompact metric manifold carry continuous linear local responses W_i, each defined on every compactly supported metric probe in U_i. Suppose their actual comparison maps obey the overlap cocycle and, in the resulting common metric presentation, the responses agree on all compact probes in every overlap. Then there is a unique global continuous response restricting to all W_i. For a smooth subordinate partition of unity $\{\rho_i\}$, it is*

$$W(h) = \sum_i W_i(\rho_i h).$$

The value is independent of the partition. The corresponding source distributions glue uniquely; local regularity and local distributional Ward identities persist globally. If the actual global protocol family admits these finite local assemblies and every global protocol is localized by these assemblies *with the same output, then this global response is attached to that family. Without that last source-assembly condition the conclusion is a global mathematical response, not a new physical attachment.*

Proof. For compactly supported h, local finiteness makes the sum finite and each $\rho_i h$ has compact support inside U_i. If $\{\sigma_j\}$ is another subordinate partition, expand $\rho_i h = \sum_j \sigma_j \rho_i h$. Every product is supported in the overlap and hence has the same value under W_i and W_j. Interchanging the two finite sums proves partition independence. The same argument shows that a probe supported in U_k is evaluated by W_k. Any global linear response with these restrictions must have the displayed value, proving uniqueness.

On a fixed compact support only finitely many local responses occur. Multiplication by each smooth ρ_i is continuous on test sections, so their finite sum is continuous there; the LF topology gives global continuity. Distributional duality identifies the resulting source with each local source. Regularity is local. If the local Ward equations have exchange distributions J_i, these agree on overlaps as the divergences of the same local source and glue to J. The equality $\operatorname{div} \mathsf{T} = J$ holds locally everywhere, hence globally as an equality of distributions. No derivative of a partition is discarded to obtain closure. Finally, source-assembly admission gives a protocol at every global probe, and the localization condition makes all its outputs equal to the displayed value. Thus the global protocol relation is exactly its graph. □

The comparison cocycle and protocol assembly preserve the same source instance. Separate overlap agreements chosen without a coherent joint comparison need not give such a global attachment. This result therefore extends local response reconstruction without assuming a global action or using numerical equality as a substitute for source identity.

11.3 An explicit color source and exchange identity

For the canonical Lorentzian branch of Section 2, choose the conventional sign

$$S_c[g, A] = -\frac{1}{4g_c^2} \int_M \kappa(F_{\alpha\beta}, F^{\alpha\beta})\, \mathrm{dvol}_g.$$

Only compactly supported variations are used. If the full integral is infinite, its variation means the difference of the densities on the compact changed region, not subtraction of two divergent actions. The response subinterface considered here consists of all and only the compact metric first variations of this retained functional, with its actual source incidences. Assume its full finite-deformation and first-response protocol family is admitted by the local-law source. This fixes equality, not merely inclusion, between the protocol response graph and the derivative graph. Other metric-responsive terms remain separate parts of the joint target treated below. The connection, matter fields and Higgs field are held fixed in this first metric variation. A shared global quotient causes no ambiguity: curvature is an adjoint-valued two-form and κ is invariant.

Theorem 11.4 (Canonical color source from the local response law)**.** *For smooth admitted connection data, the compact metric variation is a complete smooth metric response, represented by*

$$T^c_{\mu\nu} = \frac{1}{g_c^2}\left[\kappa(F_{\mu\alpha}, F_\nu{}^\alpha) - \frac{1}{4} g_{\mu\nu} \kappa(F_{\alpha\beta}, F^{\alpha\beta})\right].$$

With the Levi-Civita and color covariant derivatives acting together, its exact off-shell divergence is

$$\nabla^\mu T^c_{\mu\nu} = \frac{1}{g_c^2} \kappa(D^\mu F_{\mu\alpha}, F_\nu{}^\alpha).$$

In particular, an actual coupled solution satisfying $g_c^{-2} D^\mu F_{\mu\alpha} = J_\alpha$ has color-source exchange $\kappa(J_\alpha, F_\nu{}^\alpha)$. Closure of the color subsystem is equivalent to the vanishing of this exchange, not to the vanishing of its color charge label.

Proof. For $h_{\mu\nu} = \delta g_{\mu\nu}$, $\delta g^{\mu\nu} = -h^{\mu\nu}$ and $\delta d\mathrm{vol}_g = \frac{1}{2} g^{\mu\nu} h_{\mu\nu} d\mathrm{vol}_g$. Curvature with both indices lowered is unchanged when the connection is fixed. Varying its two inverse metric contractions gives

$$\delta\kappa(F_{\alpha\beta}, F^{\alpha\beta}) = -2\kappa(F_{\mu\alpha}, F_\nu{}^\alpha) h^{\mu\nu}.$$

Substitution gives $\delta S_c = \frac{1}{2} \int (T^c)^{\mu\nu} h_{\mu\nu}\, d\mathrm{vol}_g$. The derivative is linear and continuous on every compact test-section space, with compatible restrictions; it is thus LF-continuous. Every compact variation remains Lorentzian for sufficiently small parameter, since the signature is open and the varied support is compact. The stipulated protocol attachment identifies this derivative with the original response relation. Theorem 11.2 then gives the unique source.

Differentiate the tensor using metric compatibility and the invariance of κ. The Bianchi identity gives

$$\kappa(F^{\mu\alpha}, D_\mu F_{\nu\alpha}) = \frac{1}{2}\kappa(F^{\mu\alpha}, D_\nu F_{\mu\alpha}).$$

This cancels the derivative of the scalar curvature contraction, leaving exactly the displayed divergence. Substituting the actual current equation gives the exchange identity. □

The formula is a global gauge-invariant response on all four admitted Standard Model global forms. It introduces neither an extra bosonic carrier nor a new particle. Its metric dependence makes explicit how color enters a licensed gravitational source description. The earlier premetric gravitational necessity fixes the relational localization and continuation setting; the source tensor represents an admitted response within that setting. It does not supply the premise from which that necessity is inferred.

Proposition 11.5 (A nonzero closed classical color response)**.** *On a trivial Minkowski chart, choose a nonzero constant Cartan element $C \in \mathfrak{su}(3)$ and a real constant antisymmetric form $f_{\mu\nu}$. The connection*

$$A_\mu = -\tfrac{1}{2} C f_{\mu\nu} x^\nu$$

has $F_{\mu\nu} = C f_{\mu\nu}$ and $D^\mu F_{\mu\nu} = 0$. Taking only $f_{12} = -f_{21} = B \neq 0$ gives

$$T^c_{00} = \frac{\kappa(C,C)B^2}{2g_c^2} > 0, \qquad \nabla^\mu T^c_{\mu\nu} = 0.$$

These data give an explicit nonzero, gauge-invariant, closed classical local metric response. Its admission to a particular source-restricted target requires that target to contain this connection and its complete metric protocols; the construction does not assert their presence in every canonical source fibre.

