The Bosonic Carrier–Response–Projection Atlas

Common-source reconstruction, gauge–Higgs coherence, and physical realization under structural constraints

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Abstract

The electromagnetic, weak, color, and Higgs sectors belong to one coupled structure, whose joint responses contain information absent from its separate sector descriptions. We construct a bosonic carrier–response–projection atlas from the premetric source laws of the AASC structural framework and their licensed realizations, distinguishing rigidity of a complete law from variation of its marked responses. The complete mixed observation algebra gives an exact quotient and identifies the relative data required for joint reconstruction. On the nonzero minimal single-doublet branch we determine the color-correlated electromagnetic stabilizer and the five graded reducing-support components. Five invariant response coordinates reconstruct the stipulated joint local interaction; a sixth is required when the vacuum contribution is observed. Complete mixed polynomial data extend this reconstruction to admitted higher operators. A fixed Wheeler–DeWitt matter branch provides a simultaneous classical gauge–Higgs–gravity history, with a retained massive scalar clock, exact coupled constraints, and local covariant development. State kernels, particle polarizations, physical poles, neutral-composite channels, and metric exchange enter through their corresponding source constructions. The resulting atlas combines structural closure and a populated classical realization while specifying the independent joint-state and asymptotic information required for stronger physical projections.

Contents

1 Source scope and the integrated bosonic target

The bosonic Standard Model is a coupled structure. Its Higgs field participates in electroweak reduction and matter interactions, its global gauge form can correlate color and hypercharge, and its sectors contribute to one metric source. An integrated description must retain not only the individual laws but also the relative data through which those laws act together. In a premetric structural account, this is the passage from separately determined carrier roles to a common response and realization theory.

The preceding papers establish the source-side bosonic architecture and the separate electroweak–Higgs and color realization theories [1, 2, 3]. Here we determine which comparisons, operations, and physical projections are preserved when those sectors belong to the same construction. The distinction between a carrier, its responses, and their physical realizations is fundamental to this question. One color carrier supports an adjoint field; one Higgs bridge supports several response directions; an independently established pole belongs to a realized response. The representation dimension and the number of response directions do not count additional source carriers.

Throughout the paper, a *source* is the common law together with its identity-preserving relations, and a *target* fixes the questions and comparisons under consideration. A *marked argument* is the actual field, configuration, or preparation on which the law is evaluated. An admitted object belongs to the domain established by its source; admission alone does not assert its experimental observation. The *authority* of a projection is the mathematical construction that establishes it, including its hypotheses, not an institutional judgement. These distinctions allow a rigid law to support many different responses.

The main results follow this dependency. First, the complete mixed observation algebra identifies exactly what separate sector signatures omit, and actual relative attachments classify the joint source. The shared global gauge and Higgs structure then yields a residual stabilizer, a graded support decomposition, and an explicit inverse from local responses to the joint interaction law. A coupled classical construction supplies a nonempty common-source branch. Finally, state, pole, composite, and process constructions determine the exact domains of the corresponding physical projections and the information needed to reconstruct them.

1.1 The fixed source and its actual image

Definition 1.1 (Integrated source target)**.** A target T specifies a source scope, its admitted continuation and boundary comparisons, the gauge and Higgs mechanism families, matter incidences, the observable operation families, and the authority at which each physical projection is requested. A source instance $x \in X_T$ is an actual jointly witnessed configuration at that target. It retains

$$x = (J; \mathfrak{L}_Q, \mathfrak{L}_W, \mathfrak{L}_C, \mathcal{B}_H; \mathcal{A}, \mathcal{D}, \mathcal{P}),$$

where J is its common source ledger, $\mathcal{A}$ its full relative attachments and matter incidences, $\mathcal{D}$ its boundary, continuation, guard, and partial-domain data, and $\mathcal{P}$ its licensed realization data. A metric, global bundle, parameters, state, or asymptotic structure is included only at its actually admitted source level. When present it is retained, not chosen again in another sector.

An instance may contain a finite family of source charts joined by actual interface witnesses. Its common packet-law ledger does not imply that every internal constituent is a restriction of one universally amalgamated pointed record. The allowed source comparisons preserve the entire relative diagram, including their domain reflection laws.

The source reception and decorated-network coverage of BCRS–I [1, Secs. 12–13] provide two ways to obtain an instance: an intrinsically given joint source or a finite construction established by the source theorems. A formal tuple of independently realizable marginal values is not a third existence construction. The complete operation families precede the response quotient, so the source domain is independent of the response count eventually obtained.

Write $b(x)$ for the retained target–parameter state of an instance. It contains its source and metric branch, global gauge form, boundary, normalizations, and authority data. Put

$$B_T = \mathrm{im}(b : X_T \longrightarrow \mathcal{B}_T), \qquad X_{T,b} = b^{-1}(b).$$

Thus B_T is an actual source image. A parameter range declared by a coordinate formula need not equal this image. Every positive existence statement below either starts with $x \in X_T$ or independently constructs such an instance.

1.2 Source results used in the integration

Three source results determine the starting point of the integration.

First, BCRS–I receives all admissible standing-semantic source uses, forms their full context algebra before taking its quotient, and proves fixed-input source covariance even for mixed Higgs–gauge operations. Its final source-law theorem determines one intrinsic complete gauge-law reduct over each fixed nonempty source-realized extension fibre. With its fixed Yang–Mills and canonical Higgs families the whole-law inventory is $(1, 1, 1; 1)$. Every compatible assignment is represented by actual finite decorated source-law networks, with its marks and relative charts retained [1, Secs. 12–13, especially Thms. 13.3, 13.9 and Cor. 13.10]. The inventory does not count marked response values.

Second, BCRS–IIEW/H reconstructs the stipulated local electroweak–Higgs law from four invariant coordinates, or five when its vacuum value is observed. Its complete native observation algebra includes all admitted interaction and continuation operations. Additional state, spectral, pole, and asymptotic inputs retain their own domains and are not generated by the local quadratic form [2, Secs. 9–13].

Third, BCRS–IIC reconstructs the canonical local color law, proves finite invariant-network generation and the precise boundary of orbit separation, and retains the correlated global-form, electric, magnetic, dyonic, screening, and phase data. Its compact metric response determines the full attached stress distribution on the stated linear protocol domain. Its confinement and neutral-composite results preserve the authority supplied by the actual Yang–Mills source [3, Secs. 2, 5, 7, 9–12].