Proof. All commutators vanish because the connection takes values in one Cartan direction. Ordinary differentiation gives the stated constant curvature; its ordinary and connection derivatives vanish. In the tensor formula, $F_{0\alpha} = 0$, $g_{00} = -1$, and the scalar contraction is $2\kappa(C,C)B^2$, proving the assertions. □

This construction establishes the response and Ward identities within the declared classical local branch. It is not a finite-total-energy vacuum state, a hadron, or a coupled Einstein solution. Those interpretations require their own source and boundary attachments.

11.4 Joint matter, Higgs and color exchange

For a tensor-density distribution T, use the smooth volume density of g to write $\mathsf{T} = T^{\mu\nu} d\mathrm{vol}_g$, and lower indices with g. This defines the distributional notation $T_{\mu\nu}$ below without assuming a smooth tensor density.

Let a range over a finite decomposition of one actual joint source response, including each admitted interaction, binding, vacuum and boundary term exactly once. Individual terms need not be separately closed. Write W_a for their attached metric responses and T_a for their distributions. Declare all internal exchange currents j_e, external currents J_a^{ext}, and a signed incidence matrix B_{ae}. The sector response equations are actual identities

$$W_a(\mathcal{L}_X g) = -\left\langle J_a^{\text{ext}} + \sum_e B_{ae} j_e, X \right\rangle, \qquad X \in \Gamma_c(TM).$$

Merely listing terms does not prove these identities.

Theorem 11.6 (Mixed-source closure and joint transport)**.** *If every internal column satisfies* $\sum_a B_{ae} = 0$*, then*

$$W_{\text{tot}} = \sum_a W_a, \qquad \mathsf{T}_{\text{tot}} = \sum_a \mathsf{T}_a, \qquad \nabla^\mu T^{\text{tot}}_{\mu\nu} = \sum_a J^{\text{ext}}_{a,\nu}.$$

The total source is closed if and only if the external sum vanishes. These constructions commute with every lawful comparison preserving the actual mixed source incidences, metric protocols and boundary data. A comparison with specified matter or Higgs data held fixed uses their identity comparison and must preserve their incident maps.

Proof. Finite sums preserve continuity. Linearity of distributional pairing and uniqueness in Theorem 11.2 give the source sum. For a symmetric distribution,

$$\frac{1}{2}\langle \mathsf{T}, \mathcal{L}_X g\rangle = -\langle \operatorname{div} \mathsf{T}, X\rangle.$$

This follows from $(\mathcal{L}_X g)_{\mu\nu} = \nabla_\mu X_\nu + \nabla_\nu X_\mu$ and integration by parts, defined by duality for distributions. Summing the sector equations cancels the internal terms by column balance. Testing every compact vector field proves the exact divergence equation and both directions of the closure criterion. Transport commutes with each attached response and its actual incidences, hence with the finite sum; duality transports the representing distribution. Identity on fixed sibling data is precisely the corresponding specialization of this simultaneous comparison. □

For a diffeomorphism-covariant joint local functional, first variation supplies these responses without an independent stress-tensor choice. On an actual joint solution the compactly supported diffeomorphism variation has no remaining Euler–Lagrange terms, giving total closure. In particular, the color exchange in Theorem 11.4 is balanced by the response of the other coupled terms, not by declaring the color tensor separately conserved. Module A's finite cancellation theorem applies directly to this common source [23, Thms. 8.2, 11.4, 12.1].

11.5 Observation, redundancy and distinct physical authority

Proposition 11.7 (Metric-response information and minimality)**.** *On a target admitting the complete metric probe family, inequivalent source distributions in the same metric envelope are separated by an admitted metric response. A stored source tensor is redundant when the full attached response law and its fixed metric inputs remain. Conversely, deleting all data that distinguish two actually admitted source responses is not lossless. These conclusions extend to the native color quotient with the exact attachment, regularity, boundary and failure tests retained.*

Proof. A nonzero difference distribution has a nonzero pairing with some compact test section, which supplies a separator. The duality theorem reconstructs the tensor from the complete response; the explicit local formula does so from the canonical attached law. The separating pair therefore violates the kernel criterion of Theorem 13.4 precisely when the retained description has erased its response difference. The attachment relation is source-defined and the comparison preserves all its inputs, so adjoining its evaluations and status tests to the native grammar preserves the reception, factorization and contextual-congruence proofs. The claim concerns actual admitted pairs, not arbitrary tensors outside the source image. □

Neither first metric response nor its Ward identity constructs a configuration-to-hadron channel. Metric-source closure and the confinement interface consequently remain separately typed parts of the physical realization atlas. A future coupled gravitational realization may use the derived source only at its proved regularity, boundary and continuation scope.

12 The complete native color-response algebra

Reconstruction and observation identify different aspects of a response. A reconstructed object can contain distinctions inaccessible to the admitted observations, while gluon components, singlet dimensions or screened charges alone may omit essential joint data. The native algebra consists of the independently received operations and their admitted compositions. Its observational quotient retains exactly the distinctions these operations can detect.

12.1 Operations and their source domain

Fix a small target schema, or a chosen small skeleton of its source categories. At a target T, the received operation types are the following:

1. the BCRS-I source restrictions, color transport and representation operations, primitive-status tests, and actual mixed matter/Higgs incidences;
2. the declared local law, its admitted variations, jet evaluations and contractions, with the bundle, boundary and metric data used by those operations;
3. classical path and network transport, composition, source-admitted invariant tensors and their evaluations;
4. global representation descent, admitted screening, line/phase/boundary operations, and the actually received closed-support sector actions and comparisons;
5. the supplied invariant observable algebra, state, bounded actions, closed operator graphs, represented continuations, and actual physical ports or analytic channels;
6. the joint incident maps, admissibility and exact-domain tests, boundary-authority tests, obstruction and failure readouts for these operations.

Each family is restricted to its received operations. In particular, tensor-product formation does not itself authorize every invariant covector as an observation, and a support-sector diagram requires a separate represented realization before it carries a Hamiltonian.

A primitive readout is a target-authorized value or status of these operations after the required source or gauge descent. Gauge-frame coordinates and documentary identifiers are not additional physical observations. A many-input operation retains its actual mixed parameters and all sibling inputs. Each partial operation has its exact domain and its source-defined failure outcomes; an absent operation, an unresolved required premise, a defined zero and a proved obstruction remain distinct.

Proposition 12.1 (Reception of finite native uses)**.** *Every finite response construction on the stated received color target is a typed diagram of the preceding operations, with its original source and physical authorities. This assertion covers all admitted arguments, not a finite set of tested configurations.*

Proof. The source part is received through the actual restriction of Proposition 1.2 and BCRS-I's source-generation theorem [1, Secs. 16–17]. A local-law vertex uses the declared field/operator grammar and its evaluations or variations. A transport vertex uses an actual path, graph, representation or invariant contraction from Sections 3–4. A global or extended-support vertex uses the received descent, screening, sector or continuation data of Sections 6–8. A represented or physical vertex uses its actual algebra, state, graph, channel or boundary map from Sections 9–10. Incident and status vertices belong to the last family. Replacing each vertex by its received operation preserves all its arguments and edges. Induction over the finite diagram proves reception. New field operators, new boundary types or an unproved extension of the source domain are not instances of this argument. □

This coverage is relative to the independently specified source family, not to a preselected list of classified outputs or to every mathematical model with the same Lie algebra. Analytic and Hilbert completion operations require the additional convergence, cyclicity and domain structure considered next.

12.2 Joint represented reconstruction

For a represented interface let $\{v_\lambda\}_{\lambda\in L}$ be its full labelled source-generated family, with dense linear span in its Hilbert space. These labels are actual source words or preparations, not an arbitrary preferred basis. Retain the joint Gram kernel

$$K(\lambda,\mu) = \langle v_\lambda, v_\mu\rangle.$$

For a bounded incident map $A : \mathcal{H} \to \mathcal{H}'$, retain its mixed matrix elements against both dense source families. For a closed operator or unbounded incident map, retain the labelled pairs in a graph-norm dense subspace of its actual graph, with both components included in the joint Gram data. Its closed graph, not the choice of core, is the intrinsic datum.

Theorem 12.2 (Joint state, operator and incidence reconstruction)**.** *Equality of these elementary data for two cyclic presentations yields a unique unitary carrying every labelled source vector to its counterpart. It intertwines all retained bounded actions and incident maps and carries every retained closed graph onto its counterpart, including its entire domain. For a family of interfaces the comparisons commute simultaneously with the retained mixed incidences.*

Proof. On finite linear combinations define

$$U\left(\sum_\lambda c_\lambda v_\lambda\right) = \sum_\lambda c_\lambda v'_\lambda.$$

Equality of Gram kernels gives equality of the squared norm of every such combination. The map is well defined, isometric and has dense range; it extends to a unitary. Its values on the dense span prove uniqueness.

For a bounded incidence A, equality of mixed matrix elements gives $\langle U_2 w, A'U_1 v\rangle = \langle w, Av\rangle$ on the two dense spans. Density in the second interface gives $A'U_1 v = U_2 A v$; boundedness extends the identity to the full first interface. For a closed incidence, the product unitary $U_1 \oplus U_2$ identifies the retained dense graph pairs. Taking graph closures identifies the full graphs and therefore their domains and actions. Source equalities among labels remain equalities under the same unitary, so all incidences are reconstructed jointly, not by independent choices of marginal unitary maps. □

Reconstruction is determined by the positive kernel on the full source-generated domain. An incomplete vector family leaves noncyclic summands or alternative self-adjoint extensions undetermined. When the target includes such data, they require their own source attachments and determining domains before this theorem applies.

For source-admitted analytic channels, retain their convergent germs on the specified connected marked domains or sheets and their continuation paths and statuses. Equality of the full germs gives local equality by convergence; the identity theorem extends it along the supplied connected domain. The same argument through overlapping charts preserves each admitted continuation, its poles and Laurent coefficients. Different sheets and unprovided paths are not identified by equality on one patch [2, Secs. 11.1, 12].

12.3 Intrinsic reconstruction before observation

Let $b(x)$ be the intrinsic reconstruction datum of an actual instance $x \in \mathcal{X}_T$. It contains the source-restricted operation graphs and the following activated data:

1. the complete intrinsic local law and its source attachments, or its proved reconstructive coordinates;
2. the classical transport and joint tensor orbit data with all boundary anchors, together with separate global, phase and nonnumerical records;
3. the actual closed-support representation and deformation diagram, including observed path incidence;
4. the full admitted channel germs, joint cyclic Gram and operator-graph data, and their actual mixed incidences and physical authority records.

The equivalence of these data is simultaneous and source-preserving; separate isomorphisms of marginal objects are insufficient. On the local canonical branch the coupling inverse and full law give an explicit description. The forest and invariant-polynomial theorems provide the corresponding classical transport description, extended to full paths under their stated hypotheses. A sector beyond cyclic reconstruction is retained as its represented object with the actual incident comparison class; no smaller determining invariant is asserted for that case.

Write

$$\mathcal{B}_T = b(\mathcal{X}_T)$$

for the actual reconstruction image. Derived entries may be useful for display without being necessary to reconstruction. Any distinction in $\mathcal{B}_T$ that the admitted operations cannot observe is removed by the quotient below.

Theorem 12.3 (Reconstruction and complete-use factorization)**.** *Every native finite use on the received target can be evaluated from $b(x)$, on its exact domain. A lawful simultaneous comparison of reconstructive data preserves all such values and domain/failure outcomes, including comparisons with specified siblings held fixed.*

Proof. At a local vertex use the whole action or its inverse reconstruction from Section 2. A general finite-degree differential-polynomial law is recovered from its coefficient tensors as sections on the full base, by its finite Taylor expansion in jets; these coefficients are not replaced by a sample at one spacetime point [2, Sec. 9.2]. At a tensor vertex the retained coefficient tensor determines the actual operation, while Theorem 3.1 identifies its invariant intertwining space. Closed equivariant operators retain their full graphs and use Proposition 3.2 for neutral restriction; nonlinear equivariant maps use orbit descent, not commutation with averaging.

Theorems 4.1, 4.5 and 4.6 supply their respective classical comparisons. The global records use the joint descent of Section 6. A supplied sector vertex uses its actual representation, word actions and retained deformation maps, not just an isomorphism-class name. Its mixed use is governed by Theorem 8.6. At a physical vertex Theorem 12.2 or the analytic reconstruction supplies the whole operation; otherwise the actual supplied operation graph is retained unchanged. The physical channel construction of Theorem 10.6 uses the joint attachment, including its nonzero test.

These cases cover all vertices by Proposition 12.1. Exact pullback domains compose by Lemma 10.3. Induction therefore evaluates every finite use and its first source-recorded failure. Each comparison commutes with the actual generator incidences and reflects their domains. The same induction proves covariance, using identity on fixed siblings. An independent second diagram requires these joint incidence equations; they are not created by renaming its arrows. □

12.4 The observation algebra and its quotient

The operation schema is many-sorted. For each finite list of port sorts $\boldsymbol{s} = (s_1, \ldots, s_r)$, let $\mathcal{P}_{T,\boldsymbol{s}}$ be the actual jointly admitted tuples of pointed port values over their retained common source anchors. Let $\mathcal{U}_{T,\boldsymbol{s}}$ be the native contexts with one distinguished compatible tuple of these sorts and an authorized readout at the output. All admitted choices of other arguments and continuations are included. For a one-port fixed-sibling context, the siblings and every incident map are part of the fixed source record. Repeated occurrences of a distinguished port are allowed. The status test of every admitted partial operation, including its exact domain and failure alternatives, is itself one of these received readouts.

For whole source instances write $\mathcal{U}_T$ for the corresponding context family and define the tagged evaluation

$$R_T(b) = \big(\mathrm{status}(u, b), \mathrm{value}(u, b)\big)_{u \in \mathcal{U}_T}.$$

Each value is paired with its specified status, preserving all required status distinctions. This family is obtained by evaluating the reconstructed operations; its completeness follows from their reception and factorization rather than an assumed signature.