These results apply at their stated targets. In the AASC framework, admissibility, standing, reference, and irreversibility govern nondegenerate determinate construction. This structural kernel is not an optional additional condition on such instances. It is also distinct from the observational equivalence kernels, linear null spaces, and state kernels used below. Each of those has its own mathematical type. Premetric gravity constrains relational localization; its licensed metric expression and the subsequent classical field equations retain the hypotheses of their source results. Structural governance does not by itself provide a spectral reconstruction or arbitrary global gluing.

Proposition 1.2 (Conservative integrated reception)**.** *For an actual integrated source instance, restriction to each predecessor interface and evaluation of each received source operation are well-defined under lawful full-diagram source comparison. All admitted predecessor observations, exact failure values, and definedness predicates are retained. Every finite integrated operation network made from the received families has a source evaluation independent of presentation.*

Proof. The predecessor reception maps use actual source witnesses. Restrict a licensed comparison to a pointed port and retain all incident maps. The fixed-input covariance theorem transports a replaced presentation together with its attachment, leaving the intrinsic sibling inputs fixed. In particular it reflects, rather than merely preserves, the domain of a mixed operation. Color support comparisons are used over their fixed local object; no arbitrary marginal automorphism is presumed to lift to them. For a primitive vertex its source-proved partial graph and admission predicate are therefore unchanged. Induct over a finite network, applying this statement to every vertex with the actual common-interface witness used to join its inputs. Each output and each failure value agrees. This is the decorated-network argument of BCRS–I with the complete successor operations received at their own source domains. It never requires a universal amalgamation of the intermediate charts. □

1.3 Three realization questions

The structural common-source problem asks for jointly witnessed laws and their actual incidences. The classical problem asks in addition for one licensed metric configuration obeying the coupled constraints and evolution. The physical-projection problem adds the state, representation, spectral, pole, scattering, and confinement properties appropriate to the requested observables. This last designation does not deny the physical meaning of a classical solution: it names the additional constructions required for particle and channel claims.

Section 7 constructs a populated classical branch from the coupled Wheeler–DeWitt theory. Its scalar clock, global form, and boundary scope are retained, so the branch is not a pure Standard Model vacuum. The construction also exhibits the metric's dynamical role: adding gauge–Higgs energy requires a correction of the same gravitational constraint. The metric is not a passive label attached after solving separate sector equations. Sections 8 and 11 describe the additional physical-projection domains. The mixed quotient and relative-attachment classification are established without presuming those larger domains populated.

For clarity, $\mathsf{NotReady}$ means that the demanded authority is not supplied at the given source. It is distinct from a proved negative result, from an inactive input, and from a mechanism that is not applicable to that kind. The distinction is necessary for the final classification: a confinement-qualified color projection is not a failed attempt to define a free asymptotic gluon.

2 Boson kinds and mechanism-preserving naturality

Bosonic mechanisms can share a response construction without becoming the same mechanism. We first specify the comparisons that preserve a kind and then the common interfaces on which different kinds can be described together. Coupling belongs to the operations between these interfaces, not to an identification of their sources.

2.1 Kind-preserving comparisons and cross-kind incidences

The four physical designations used here are electromagnetic, weak, color, and Higgs. They label different fibres, denoted

$$\mathsf{Kind}_B = \{\mathsf{EM}, \mathsf{W}, \mathsf{C}, \mathsf{H}\}.$$

Before electroweak reduction, the electromagnetic designation refers to its charge-law ancestry and dependent residual interface, not to a photon already present in the premetric source. The gauge roles Q, W, C and the bridge role H retain their source meanings throughout.

Definition 2.1 (Full kind comparison)**.** For a fixed target T, let $\mathcal{K}_k(T)$ be the groupoid of actual intrinsic source records of kind k, with their complete mechanism fields and source-proved reversible presentation comparisons. The full boson-kind groupoid is

$$\mathcal{K}_B(T) = \coprod_{k \in \mathsf{Kind}_B} \mathcal{K}_k(T).$$

An arrow preserves the kind, the source target, the mechanism, and all standing fields. A lawful quotient comparison is represented at its common intrinsic record, not inverted as a raw noninvertible map. Cross-kind operations are separately typed many-input incidences on actual relative source diagrams. They are not arrows identifying their input kinds.

The disjoint union is not a denial of physical coupling. It distinguishes coupling from identity. The covariant derivative of the Higgs field uses both weak and hypercharge connections; a quark current may involve color, weak, and Higgs data at once. Such a vertex has several input ports and its own output type. Its existence does not identify a connection with a bridge or a color carrier with a weak carrier.

Definition 2.2 (Lawful common interface)**.** A common interface for a family of kinds is a source-specified subfamily of their sorts, observations, and partial operations, together with source-proved identifications of those types and their full operand domains. It retains the shared target, boundary, and continuation records. Its receiving functors U_k forget only fields outside that interface. The common-interface atlas consists of all such received interfaces and their actual restriction maps. It is not a chosen coarsest quotient of the full mechanism records.

A common interface retains precisely the information that can be compared with its operations and domains intact. For a fixed pair of kinds, the maximal atlas is the collection of all such interfaces. A single greatest interface requires the union of the whole family to be compatible and source-licensed. Pairwise compatible unions alone do not establish that conclusion. The atlas retains the actual interfaces without imposing an unsupported greatest description.

Theorem 2.3 (Naturality on the exact common interface)**.** *Let I be a lawful common interface and $u : x \to x'$ an actual source comparison in one of its kind fibres. Let R_I evaluate the received carrier–response incidences on I. Then response evaluation commutes with the induced intrinsic source and response comparisons, with preservation and reflection of every partial domain. More explicitly, each retained partial operation g satisfies*

$$u_o g(\mathbf{a}) = g'(u_1 a_1, \dots, u_n a_n), \qquad \mathbf{a} \in \operatorname{dom} g \iff (u_1 a_1, \dots, u_n a_n) \in \operatorname{dom} g'.$$

These equations compose for finite networks. No equation is inferred for a mechanism field erased by U_k, or between distinct full kind fibres.