Theorem 12.4 (Exact terminal color quotient and congruence)**.** *On the received target,*

$$x \sim_T y \quad \Longleftrightarrow \quad R_T(b(x)) = R_T(b(y))$$

is exactly native observational equivalence. Hence

$$\mathcal{X}_T/{\sim_T} \;\simeq\; R_T(\mathcal{B}_T).$$

The corresponding source-pointed contextual relations form a congruence for every admitted operation and continuation in the following source-fibred sense. Equality under all contexts in $\mathcal{U}_{T,\boldsymbol{s}}$ defines contextual equivalence on each actual jointly admitted tuple sort. Every admitted partial operation preserves this equivalence on its exact source fibre and reflects its domain/status alternatives. Every distinct terminal pair is separated by a particular authorized readout after a particular native context.

Proof. The factorization theorem evaluates all native contexts from b. Equal displayed evaluations therefore preserve every native observation. Conversely every component of R_T is such an observation, so native equivalence forces equality of all components.

For congruence, let an admitted operation have a jointly admissible input tuple of sorts $\boldsymbol{s}$. Its domain/status test is a context of those sorts, so contextual equivalence reflects definedness and the typed failure value. Insert the operation into the distinguished tuple input of any output context. The resulting diagram is again a native $\boldsymbol{s}$-context and has the exact composite domain. Equality at all these contexts gives contextual equivalence of the outputs. This proves the many-sorted congruence without assuming that arbitrary mixtures of ports from unrelated instances are admissible. A one-port replacement with siblings fixed is the special case where both pointed tuples occur over that same fixed joint source record; simultaneously transporting the siblings is a different context. Repeated inputs are handled by contexts in which the pointed tuple carries the required diagonal incidence. The quotient-to-image map is well defined and injective by its kernel equation and surjective by definition of the image. Unequal terminal evaluation tuples differ in a component, whose context and readout give the claimed separator. □

Proposition 12.5 (When reconstruction itself is observable)**.** *If the activated target admits determining local-law responses, a separating joint network probe family, all required global/status tests, and the determining channel, joint-correlation and operator-domain probes of its physical interfaces, then*

$$\ker R_T = \textit{equality on } \mathcal{B}_T.$$

Without these probe admissions the terminal quotient remains exactly $R_T(\mathcal{B}_T)$, which can be strictly coarser.

Proof. On the canonical normalized local branch, the quartic witness recovers the variable coupling when the other inputs are fixed. More general admitted tensor evaluations recover the whole declared polynomial law. The complete joint invariant family separates classical transport orbits by Theorem 4.2, with the fixed nonnumerical fibre and global qualifications of that theorem. The stated independent tests recover the remaining source and global records.

In a cyclic observable interface, $v_\lambda = A_\lambda \Omega$ gives $K(\lambda, \mu) = \omega(A_\lambda^* A_\mu)$. These are native correlations only when their products and adjoints are source-admitted. Under the stipulated probe admission they recover the joint Gram kernel; the mixed and domain probes recover the incident maps and full closed graphs. Theorem 12.2 then applies. Admitted full analytic responses recover their germs and retained continuation statuses. Any remaining retained sector or authority datum must likewise have the stipulated determining readout; an untested sector label is not made observable by its name. Thus equality of readouts recovers all of b. The converse is factorization. □

The resulting standing interface incorporates source status, continuation, joint consequences and failures on the complete received domain, as required before cardinality is considered [15]. Its identity is determined by the native operation algebra, not by the local adjoint dimension.

13 Whole-interface closure and no-repair minimality

The native quotient determines which response distinctions survive all received operations. Role-occupancy closure addresses a separate question: which complete interfaces perform the independently required work, and whether their joint realization is unique. An adjoint basis, a set of invariant coordinates, or a list of physical channels does not by itself answer that question.

13.1 Role work and candidate families

Let I_T index the independently activated operation-bearing interfaces of the target. Their generic work comprises the local color law; the classical transport/tensor interface; each required global or extended-support interface; and each activated state, continuation or physical-channel interface. All repeated evaluations of one such interface share its object. A derivative, matrix entry, invariant projector or residue is its output, unless a separate source requirement establishes additional interface work. Role indices are typed interface requirements, not proposed elementary particle species.

A generic candidate performs the specified work within the independently received source family; candidate admission is not defined by a proposed normal form. At $b \in \mathcal{B}_T$, eligibility tests its actual source attachments, determining local/transport data, elementary joint Gram and channel data, exact graphs, and global/boundary records against the corresponding restrictions of b. Where no smaller reconstruction is proved, the actual full represented interface and its incident comparison class are retained. In that case the source description is retained faithfully, without asserting a new finite classification of its representations.

Write S_b for the disjoint typed union of eligible candidates modulo lawful simultaneous source-preserving comparisons. A complete injection $a : I_T \to S_b$ is compatible when its generating incidences assemble on the actual common source and satisfy all mixed, continuation, domain and boundary equations. The disjoint typing does not claim that different roles are different primitive carriers.

Let $\mathcal{X}_T^+ \subset \mathcal{X}_T$ be the independently determined subdomain whose every required interface is supplied by an actual received object, with all its generating incidences, exact domains, failures and physical authorities on one common source instance. This condition is checked on the received source records; it does not require a terminal signature or a preselected assignment orbit. Put

$$\mathcal{B}_T^+ = b(\mathcal{X}_T^+).$$

An unresolved required interface remains in the general native status quotient of Section 12, but it does not belong to this positive complete-realization image. A proved negative or inapplicable branch retains its own status and is not silently recoded as positive occupation.

Lemma 13.1 (Joint realization at a reconstruction point)**.** *For $b \in \mathcal{B}_T^+$, any two compatible eligible whole-interface presentations at b admit a simultaneous source-preserving comparison. For each role i, its lawful eligible class $s_{b,i}$ is unique. These comparisons preserve every native use.*

Proof. The local action inverse, forest and invariant separation theorems give their specified whole objects and lawful comparisons. Full classical transport uses its all-finite-path compatibility and smoothness hypotheses, not a finite sampling argument. The global data and any retained nonreconstructed source-sector objects have the same actual comparison class by eligibility.

For represented and analytic interfaces, Theorem 12.2 and analytic continuation reconstruct the entire received object. The same theorem uses the joint mixed matrix elements to fix each incident map; full graph data treat unbounded incidences. Other partial incidences retain their actual graphs and exact domains. Thus rolewise comparisons fit the same generating diagram, including the source anchors and fixed siblings. This proves the simultaneous comparison and rolewise uniqueness. The factorization theorem then preserves every composite native use. □

Theorem 13.2 (Full target-fibred role-occupancy closure)**.** *For every complete-realization reconstruction point $b \in \mathcal{B}_T^+$, the received whole-interface diagram satisfies all seven role-occupancy completeness conditions. If $\mathcal{C}_b$ is the family of compatible complete eligible injections, then*

$$S_b^{\mathrm{live}} = \bigcup_{a \in \mathcal{C}_b} \mathrm{im}(a) = \{s_{b,i} : i \in I_T\}.$$

Every such assignment is a bijection onto this live residue, and its quotient by lawful full-diagram automorphisms is a singleton. On the detectable branches of Proposition 12.5, this is terminal role-occupancy closure. Otherwise terminal identity is the further native quotient $R_T(\mathcal{B}_T)$.

Proof. We verify the seven conditions in their source order [14, Def. 7.1; Thms. 7.2–7.4].

Candidate completeness. Proposition 12.1 receives the independent operation types before normal forms are formed. Local, classical, sector and represented candidates remain in their respective actual source families. The separate reconstruction theorems cover the infinite path, analytic or Hilbert operations only on their specified domains.

Quotient completeness. The reconstruction theorems and actual retained source comparisons identify two eligible presentations precisely at their intrinsic reconstructive data. Conversely lawful comparisons preserve those data. This proves the reconstructive quotient; its equality with the observational quotient is used only where detectability was proved.

Role completeness. The roles are all the independently activated interfaces and their required work. Every received incidence appears in the joint diagram. Derived outputs do not create a second copy of the interface performing that work. A genuinely new operation requirement would change this target and is not covertly admitted here. An already required but unresolved interface stays in $\mathcal{X}_T$ with that status and lies outside $\mathcal{X}_T^+$.

Eligibility completeness. At b, every candidate has its exact source-attachment and elementary-data test. Lemma 13.1 identifies its unique lawful role class whenever eligible. This test does not presume terminal-signature equality.

Compatibility completeness. Since $b = b(x)$ for an actual $x \in \mathcal{X}_T^+$, that instance supplies one complete simultaneous assignment and all its incident maps. Any other compatible eligible presentation lies in its joint comparison class by the lemma. Marginal eligibility without those equations was not used to assert compatibility.

Live-residue completeness. Every compatible assignment takes i to $s_{b,i}$, and the actual simultaneous assignment attains all those classes. Taking the union of their images gives the displayed residue. Each assignment is therefore onto that residue. There is already one assignment at the lawful class level, so quotienting by the allowed whole-diagram automorphisms leaves one orbit. No unrelated graph symmetry is promoted to a lawful source comparison.

Failure completeness. The primitive operation graphs and their composites retain exact admission and failure records. Inactive physical work differs from required work with unresolved authority. The latter does not furnish a positive eligible occupant or a populated complete instance. Defined zero channels and proved obstructions retain their own outcomes.

These checks prove the stated closure. Detectability identifies reconstructive and native identity on its exact branches; Theorem 12.4 supplies the native quotient in every received branch. □

Uniqueness is relative to the specified target and reconstruction point. It does not imply $|\mathcal{B}_T| = 1$, select a global form, determine the strong coupling, or assign one hadron to each quark. The number of activated interfaces can depend on the target. The single invariant-support atom of the adjoint is a distinct internal result.

13.2 Information that can and cannot be deleted

Definition 13.3 (Lossless deletion)**.** A retained description $D : \mathcal{X}_T \to Z$ is sufficient when a map $h : D(\mathcal{X}_T) \to R_T(\mathcal{B}_T)$ satisfies

$$R_T(b(x)) = h(D(x)) \quad (x \in \mathcal{X}_T).$$

This map may use only the retained description and fixed target data. It may not consult the deleted information under another name, a representative-specific oracle or a new source measurement.

Theorem 13.4 (Exact no-repair minimality criterion)**.** *A deletion D is sufficient exactly when*

$$\ker D \subseteq \ker(R_T \circ b).$$

If it is not sufficient, two actual admitted instances with equal retained data differ in a particular native context. Conversely the terminal image $R_T(\mathcal{B}_T)$ retains exactly the native distinctions and no invisible reconstructive distinction.

Proof. If the factorization exists, equality of D implies equality of the response tuple. For the converse define $h(z)$, on the actual image of D, to be the response tuple of any instance with retained value z. The kernel inclusion makes it independent of the chosen instance; no choice of representatives enters its value. No extension outside the actual image is needed. If inclusion fails, the witnessing pair has unequal response tuples. A differing coordinate identifies an actual native context by Theorem 12.4. The terminal image is its own evaluated observation object, so no further distinct points can be merged without losing such a coordinate. □

The criterion has concrete consequences:

1. A change of adjoint basis or forest presentation is lossless when all boundary and incident data are transported. Stored curvature and vertex copies are removable if the complete law remains. The reconstruction theorems explicitly provide the recovery maps.
2. On an admitted fixed-target family containing the two relevant loop configurations, adjoint holonomy alone can miss a central distinction detected by a lawful fundamental probe. Individual loop conjugacy classes can also miss the mixed-loop response of Proposition 4.3. These are distinct failures: center blindness and loss of relative attachment.
3. Across an admitted family of global targets observing the line-admission tests, Proposition 6.4 shows that the common local law cannot replace the global record. At a fixed known global form its separate numerical label can instead be redundant.
4. On an admitted canonical local family with different g_c, the quartic witness detects coupling loss unless other retained data already determine that coupling.
5. An actual difference in domain, boundary authority or joint port response is a native separator even if successful values agree on their common domain. Such a pair must belong to the actual admitted comparison family; formal variations outside it do not prove an essential coordinate.

Minimality is therefore relative to the actual source image. A displayed coordinate need not be essential on every restricted target: derivable copies may be removed, whereas a genuine loss is witnessed by a pair of admitted instances separated by a native response. The kernel criterion characterizes both cases without fixing a preferred presentation.

14 Classification and the physical branch atlas

Theorem 14.1 (Color carrier–response–realization classification)**.** *For the exact admitted joint source target, the color construction has the following endpoint.*

1. *The premetric color-transport carrier and its matter/Higgs incidences remain those of the accepted source classification. On its $SU(3)$ adjoint realization, the eight real coordinates form one irreducible invariant support.*
2. *The licensed local law determines its curvature, covariant matter action and canonical self-interaction responses. Joint compact-group invariant theory constructs neutral tensor and observable responses and their exact linear, closed-domain and nonlinear continuations.*
3. *The classical transport interface has a boundary-respecting forest normal form, separating invariant probes on the stated fibres, and a full-path reconstruction on the stated compact and smooth branches. The shared global target has the four quotient forms and exact electric descent/screening classification on the received Standard Model representation family.*
4. *Actually received closed-support sectors, represented local theories and composite boundaries retain their distinct source authorities. Their joint incident maps construct the two-level configuration-to-neutral-boundary profunctor, productive amplitude tests and lawful continuation.*
5. *The complete received native algebra factors through the reconstructed whole-interface image. Its terminal quotient is $R_T(\mathcal{B}_T)$; its full role occupation is unique over each complete-realization point $b \in \mathcal{B}_T^+$ and is terminal where the determining probes are admitted. No-repair minimality is the exact kernel criterion of Theorem 13.4.*

These conclusions form a conservative common-source color extension of the accepted fermionic and electroweak/Higgs response chains. The positive physical consequences below apply to their exact matched branches, not to unrelated marginal realizations.