Proof. The source comparison on an interface transports its entire typed graph, including its admission predicate, operands, and output. The displayed equation is the commuting square of that graph. The reverse comparison reflects admission. At a mixed vertex the maps act on the actual relative diagram, so the same square holds while unreplaced intrinsic siblings remain fixed. Apply Proposition 1.2 inductively to each finite composite. The receiving functor has no value on erased mechanism fields; hence the proof neither produces an equation there nor an arrow in the disjoint full-kind groupoid. □

2.2 The common formation grammar

All kinds admit the following structural organization:

$$\text{source use} \longrightarrow \text{standing activity} \longrightarrow \text{contextual signature} \longrightarrow \text{compatible response class.}$$

The arrows have different source realizations. Gauge transport and loop response, Higgs order/fixation response, and a confinement-qualified color continuation are not interchangeable instances of one physical action.

Theorem 2.4 (Kind-indexed universality)**.** *The received bosonic families share a source-first, domain-reflecting response-formation construction on every lawful common interface. Kindwise application commutes with lawful source redescription and interface restriction whenever the* *restricted observation algebra is received from the larger one. The common construction does not imply equal response cardinalities, equal physical mechanisms, or equal projection authority among kinds.*

Proof. Source reception is Proposition 1.2. Activity is tested by its independently fixed standing predicates, so lawful redescription preserves it. The full contextual signature is built from received partial observations and their admission domains; naturality preserves every coordinate. Therefore source comparison descends to the response quotient. If an interface observation is the restriction of a larger one, equality of the larger signatures implies equality of the smaller signatures, giving the unique induced map on the actual quotient image. None of these arguments compares the sizes of different kind fibres or constructs a missing physical realization. Such assertions require additional results about those fibres. □

Proposition 2.5 (Mechanism distinctions survive universality)**.** *On the full target, gauge transport, Higgs fixation, polarization, unstable resonance, and confinement are distinct typed operations or response interfaces. In particular, shared bosonic statistics cannot identify the Higgs bridge with a gauge-law root, nor convert a confinement-qualified field response into a Wigner one-particle state.*

Proof. A full-kind comparison preserves each mechanism's source and output types. Gauge transport takes continuation data to comparison arrows. Higgs fixation takes an admitted order/bridge incidence to a residual or fixation response. Polarization requires the appropriate realized kinematical representation. A pole requires its spectral/analytic source; confinement requires its sector and continuation source. These are different partial graphs, with different antecedents. Replacing one by another would fail preservation of at least its domain or output type. Statistics supplies none of the missing maps. A common erasure can remove a distinction from a selected description, but it cannot prove equality of the unerased records. □

Universality therefore provides common natural operations and exact maps between response descriptions while preserving their mechanisms. The common grammar specifies which questions can be asked of each kind. Integration adds questions whose arguments belong to several kinds; the response quotient developed next preserves those questions as well.

3 The complete mixed observation algebra

The common grammar becomes a coupled response theory when it includes observations with arguments from more than one sector. Two configurations can agree on every isolated-sector test yet differ on a mixed one. Their response quotient must therefore retain the relative information seen by those additional tests. We construct the complete mixed quotient before asking how many response classes it contains.

3.1 Source operations and complete observations

Fix T. Let S_T be the received sorts, including gauge, Higgs, source, relative-incidence, boundary, continuation, obstruction, and physical-output sorts. Each primitive operation is an actual typed partial graph

$$g_\lambda : D_\lambda \subseteq X_{s_1} \times \cdots \times X_{s_n} \longrightarrow X_{s_o}.$$

The admissible input combinations are specified by the domain: D_λ retains the actual common-source and interface conditions, including any restrictions on the displayed Cartesian product. The intrinsic parameter λ includes the operation's relative attachment, boundary, and all fixed sibling inputs in its source record. A change of physical attachment changes this parameter; a lawful change of its presentation preserves it.

The primitive families are supplied by Proposition 1.2: gauge transition, transport and loop laws; representation, charge and anomaly tests; actual product and residual operations; the one Higgs bridge and its order/fixation maps; the color support, network and sector maps; and the admitted local-law, metric, state, spectral, process and record operations. A conditional physical interface retains both its required hypotheses and its conclusion as typed data. Evaluation at that interface requires the actual state or spectral input specified by those hypotheses.

Definition 3.1 (Source contexts)**.** A context with one input hole is a finite composition of received operations, restrictions and probes, with actual intrinsic parameters and fixed sibling inputs. Its domain is the exact domain of that composite. Source comparison can re-present the whole pointed diagram; it cannot replace a missing physical attachment with an arbitrary one. Let $\mathcal{C}_T(s)$ be all such contexts with input sort s, including the identity context.

Every primitive partial probe $p : D_p \to V_p$ is totalized as

$$\widehat{p}(x) = \begin{cases} (\mathsf{defined}, p(x)), & x \in D_p, \\ (\mathsf{undefined}, \delta_p(x)), & x \notin D_p, \end{cases}$$

where δ_p retains the source-defined failure or missing-domain diagnostic, if any. Each operation's failure diagnostic is itself a received primitive observation on that operation's full input record. Composite evaluation follows the declared order of its source diagram: it records the first failed primitive vertex and that vertex's received diagnostic. If the source gives an order-independent failure convention, that convention and its proof are retained instead. No additional diagnostic depending on an unobserved input is introduced at a composite. In the absence of a finer source diagnostic there is one formal undefined value. Defined negative, inactive, not-applicable and not-ready values remain distinct values in their declared output sorts.

Whether an operation is defined and whether a response is active are themselves observations. Including these tests ensures that the quotient preserves the distinction between an executable operation and a missing input or failed condition. Totalization permits their partial graphs to be compared while keeping failure values distinct from physical responses.

Probe values are compared in their fixed intrinsic target sorts, with coordinate changes handled by the actual transport maps. For each sort define

$$\Sigma_s(x) = \left(\widehat{p \circ C}(x)\right)_{C,p}, \qquad x \equiv_T y \iff \Sigma_s(x) = \Sigma_s(y), \tag{1}$$

with the following indexing conventions. When the input sort is the full joint-source sort, write Σ_T for this same Σ_s. Here C ranges over all licensed one-hole contexts and p over the matching probes. A finite set can replace this family only when a generation or reconstruction theorem proves its completeness on the specified target. The inherited color-network generation theorem and the joint local-law reconstruction proved below give such reductions on their respective branches.

Theorem 3.2 (Complete mixed congruence)**.** *The relations $\equiv_T$ are the greatest many-sorted congruence contained in the kernels of the primitive totalized observations. For every received operation g_λ, replacing any input by an equivalent input while fixing its intrinsic siblings preserves and reflects admission and preserves the equivalence class of the output. Hence all received operations descend uniquely to their actual quotient domains. Activity is constant on each equivalence class, so active classes form distinguished subsets of the full quotients. Operations on active inputs still take values in the full output quotient; they restrict to active output subsets only on their exact active-output subdomains.*

Proof. Equality of signatures is an equivalence relation. Put a one-hole context into a fixed input slot of g_λ. The test for admission of that vertex is among the received observations.