Proof. The first conclusion combines source reception with Theorem 2.1. The second is Section 2 and the invariant, operator and GNS constructions of Section 3. The third is Sections 4 and 6, with their explicit probe and global-family conditions. The fourth follows from the actual sector composition and physical continuation theorems, together with Theorem 10.4. The last conclusion is Theorems 12.3, 12.4, 13.2 and 13.4. All maps in this chain restrict from the same source instance. Proposition 1.2 therefore proves conservative attachment. □

14.1 Constructive completion of the response atlas

The preceding classification admits a more explicit realization on the source-generated branches proved here. Invariant observations are derived from elementary contractions; the magnetic and phase coordinates are calculated from their coupled source data; and a complete metric response has a unique distributional representative. These constructions concern different parts of one source and must be compared together.

Theorem 14.2 (Source-generated color realization and closure)**.** *Let $T^\sharp$ be a joint color target activating any of the following independently specified interfaces on its actual source domain $\mathcal{X}^\sharp$:*

1. *the contraction-generated network interface of Definition 5.3, with the full global qualifications of Proposition 5.6;*
2. *an admitted magnetic/dyonic and phase interface of Section 7, retaining its actual line branch, endpoint maps, theta monodromy and joint matter/Higgs actions;*
3. *the attached metric-response interface of Section 11, with its complete probes, source assembly where local gluing is used, and declared exchange.*

All activated interfaces and their mixed maps belong to the same source instance. Let $\mathcal{X}^{\sharp,+}$ be the independently determined subdomain with every required interface, physical authority and mixed incidence actually supplied, and put $\mathcal{B}^{\sharp,+} = b^\sharp(\mathcal{X}^{\sharp,+})$, where $b^\sharp$ is specified below. Then the color classification has the following constructive refinements. Each of the first three applies only to its activated interface and on the exact branch hypotheses stated there.

1. *The elementary contractions generate every finite-network Reynolds observation. Thus complete network detectability is proved from generator admission and its closure laws. Equality on every finite restriction gives the full pointwise comparison, and gives a smooth comparison on the specified connection branch.*
2. *Electric descent determines the cyclic magnetic group. The marked dyonic subgroup, the actual screened charge quotient, and the phase quotients on their stated reference branches and torus strata have the explicit reconstruction formulas above. These do not replace their additional physical incidence maps.*
3. *Complete metric response determines exactly one source distribution. Compatible local responses glue uniquely under the stated assembly conditions. On the canonical local-functional branch the color source and its exchange are explicit, and completed joint sources are closed exactly when their external exchange sum vanishes.*
4. *Let $b^\sharp$ retain these reconstructed objects and all remaining actual operation graphs and common-source maps. Every admitted use of the enlarged interface factors through $b^\sharp$ on its exact domain. For its complete contextual evaluation $R^\sharp$,*

$$\mathcal{X}^\sharp/\sim_{T^\sharp} \simeq R^\sharp(b^\sharp(\mathcal{X}^\sharp)).$$

   The contextual relations are source-fibred congruences, including the fixed-sibling comparisons actually admitted. On the image $\mathcal{B}^{\sharp,+}$, all seven whole-interface role-occupancy conditions hold, with one lawful complete assignment orbit at each point of that image.
5. *A retained description D is lossless exactly when*

$$\ker D \subseteq \ker(R^\sharp \circ b^\sharp).$$

   Where all activated reconstruction data have determining native readouts, the response image is the reconstruction image itself. Otherwise the further observational quotient remains necessary.

Forgetting the additional interfaces gives the actual underlying source and preserves every use depending only on that restriction. No surjective physical lift over all predecessor points is asserted.

Proof. The first refinement is Theorem 5.4 and Corollary 5.5, with full joint transition covariance supplied by Proposition 5.6. The second is Theorems 7.1, 7.2, 7.3, 7.5 and 7.6, together with the endpoint normal form. Reference-line and joint-phase coordinates are not treated as independently variable physical parameters. The third follows from Theorems 11.2, 11.3, 11.4 and 11.6.

For the fourth, replace each added operation by its proved reconstruction: a finite contraction expression, a charge or phase formula, a complete response functional, or its unique source distribution. Retain all other actual graphs, including those of physical channels not reconstructed by these formulas. Every generator comparison commutes with its entire incident tuple and reflects its actual domain. Induction on a finite native context then gives its value and first recorded failure, exactly as in Theorem 12.3. Insertion of an operation into any output context proves contextual congruence. The kernel of the complete evaluation is therefore precisely observational equivalence, giving the displayed image bijection.

For positive role closure, the independent source requirements are the activated interfaces and their work, not a proposed number of particles or tensor expressions. Their admitted candidates are received before reconstruction. The new formulas give their complete reconstructive comparisons; any additional sector, background or physical maps remain in their actual joint comparison class. Eligibility is agreement at those determining data, with every mixed map retained. An independently complete source instance supplies one simultaneous eligible realization. Any other eligible whole presentation is simultaneously compared to it by the reconstruction theorems and retained incidence equations. Thus candidate coverage, quotient coverage, required work, eligibility and compatibility are complete. The live residue consists precisely of those unique lawful interface classes; every complete assignment reaches them. Exact source domains and failure records give the seventh condition. This is the seven-part argument of Theorem 13.2 on the enlarged actual domain, without treating an unresolved required interface as a positive occupant.

Finally, the proof of Theorem 13.4 applies to this complete evaluation: a response factors through D precisely when it is constant on each actual D-fibre. The specific determining probes identify the reconstruction image with that response image exactly when no reconstructed distinction is unobserved. Restriction to the underlying instance uses the original common-source maps and hence preserves all uses factoring through them. □

The theorem establishes more of the realized response structure from its source operations. It does not identify those operations with an all-sector confinement law. Configuration coverage, physical neutral channels or their proved exclusions, and the requisite phase-dependent asymptotic results retain the exact requirements of Definition 10.8.

14.2 Physical interpretation of the classified branches

Received branch	Established consequence	Distinct claim requiring further authority
Joint invariant tensors or invariant algebra	Exact neutral projections, invariant observables and equivariant descent	A bound-state mass, hadronization law or asymptotic completeness
Witnessed elementary tensor operations and complete readouts	Generation of every invariant polynomial observation and the stated transport reconstruction	Automatic admission of all tensor products or a physical state for every invariant
Primitive shared global form with actual line and phase data	Magnetic duality, marked dyonic charge branches, screening normal forms and determining joint phase characters	Physical existence of every algebraic line branch or a confinement law from screening
Actual closed-support sector diagram	Fixed-local-root sector actions and admitted support-deformation transport	A new realization map for every formal support label or an open quantum junction web
Matched local Yang–Mills vacuum theory	Dynamical GNS equivalence, local subgap rigidity and, with the actual color action, local-vacuum neutrality	Exclusion of all massive colored sectors or confinement of a different matter-coupled theory
Admitted same-domain Yang–Mills completion	Complete subgap rigidity using the exact source completion theorem	A gap for the full electroweak–color theory or unrestricted QCD scattering completeness
Actual hadronic admission or registered infrared branch	Its neutral persistence boundary, histories and source-qualified responses	A channel from every colored configuration or a complete hadron spectrum
Joint operator realization and independent physical port	Explicit amplitude, nonzero-overlap criterion and full matched continuation	Nonzero coupling on every input or physical authority for a different boundary type
Complete metric response with its joint source attachment	Unique source distribution, compatible local-to-global gluing and balanced exchange; explicit Hilbert source on the canonical local branch	Universal admission of local configurations, unrestricted gravitational dynamics or hadron-channel existence
Licensed partonic approximation	Color-bearing effective descriptions within the admitted factorization, resolution and kinematic domain	A free asymptotic quark/gluon or unrestricted domain of validity
Jet or detector observation	The actual process-dependent reconstruction and response map	A new primitive carrier or an independently established pole

The atlas is target-dependent and includes higher-arity responses: the color-neutral contraction and its physical channel can depend on several matter, gauge and Higgs inputs at once. Such a joint response is not generally recoverable from its marginal responses.

The branches establish complementary results with different realization domains. In particular, Theorem 9.12 excludes a standard massless one-particle embedding on the matched gapped Yang–Mills branch. This is not a gap for the full Standard Model: a theory admitting a massless photon channel cannot be identified with that completion. A common premetric carrier source does not identify the two represented theories.

Local-vacuum color neutrality similarly excludes nonzero equivariant colored channels into that observable-generated sector, without identifying all nonlocal or charged representations with the same vacuum representation. The two-level interface preserves these distinct physical possibilities and their boundary types.

14.3 Scope and implications

The color atlas is organized by structures that determine responses, not by a list of particle names. Elementary tensor operations determine invariant observations on their witnessed branches; global charge and phase data determine line-admission and joint phase responses; complete metric probes determine the source distribution. Represented sectors and physical channels retain their actual preparation, boundary and continuation maps. These are complementary parts of one source, and their matter and Higgs incidences cannot be recovered from representation dimensions or separate marginal equivalence classes.

The native classification and closure are complete on the received structural target. Where the stated generating operations and determining probes are present, the reconstruction formulas give explicit coordinates for that classification. Complete metric attachment adds color's geometric response and its joint exchange balance while preserving the premetric gravitational role and licensed continuation. Unrestricted physical confinement and asymptotic-sector completeness require the additional source results specified in Definition 10.8. Configuration coverage, all-sector confinement and physical boundary authority are separate existence and comparison claims; they do not follow from the source-witness category alone. Where those results are required but unavailable, the corresponding status is $\mathrm{NotReady}_{\mathrm{Confinement}}$, not a structural obstruction and not a successful zero response.

The positive nonlocal results retain the provenance stated in Section 1; the independently proved network, global-response and metric constructions do not replace their general sector-realization dependency. Extending the classification to a broader physical domain requires the corresponding source comparison, not an identification of algebraic labels with physical states.

The result is a premetric, constraint-derived re-presentation of color with explicit realization, comparison and observation maps. One carrier supports distinct field, network, line, phase and metric responses; these responses acquire physical meaning through their actual source attachments. The exact observational quotient identifies precisely what the admitted uses cannot distinguish, while the lossless-deletion criterion identifies which information those uses genuinely require. This supplies a common mathematical basis for the color sector and its joint matter, Higgs and physical-process descriptions.

A Charge ratios from joint operator and anomaly constraints

The global-form calculation in Section 6 uses the primitive integer charge table. Its ratios can also be derived from the joint matter–Higgs incidence, rather than entered as independent numerical assignments. The source charge-normalization and charge-ratio theorems distinguish witnessed charge information from a change of its coordinate unit [16, 17]. The charge–anomaly–residual network retains the actual Yukawa operator availability and anomaly obstruction together [18, Secs. 6, 8]. The following calculation makes their interaction explicit on the specified minimal chiral branch. It also gives a direct polynomial proof of the charge-lattice source's hypercharge collapse, with its precise field and operator premises retained [19, Lemmas 9.1–9.2; Theorem 9.3].

Fix the non-Abelian representations

$$Q:(3,2),\quad u^c:(\overline{3},1),\quad d^c:(\overline{3},1),\quad L:(1,2),\quad e^c:(1,1),\quad H:(1,2).$$

Fermions are left-handed. Their real Abelian charges are

$$(a,b,c,d,e;h).$$

The same charges are used for every admitted generation. The target admits the three gauge-invariant operator incidences QHu^c, $QH^\dagger d^c$, $LH^\dagger e^c$ with their actual non-Abelian contractions.

Operator availability, not a nonzero numerical Yukawa coefficient, is the input. No extra chirally charged fermion is included; a sterile fermion with zero Abelian charge changes none of the equations below. Extra charged singlets or independent family charges change this calculation.

Lemma A.1 (Linear joint-incidence reduction)**.** *Gauge neutrality of the three admitted Yukawa operators, together with the mixed weak–Abelian anomaly equation, gives*

$$b = -a - h, \qquad c = -a + h, \qquad d = -3a, \qquad e = 3a + h.$$

The mixed color–Abelian anomaly then vanishes identically. The mixed gravitational and cubic Abelian anomaly coefficients, with multiplicities from the stated non-Abelian representations, are respectively

$$\begin{aligned} A_{\text{grav}} &= 6a + 3b + 3c + 2d + e = h - 3a, \\ A_{\text{cub}} &= 6a^3 + 3b^3 + 3c^3 + 2d^3 + e^3 = (h - 3a)^3. \end{aligned}$$

Proof. The three operator charges are $a + h + b$, $a - h + c$ and $d - h + e$. Setting them to zero gives the first, second and fourth charges in terms of a, d, h. The three color copies of a weak doublet and the lepton doublet give the weak anomaly equation $3a + d = 0$, giving $d = -3a$ and then $e = 3a + h$. The color anomaly coefficient is proportional to $2a + b + c$, which is zero after substitution.

The gravitational coefficient counts six components of Q, three of each color antitriplet, two of L and one of e^c. Substitution gives $h - 3a$. For the cubic coefficient use

$$(-a - h)^3 + (-a + h)^3 = -2a^3 - 6ah^2$$

and expand $(3a + h)^3$. The sum is $h^3 - 9ah^2 + 27a^2h - 27a^3 = (h - 3a)^3$. These are exact polynomial identities. □

Theorem A.2 (Charge-ratio determination on the minimal joint branch)**.** *Under the hypotheses of Lemma A.1, cancellation of either the mixed gravitational anomaly or the cubic Abelian anomaly implies*

$$(a, b, c, d, e; h) = a(1, -4, 2, -3, 6; 3).$$

Conversely every point on this line satisfies all the displayed Yukawa-neutrality and anomaly equations. On the nontrivial Abelian matter-action branch $a \neq 0$; there the charge ratios are unique. Repeating the same chiral package a positive number of times does not change this ratio conclusion.

Proof. The gravitational equation is $h - 3a = 0$. Over the real numbers the cubic equation $(h - 3a)^3 = 0$ has the same unique solution. Substituting $h = 3a$ into the lemma gives the displayed line. Direct substitution proves the converse for the stated equations. If $a = 0$, every charge, including h, is zero, so the Abelian matter action is trivial. Conversely $a \neq 0$ acts nontrivially on Q. For repeated identical generations the fermionic anomaly coefficients are multiplied by a positive integer, leaving their zero loci unchanged. □

The theorem proves a ratio, not the primitive period of a compact Abelian group. A simultaneous change of charge unit, period and coupling can be a lawful redescription, but rescaling charges while keeping a fixed primitive character and its probe family unchanged can change the global theory. This is the charge-witness distinction required by the normalization source [16, Sec. 11].