Consequently equivalent inputs give the same domain verdict. When admitted, following its output by any context and probe is another allowed context of the replaced input. Equation (1) therefore gives equivalent outputs. Replacing finitely many inputs one at a time preserves admission at every step and proves the many-input assertion. The actual gluing predicate is included in the admission test; it is never inferred from equality of unrelated marginal root names.

Conversely, let R be a many-sorted congruence preserving all primitive totalized observations and partial-operation domains. Induction on a context shows that R-related inputs have matching definedness and related outputs at every vertex. Applying a primitive probe gives equal totalized values, including failure data. Thus $R \subseteq \equiv_T$. This proves greatestness. For quotient descent, define the output class using any admitted representative. Congruence makes it independent of that choice; domain reflection makes the quotient domain exact rather than existentially enlarged. Activity tests are included, so every class lies in a single activity disposition. Restriction to active inputs therefore gives distinguished subsets of the full quotients. It does not in general give a subalgebra: an admitted operation on active inputs can have an inactive output. Retain that output in its full sort, or restrict the operation to the inverse image of the active output subset. Congruence makes this latter subdomain saturated, so its quotient restriction is again exact. □

Corollary 3.3 (Source descent precedes observation descent)**.** *Every lawful full-diagram source comparison lies in $\equiv_T$. The complete contextual quotient factors through the intrinsic source quotient. It need not be equal to that quotient.*

Proof. Proposition 1.2 preserves every source operation and primitive probe with its exact domain. It therefore defines a congruence of the kind considered in Theorem 3.2. Greatestness gives the inclusion and the quotient factorization. Observational indistinguishability can identify more than source skin if the target does not observe every source distinction; the reverse inclusion has not been used. □

3.2 Why the marginal quotients need refinement

Let Σ_E and Σ_C be the complete predecessor signatures on the electroweak–Higgs and color restrictions, and let Σ_J retain all independently observed shared-source coordinates. On X_T put

$$m(x) = (\Sigma_E(x|_E), \Sigma_C(x|_C), \Sigma_J(x)).$$

Let $\mu(x)$ collect precisely the remaining integrated contexts, including every mixed operation and all its downstream probes. This family is specified by the source operations before forming m; it is not selected afterward to distinguish particular representatives.

Theorem 3.4 (Exact mixed refinement)**.** *On the actual joint source image,*

$$\ker \Sigma_T = \ker m \cap \ker \mu. \tag{2}$$

Consequently the marginal packet m reconstructs the integrated observational quotient if and only if every mixed signature μ is constant on each actual m-fibre. Equivalently, there is a unique map $\bar{\mu} : \operatorname{im} m \to \operatorname{im} \mu$ with $\mu = \bar{\mu} m$. No value of this map is assigned outside $\operatorname{im} m$.

Proof. The complete observation family is the union of the received marginal, shared, and remaining mixed families. Equality of all its coordinates is exactly equality of both m and μ, proving (2). If m determines the integrated class, any two members of one m-fibre have equal full signatures and therefore equal μ. Conversely constancy defines $\bar{\mu}(m(x)) = \mu(x)$, independent of the chosen actual representative, and $(m, \bar{\mu} m)$ recovers the full class. Every point of $\operatorname{im} m$ has such a representative, proving uniqueness there. The argument supplies neither an extension to unrealized parameter tuples nor its existence. □

Corollary 3.5 (Mixed completeness is a proof obligation)**.** *If actual $x, y \in X_T$ satisfy $m(x) = m(y)$ but one mixed context has different domain or value on them, no reconstruction of the full response from m exists. Conversely a proved complete mixed factorization closes that obstruction without identifying the distinct mechanisms involved.*

Proof. The first assertion contradicts constancy in Theorem 3.4. The second is its converse. □

Example 3.6 (Relative data invisible to two marginals)**.** Consider two nonzero vectors $u, v \in \mathbb{R}^2$ with one common orthogonal action, and suppose the marginal probes observe their norms. The pairs (e_1, e_1) and (e_1, e_2) have the same two marginal orbits, but the lawful joint probe $u^\top v$ has values 1 and 0. The relative angle is not removed by a simultaneous orthogonal redescription. Quotienting the two vectors independently would discard it. This is a mathematical example of the gluing problem, not an assertion that Higgs and color fields are two vectors in the same representation.

The example isolates the information carried by a relative attachment. For a bosonic source, a corresponding physical distinction requires actual admitted instances and a mixed probe that separates them. Conversely, reconstruction from marginal data requires factorization of the complete mixed family. A selected finite display establishes that conclusion only if its completeness has also been proved.

3.3 Observation-preserving changes of target

Proposition 3.7 (Exact restriction and extension law)**.** *Suppose a successor target T' retains the full source of T through an actual restriction $r : X_{T'} \to X_T$, and every T observation pulls back to a T' observation with its domain. Then*

$$x \equiv_{T'} y \implies r(x) \equiv_T r(y).$$

There is a unique induced map on actual observational quotient images. It is injective precisely when the successor observations factor through the predecessor quotient on $X_{T'}$. It is surjective precisely when every predecessor observational class is met by r.

Proof. Equality of the larger signature implies equality of each pulled-back coordinate, including its domain value, proving the implication and well-defined quotient map. Its injectivity is equivalently equality of the two induced kernel relations on $X_{T'}$; the same fibre argument as in Theorem 3.4 gives the factorization criterion. Surjectivity is exactly the stated image condition. Neither follows solely from retention of predecessor formulas. □

The restriction map relates descriptions at different levels of resolution while preserving their source ancestry. A populated classical extension and a stronger physical extension with unresolved requirements can therefore occupy distinct, consistently related parts of the atlas. The next question concerns the source itself: which relative attachments describe the same joint construction?

4 Relative attachments and joint-source reconstruction

The mixed quotient classifies responses. To classify the joint source itself, one must also retain how its sector descriptions are attached. The comparison groupoid records those attachments and the transformations that preserve the whole construction. It distinguishes a common source from two marginal descriptions whose shared entries merely agree.

4.1 The actual comparison groupoid

Let $\mathcal{E}$ and $\mathcal{C}$ be the intrinsic electroweak–Higgs and color source groupoids at a common target, and $r_E : \mathcal{E} \to \mathcal{J}$, $r_C : \mathcal{C} \to \mathcal{J}$ their source-proved shared-interface restrictions. Their isomorphism-comma groupoid has objects

$$(e, c, \alpha), \qquad \alpha : r_E(e) \xrightarrow{\sim} r_C(c),$$

and arrows (u, v) satisfying

$$r_C(v)\alpha = \alpha' r_E(u). \tag{3}$$

This space records compatibility at the displayed shared interface. Simultaneous realization can also depend on mixed currents, anomalies, boundary conditions and metric data. Those additional conditions belong to the joint source; agreement at the displayed interface alone does not establish that a triple is realized.

Definition 4.1 (Actual joint source groupoid)**.** Let $\mathcal{X}_T$ have the actual configurations of Definition 1.1 as objects and their actual licensed full-diagram source comparisons as arrows. Restriction gives a functor

$$F : \mathcal{X}_T \longrightarrow (r_E \downarrow_{\cong} r_C).$$

Any additional joint standing fields remain in $\mathcal{X}_T$, even if the displayed functor forgets them. For fixed marginal records (e, c), the relative source fibre consists of actual configurations with witnessed identifications of their restrictions with e, c. Its framed comparison arrows commute with those identifications using identity maps on the fixed endpoints. Separately, the unframed relative groupoid permits the source-licensed endpoint automorphisms, retaining their actions on the identifications and on every joint field. These are different groupoids. Write π_0 for the comparison classes of whichever groupoid is explicitly specified.

Theorem 4.2 (Exact joint reconstruction criterion)**.** *The triple (e, c, α) reconstructs the intrinsic joint source class on the actual image of F if and only if*

$$F(x) \cong F(y) \quad \Longrightarrow \quad x \cong y \quad (x, y \in \mathcal{X}_T). \tag{4}$$

If this condition holds, $\pi_0(F)$ is a bijection onto the comparison classes met by F. If, additionally, every object in a specified gluing subdomain is isomorphic to an actual image object, the bijection covers that subdomain. A sufficient condition for (4) is that every comparison between two image objects lifts to a whole-diagram source comparison. Merely covering the objects is not sufficient.

Proof. The functor sends isomorphic source objects to isomorphic triples, so it induces a map on π_0. Its image is by definition the classes met by F. Injectivity is exactly (4). Coverage of a larger subdomain gives surjectivity there. A lift of a triple comparison gives the required source isomorphism. Nothing about existence of preimages for individual triples controls comparisons between distinct preimages, which proves the final warning. Faithfulness on automorphism groups is needed for an equivalence of full groupoids, but is not necessary for this classification of their object classes. □

BCRS–I represents every compatible assignment by finite source networks with their relative charts retained [1, Thm. 13.9 and Cor. 13.10]. The preceding criterion identifies the additional condition needed to reconstruct the joint source from a smaller description: comparisons of that description must reflect comparisons of the complete source. The relative charts remain necessary whenever they distinguish assignments with otherwise identical marginal records.

4.2 A double-orbit normal form for complete attachment data

Fix marginal records e, c. Suppose their complete joint data are determined by an admitted attachment $\alpha \in A_T(e,c) \subseteq \mathrm{Iso}_{\mathcal{J}}(r_E e, r_C c)$. Completeness here concerns what an admitted attachment determines; it does not enlarge the family of admitted attachments. Let H_E and H_C be source-licensed endpoint automorphism groups whose actions retain all other fixed data. On attachments their action is

$$(v,u)\cdot\alpha = r_C(v)\alpha r_E(u)^{-1}.$$

Theorem 4.3 (Source-exact relative double-orbit classification)**.** *Suppose that:*

(i) every attachment in $A_T(e,c)$ has an actual joint-source realization, every actual joint source in the specified branch is represented by such an attachment, and all retained joint fields are determined by it;

(ii) the displayed actions preserve and reflect the exact admission domain $A_T(e,c)$;

(iii) each action comparison lifts to an actual full-diagram source comparison; and

(iv) every actual comparison between these joint sources induces one of the displayed action comparisons.

Then the unframed intrinsic joint source classes over the fixed marginal classes are precisely

$$H_C\backslash A_T(e,c)/H_E. \tag{5}$$

Without these hypotheses the exact classification remains the π_0 of the actual unframed relative source groupoid. There is no replacement by the orbit set of arbitrary marginal automorphisms.

Proof. By (i) attachments represent all objects without losing a retained joint field. Conditions (ii) and (iii) make every generated double-orbit identification a lawful comparison within the actual source domain. Condition (iv) makes every actual source identification a double-orbit identification. Thus the two equivalence relations coincide. The quotient in (5) follows. If one of the conditions is absent, the definition of the actual relative groupoid still applies, but one of the two implications used to identify its relation may fail. □

For a complete gluing problem, these hypotheses give an explicit orbit description of its joint sources. General color support, global bundle, Higgs attachment and physical-state data need not satisfy them uniformly. An automorphism of a gauge-law reduct, for example, need not lift through its fixed extended Yang–Mills attachment. The actual relative groupoid retains such restrictions and the resulting distinctions between sources; the double-orbit formula applies only where its full hypotheses hold.

Example 4.4 (Two independent failures of an ambient orbit quotient)**.** If two joint sources have the same attachment but different retained joint response values, attachment coverage holds while comparison reflection fails. If $A = \{a, b\}$ and an ambient marginal symmetry exchanges a, b but no such symmetry lifts to a joint source comparison, the ambient orbit set is a singleton while the actual relative groupoid has two components. These examples separately test loss of joint fields and invention of comparison arrows.

4.3 The observational quotient of relative attachments

Source equivalence and observational equivalence answer different questions. Two relative sources can remain distinct under lawful redescription while agreeing on every observation available at a particular target. Let $\mathcal{A}_{T,e,c}$ denote its actual unframed relative groupoid and let Σ_T be the full mixed signature from Section 3.

Proposition 4.5 (Relative response reconstruction)**.** *The signature induces a map*

$$\bar{\Sigma}_T : \pi_0(\mathcal{A}_{T,e,c}) \longrightarrow \operatorname{im} \Sigma_T.$$

The relative source class is recoverable from the response if and only if this map is injective. The relative response is recoverable from the two marginal response classes alone if and only if Σ_T is constant on every actual fibre of the marginal-response map.