Theorem A.3 (The global kernel before primitive normalization)**.** *Fix the product cover*

$$SU(3) \times SU(2) \times U(1)$$

with Abelian angle of period 2π*, and the integer charges*

$$k(1,-4,2,-3,6;3), \qquad k \in \mathbb{Z} \setminus \{0\}.$$

Their full common representation kernel is cyclic:

$$K_k = \langle \xi_k \rangle, \qquad \xi_k = (\omega_3, -1, e^{i\pi/(3k)}), \qquad |K_k| = 6|k|.$$

Its pure Abelian subgroup is

$$K_k \cap U(1) = \langle \xi_k^6 \rangle \simeq \mathbb{Z}/|k|\mathbb{Z}.$$

Thus the charge-ratio theorem alone does not imply that the common global kernel has order six.

Proof. Let $(g_3, g_2, e^{i\alpha})$ act trivially on every displayed representation. The e^c action gives $e^{6ik\alpha} = 1$, so $\alpha = j\pi/(3k)$ modulo 2π. The L action gives $g_2 = e^{3ik\alpha} I_2 = (-1)^j I_2$. The d^c action gives $\overline{g_3} = e^{-2ik\alpha} I_3$, hence $g_3 = \omega_3^j I_3$. Every kernel element is therefore ξ_k^j, and substitution into the other representations verifies the converse. The Abelian component of ξ_k has order $6|k|$, so this is the order of the generator. Its color and weak components are both the identity exactly when j is divisible by six. This gives the stated pure Abelian subgroup and its order. □

Corollary A.4 (Primitive-lattice recovery of the color global family)**.** *If the admitted matter charges generate the fixed primitive Abelian character lattice, the integer charge vector of Theorem A.3 has* $|k| = 1$*, and its common kernel is cyclic of order six, isomorphic to* $\mathbb{Z}/6\mathbb{Z}$*. For* $k = 1$ *the charge vector is exactly the table in Section 6, so its four-form and screening theorems apply with the stated orientation. For* $k = -1$*, the corresponding subgroup embeddings and screening family have the Abelian orientation reversed. Identification of these opposite-oriented physical targets requires an admitted bundle and line-probe comparison; without it the two orientations remain distinct source targets.*

Proof. The greatest common divisor of the six displayed integer weights is $|k|$, because the Q weight is k and all other weights are its integer multiples. They generate the primitive character lattice precisely when that greatest common divisor is one. For the positive sign apply Theorems 6.1 and 6.3 directly. Inversion of the Abelian factor is a product-group automorphism taking the positive-oriented representation family and kernel to the negative-oriented family and kernel. It transports the corresponding descent and screening calculation. This mathematical transport does not identify the physical targets unless their actual source comparison permits that inversion and transports the retained bundle and line-probe data. □

Proposition A.5 (Screening before primitive normalization)**.** *For a positive integer multiplier* k*, set*

$$L = (\mathbb{Z}/3\mathbb{Z}) \oplus (\mathbb{Z}/2\mathbb{Z}) \oplus \mathbb{Z}, \qquad \kappa_k(t,s,q) = 2kt + 3ks + q \pmod{6k}.$$

The subgroup generated by the displayed dynamical matter and Higgs center charges is $\ker \kappa_k$*. In the declared quotient family, the possible central subgroups have orders* $n \mid 6k$*. The electric descent and screened quotient are exactly*

$$L_{k,n} = \{\ell : \kappa_k(\ell) = 0 \pmod{n}\}, \qquad L_{k,n}/L_{\text{dyn}} \simeq \mathbb{Z}/(6k/n)\mathbb{Z}.$$

Proof. Changing the representatives of t, s changes κ_k by a multiple of $6k$, so it is well defined. Every dynamical charge lies in its kernel. Conversely, for integer representatives t, s, write

$$(t,s,q) = -t(-1,0,2k) + s(0,1,3k) + \frac{q + 2kt - 3ks}{6k}(0,0,6k).$$

The last coefficient is integral precisely on the kernel. The three generators are d^c, H, e^c; hence they generate it. The cyclic common kernel has one subgroup of each order dividing $6k$. The generator of its order-n subgroup acts on (t,s,q) by $\exp(2\pi i \kappa_k(t,s,q)/n)$, giving the descent equation. The restriction of κ_k to $L_{k,n}$ has image the multiples of n in $\mathbb{Z}/(6k)\mathbb{Z}$: the pure charges $(0,0,nj)$ give surjectivity. Its kernel is L_{dyn}, proving the screened quotient. □

Lemma A.6 (Family universality from connected operator incidence). *Consider a fixed positive number of copies of the same chiral representations and one common Higgs. Suppose the admitted neutral $Q_iHu^c_j$ incidence graph is connected and spans all Q_i, u^c_j; each d^c_j occurs in an admitted $Q_iH^\dagger d^c_j$ incidence; and the $L_iH^\dagger e^c_j$ graph is connected and spans all L_i, e^c_j. Then the charge action is family-universal within each of the five fermion kinds. The charge-ratio theorem applies after imposing the stated anomaly equations on this common source.*

Proof. An edge of the first bipartite graph gives $a_i + b_j = -h$. Two Q vertices with a common u^c neighbor therefore have equal charges; the same argument applies to the u^c vertices. Connected alternating paths propagate these equalities to every vertex of its kind. Every d^c edge then gives the same $c_j = -a + h$. The connected lepton graph likewise makes all d_i equal and all e_j equal, with $e = -d + h$. The summed weak anomaly is a positive common multiplicity times $3a + d$; the remaining anomalies have the corresponding common multiplicity. The charge-ratio proof therefore applies unchanged. □

Connectivity is an independently tested property of the admitted operator incidences. Diagonal Yukawa slots alone do not imply it. For example, the usual charge assignment can be deformed by a family-lepton difference while preserving diagonal charged-lepton Yukawa neutrality and the corresponding anomaly cancellations. Such a branch is excluded here by the declared universal action or connected incidence, not by the mere number of generations.

A quotient by a nontrivial pure Abelian kernel is an alternative only when its actual bundle and line-probe comparison is supplied. It is not an automatic normalization change in a target that observes the removed characters. Similarly the displayed local anomaly equations do not settle every global anomaly, magnetic branch or phase condition; those remain the joint target data already retained in the main classification.

This derivation strengthens the source-to-color chain at a precise point. The familiar charge ratios follow from the admitted operator and anomaly incidences, while the primitive-lattice witness supplies the separate global normalization needed for the four-form family. Neither step supplies an additional primitive color carrier or a physical confinement map.

Charge-ratio rigidity does not identify the interaction strength with another charge coordinate. The charge and coupling source distinguishes the charge-witness and lattice roles from the standing-preserving coupling envelope and renormalization transport. Thus a coupling can be physically detectable and indispensable without becoming a primitive carrier, consistently with the local response inverse in this paper [20, Secs. 7–13, 19].

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