Proof. Source-standing conservation proves constancy on source comparisons, so the first map is well-defined. Recoverability on its image is exactly injectivity. The second assertion is Theorem 3.4 restricted to the relative source domain. Neither criterion replaces a source class by a selected representative or enlarges the actual attachment domain. □

4.4 Simultaneous existence and incompatibility

Let $X^{\mathcal{R}}_{T,b}$ be the actual instances at one complete state b that realize a declared collection $\mathcal{R}$ of source, interaction, and projection requirements. Each requirement keeps its realization level. A local connection specifies geometric transport; a physical pole requires its analytic and spectral construction. Similarly, a neutral boundary construction supplies its stated boundary data, whereas complete interacting confinement requires the additional sector and continuation conditions of that stronger target.

Theorem 4.6 (Common-apex alternatives)**.** *An actual bosonic common apex at $(T, b, \mathcal{R})$ is precisely an element of $X^{\mathcal{R}}_{T,b}$. Any supplied element proves simultaneous occupancy of all its restrictions. A source-proved failure of one necessary compatibility condition excludes that specified candidate or branch. Failure to supply a required witness alone gives* NotReady*, not a physical nonexistence theorem. Separate marginal existence statements do not decide which alternative holds.*

Proof. Restriction of an actual joint instance retains its common state, relative maps and every requirement in $\mathcal{R}$. Conversely the apex is required to be that actual joint instance, not merely its marginal restrictions. A necessary condition cannot fail for an admitted instance, proving the exclusion statement. Absence of supplied evidence does not entail failure of the corresponding mathematical or physical existence predicate. Finally the implications $\exists b\, E(b)$ and $\exists b\, C(b)$ do not imply $\exists b\, [E(b) \wedge C(b)]$, still less the additional joint requirements. For example the two actual marginal images may be disjoint subsets of the common global-form or boundary-state space. □

The construction in Section 7 establishes a nonempty common source on a definite coupled classical branch. Stronger physical requirements define their own source domains whose occupancy must be established in turn. This distinction preserves both the positive classical result and the precise content of the remaining existence problem. We first examine the shared gauge and Higgs structure carried by the joint source.

5 The gauge product, one Higgs bridge, and residual structure

The global gauge structure survives electroweak reduction in a form that cannot be read from the local Lie algebra alone. A nonzero Higgs doublet leaves an electromagnetic direction whose period inherits the color-center identifications of the chosen global group. We determine its full stabilizer and relate it to the single Higgs source shared by the different response mechanisms.

5.1 The shared global source

On the admitted minimal chiral branch use integer hypercharge $q = 6Y$, color triality $t \in \mathbb{Z}/3$, and weak-center parity $s \in \mathbb{Z}/2$. The matter and single-doublet Higgs data are

	Q	u^c	d^c	L	e^c	H
t	1	-1	-1	0	0	0
s	1	0	0	1	0	1
q	1	-4	2	-3	6	3

with left-handed conjugate fields u^c, d^c, e^c. The superscript c specifies the conjugate representation while leaving its source ancestry unchanged. The primitive-lattice normalization and the full table come from the joint charge derivation, rather than from the color adjoint alone [3, Sec. 6 and App. A].

Put

$$G_0 = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_q, \qquad \xi = (\omega_3, -1, e^{i\pi/3}), \quad \omega_3 = e^{2\pi i/3}.$$

The common matter–Higgs kernel is $\langle\xi\rangle \cong \mathbb{Z}/6$. Within the retained connected central-quotient family,

$$G_n = G_0/\Gamma_n, \qquad \Gamma_n = \langle \xi^{6/n} \rangle, \qquad n \in \{1, 2, 3, 6\}. \tag{6}$$

All four groups have the same local Lie algebra. Their representation and line categories retain the additional global information and need not agree.

Proposition 5.1 (Joint descent and anomaly neutrality)**.** *Every representation in the table descends to every G_n. A center label (t, s, q) descends exactly when*

$$2t + 3s + q = 0 \pmod{n}.$$

The displayed chiral generation has vanishing local gauge and mixed gravitational hypercharge anomaly sums and even weak-doublet parity. Repeating that admitted generation preserves these cancellations. These conclusions concern the stated matter branch, not arbitrary extensions or global anomaly data absent from its source.

Proof. The generator of Γ_n acts by $\exp(2\pi i(2t + 3s + q)/n)$, giving the descent rule. Every table entry has $2t + 3s + q = 0 \pmod 6$, so it passes for every divisor n. For the Abelian anomaly coefficients, suppress common normalization factors. The color–color–Abelian and weak–weak–Abelian sums are

$$2(1) + (-4) + 2 = 0, \qquad 3(1) + (-3) = 0.$$

The mixed gravitational and cubic Abelian sums are respectively

$$6 - 12 + 6 - 6 + 6 = 0, \qquad 6 - 192 + 24 - 54 + 216 = 0.$$

The color cubic anomaly cancels between the two fundamental weak components of Q and the two antifundamentals u^c, d^c. The local weak cubic anomaly vanishes because the doublet generators have an anticommutator proportional to the identity and are traceless, so their symmetrized cubic trace is zero. Mixed sums with one traceless non-Abelian generator vanish. There are three colored weak doublets and one lepton doublet, hence even parity. The scalar Higgs is not a chiral fermion in these sums. Additivity gives the repeated-generation statement. The source's full anomaly and repair ledger remains necessary for any broader conclusion. □

The same joint charge structure governs the Yukawa contractions: $q_Q + q_H + q_{u^c} = 0$, $q_Q - q_H + q_{d^c} = 0$, and $q_L - q_H + q_{e^c} = 0$. These contractions use the same Higgs doublet. Charge neutrality is a necessary condition on the vertices; it does not fix their numerical coefficients.

Proposition 5.2 (Color neutralization preserves electroweak data)**.** *On one principal G_n-bundle, the color adjoint and all full matter and Higgs associated bundles are well-defined. A bare fundamental color factor is independently a G_n representation only for $n = 1, 2$. For $n = 3, 6$ an open color-factor presentation must retain its joint weak and hypercharge transitions, or an independently supplied lift. An admitted full intertwiner to a color-singlet output preserves its weak/hypercharge representation, which obeys $3s + q = 0 \pmod{n}$. It need not be electrically neutral.*

Proof. Central elements act trivially in the adjoint. The table representations descend by Proposition 5.1. A bare color fundamental has $(t, s, q) = (1, 0, 0)$, so descent requires $n \mid 2$. For full joint tensor factors, the color Haar projector commutes with the weak and Abelian action; its image inherits those actions. A full intertwiner carries the trivial Γ_n character to that image. With $t = 0$ the condition is exactly $3s + q = 0 \pmod{n}$, not $s = q = 0$. A merely color-equivariant map on a multiplicity space is not enough: the full intertwining condition is used in the last step. □

5.2 The global stabilizer of a nonzero doublet

The unbroken Lie algebra describes the infinitesimal response. The full stabilizer also determines which finite transformations are identified. We calculate it in the shared global family (6). Let the Higgs act as $H \mapsto z^3 g_2 H$, and fix $H_0 = (0, v)^T/\sqrt{2}$, $v > 0$. The real parameter a below uses electron-charge units in the infinitesimal electromagnetic generator. Its global period follows from the quotient calculation, rather than an initial assignment of 2π as the subgroup period.

Theorem 5.3 (Color-correlated electromagnetic stabilizer)**.** *The full stabilizer of H_0 in G_n is the image of*

$$\Psi_n : \mathrm{SU}(3) \times \mathbb{R} \longrightarrow G_n, \qquad (g, a) \longmapsto \left[g, \begin{pmatrix} e^{ia/2} & 0 \\ 0 & e^{-ia/2} \end{pmatrix}, e^{ia/6} \right].$$

Its kernel is the infinite cyclic subgroup generated by

$$(\omega_3^{6/n} I_3, 12\pi/n). \tag{7}$$

Consequently

$$\mathrm{Stab}_{G_n}(H_0) \cong (\mathrm{SU}(3) \times \mathbb{R}) / \langle (\omega_3^{6/n} I_3, 12\pi/n) \rangle.$$

Its Lie algebra is $\mathfrak{su}(3) \oplus \mathfrak{u}(1)_{\mathrm{EM}}$, but its global electromagnetic period and its color-center correlation are inherited from n. The Higgs reduction does not erase that datum.

Proof. The displayed matrix acts on the lower Higgs component by $e^{-ia/2}$, while $z^3 = e^{ia/2}$, so the image fixes H_0. Conversely if (g_3, g_2, z) fixes it, then $g_2 e_2 = z^{-3} e_2$. Unitarity and determinant one force $g_2 = \mathrm{diag}(z^3, z^{-3})$. Choose a with $z = e^{ia/6}$; this gives a preimage under Ψ_n. Since Γ_n acts trivially on the doublet, the same description passes to its quotient stabilizer.

A pair (g, a) maps to the identity in G_n exactly when its three components are $\xi^{6k/n}$ for some integer k. The Abelian equation gives $a = 12\pi k/n + 12\pi l$ for an integer l. The weak equation then holds, and the color equation gives $g = \omega_3^{6k/n} I_3$. Replacing k with $k + nl$ describes the same pair as a power of (7). Conversely every such power is in the kernel. The kernel is discrete, so the Lie algebra is the direct sum stated. The dependence of that discrete subgroup on n proves the global qualification. □

For example, at $n = 6$ a shift $a \mapsto a + 2\pi$ is correlated with the color-center generator. The pure electromagnetic path with color component fixed to the identity instead closes at 6π. At $n = 1$ its group-theoretic period is 12π, even though the given matter action is not faithful on the full residual group. These are distinct periodicity questions: one concerns the action on a chosen representation, the other the elements of the retained global group.

5.3 One bridge, distinct response incidences

The shared Higgs source appears at different input ports. It participates in matter-response incidences and electroweak fixation through different maps, each with its own remaining arguments. Keeping the source unique therefore leaves those incidences distinct.

Theorem 5.4 (One-Higgs coherence)**.** *The Higgs input ports of all integrated electroweak fixation, radial scalar, and admitted mixed matter response incidences use the single source bridge*

$$C_H = H_{\mathrm{br/fix}}, \qquad \beta_F : C_H \times D_F \rightsquigarrow Q_f^Y, \qquad \beta_{H,EW} : C_H \times D_{EW} \rightsquigarrow \mathsf{Fix}_{EW}.$$

Here the dependent domains retain all other fermionic, gauge, order, current and relative-incidence inputs in D_F, D_{EW}; the notation does not admit arbitrary products of them. The two incidence families remain distinct. On the color-singlet doublet branch, the infinitesimal color action on the Higgs vanishes, so there is no Higgs mass response in a color gauge direction from the doublet kinetic term. This neither forbids admitted induced Higgs–color interactions nor introduces another Higgs carrier.

Proof. The bridge is the fixed full canonical source tuple, including its bridge witness, order mediation and guard data, imported by BCRS–I and used without duplication by BCRS–IIEW/H. The actual joint source restrictions use that same tuple. Every corresponding finite network therefore uses that same source occurrence at its Higgs ports, while retaining every other input and the different operation graphs β_F and $\beta_{H,EW}$. A whole network is not a function of the Higgs bridge alone when its other inputs vary. Identifying the graphs would discard their different operand and output sorts, contrary to full source preservation.

For a color Lie algebra element X, color singletness gives $d\rho_H(X)H_0 = 0$. The kinetic response is the Gram form of the map $X \mapsto d\rho_H(X)H_0$, so all pairings with a color direction vanish. An induced operator, for example a source-admitted scalar multiple of a color curvature invariant, has a different operation graph and independent derivation. It does not alter the calculated doublet mass response or the bridge's identity. □

5.4 Vacuum, phases, and restoration

At $v > 0$ the preceding stabilizer and the local Hessian decomposition apply. At $v = 0$ the doublet fixation map loses rank and the restored branch has a different response decomposition. The two rank strata have their own domains; a result on the nonzero branch does not extend across that change without a further argument. Vacuum values, background curvature and phase data provide additional coordinates whenever the target observes them.

The global response data include the electric and magnetic lattices, their pairing, the admitted dyonic line branch, dynamical screening, theta monodromy, Yukawa phases, and boundary/background responses from the color analysis [3, Sec. 7]. Their transformation law is joint. In particular, whenever a fermionic rephasing changes both a theta coordinate and a Yukawa phase, both changes must be retained. The screened electric quotient, the phase law and physical confinement remain distinct properties: the first alone determines neither of the latter two.

Proposition 5.5 (Residual and phase atlas)**.** *Actual fixation, residual, restoration, and phase operations descend under complete joint source comparison and therefore belong to the mixed quotient. Their full domains, global correlations and rank strata are retained. A color-only mechanism has a typed absence of a Higgs fixation input unless an actual mixed incidence supplies one.*

Proof. For fixation use the one bridge and the full support/reduction graph from the predecessor residual law. For the displayed global branch, Theorem 5.3 transports the actual stabilizer rather than just its Lie algebra. For phases and screening, the source comparison transports the full lattices, pairings, dynamical subgroups and phase action; its inverse reflects their domains. Theorem 3.2 gives descent. Restoring $v = 0$ is a different domain/rank stratum, whose tag is an observed value. An absent Higgs input cannot appear in a color-only operation by quotienting, because exact domains are preserved. □

These global, Higgs and phase data specify the common setting for local interaction reconstruction. Within it, the different response directions can now be related to coefficients of one joint functional.

6 Reconstruction of the joint local interaction

With the common global bundle and Higgs incidence fixed, the bosonic response directions can be related to the coefficients of one local interaction. Its connection, current and Higgs maps link those coefficients, while its global and boundary data specify their domains of use. On the source-admitted minimal branch, complementary electroweak and color responses reconstruct the joint law. Both measurements refer to the same source instance, with the same independent inputs [2, Secs. 9–10][3, Sec. 2].

6.1 The common local source

Definition 6.1 (Joint normalized local branch)**.** Let $X_T^{\min} \subset X_T$ consist of the actual joint source instances whose local restriction has the following data.

1. There is one licensed smooth metric and differential envelope, one principal G_n-bundle, and the inherited honest matter and Higgs representations, with their actual transitions, boundary conditions and source attachments. Here

$$G_n = (SU(3)_c \times SU(2)_L \times U(1)_{6Y})/\Gamma_n, \qquad n \in \{1, 2, 3, 6\}.$$

The Higgs representation is the color-singlet one-doublet representation with $y_H = 1/2$. Its selected fixation has $H^\dagger H = v^2/2 > 0$. The normalized fixation reduction and its relative source incidences are fixed independently of the variable scale v, up to the actual joint presentation comparisons.

2. The local variable functional is the minimally coupled one-doublet gauge–Higgs law, with positive g_c, g_2, g_1, v, c, canonical electroweak and scalar kinetic normalizations, color pairing $\kappa_c(X, Y) = -\operatorname{tr}(XY)$, and

$$V(H) = a_0 + bs + cs^2, \qquad s = H^\dagger H, \qquad b = -cv^2.$$

The potential has a positive radial Hessian. Coupling signs and the normalized residual current are fixed through their actual common current attachment.

3. The independent matter, Yukawa, topological, boundary and background terms, their coefficients and their incident maps are retained as fixed sibling inputs. Their evaluated masses, currents or responses are not independently held fixed when a variable parameter changes. Each admitted term occurs once in the joint functional.

4. The target admits the inherited electroweak quadratic, normalized-current and radial response probes and a color quartic probe of the canonical color subfunctional. The latter

uses an actual admitted affine connection path $A_{\mathrm{ref}} + ta$ in a positive local-action or positive spatial-energy comparison, with

$$N_a = \langle a \wedge a, a \wedge a \rangle > 0.$$

Its metric, test form, normalization, source incidence and exact domain are fixed. It is not inferred from an arbitrary Lorentzian integral's sign.

A value-sensitive local target observes the potential value at fixation. A derivative-only local target instead identifies potentials differing by a constant, provided no other activated observation distinguishes that constant. These are different restrictions of the joint target.

The branch is a restriction of an independently defined source family; its actual image need not contain every positive coefficient tuple. The electroweak common-attachment construction retains the detailed response records and spectator data of one source instance. These data contain more information than its gauge-group name [2, Thm. 2.7 and Cor. 2.8]. The shared global bundle is essential. Its color adjoint always exists, whereas a bare color fundamental need not define an associated bundle for $n = 3, 6$. Actual quark representations retain their weak and hypercharge factors [3, Thm. 6.1 and Prop. 6.5].

Local-law comparisons evaluate the functional on the same complete field and current probes, or on their established source-preserving transports, over the same admitted domains. A marked connection configuration, state, holonomy or relative fixation attachment is an independent input whenever the broader target observes it. Coupling coordinates reconstruct the law acting on such inputs, not the inputs themselves.

In the formulas below the Lorentzian convention is $(- + ++)$. The positive quadratic response eigenvalues have the same normalization as the electroweak construction; the scalar kinetic sign is transported with the metric convention. Local calculations do not require a global unitary gauge.

Lemma 6.2 (Invariant kinetic separation)**.** *An invariant symmetric bilinear form on $\mathfrak{g} = \mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$ has zero cross-pairings between the three ideals. On a positive branch its restriction to either simple ideal is a positive multiple of its fixed invariant pairing. These statements descend through every shared central quotient G_n.*

Proof. If x, y belong to a simple ideal and z to a different ideal, invariance gives $B([x, y], z) = -B(y, [x, z]) = 0$. A simple ideal is its own commutator algebra, proving every such cross-pairing vanishes. This also treats its pairing with the central Abelian ideal. Relative to a fixed positive invariant pairing, an invariant symmetric form is represented by a self-adjoint endomorphism commuting with the adjoint action. Each eigenspace is therefore an invariant real subspace, hence an ideal. Simplicity leaves one eigenvalue, which is positive on the positive branch. Central quotients do not change the adjoint action or its invariant forms, proving descent. □

Invariant kinetic separation determines the block structure of the minimal law. Higher interactions between sectors remain possible on a larger operator domain. An additional Abelian factor or an admitted higher operator has its own comparison data; being nonprimitive does not make its coefficient unobservable [5, Secs. 15.2–15.4].

6.2 Joint supports before interaction coordinates

Write the real gauge coefficient space as $F = F_c \oplus F_{\mathrm{EW}}$, with dimensions eight and four. Transport each residual generator action through its actual coupling map, and retain the positive kinetic form. Since the Higgs is a color singlet, its action map and Gram response are

$$K_\phi(u_c, u_{\mathrm{EW}}) = K_\phi^{\mathrm{EW}}(u_{\mathrm{EW}}), \qquad \mu_\phi = 0_{F_c} \oplus \mu_\phi^{\mathrm{EW}}, \qquad M_\phi = \kappa^{-1}\mu_\phi.$$

The full connected effective residual action includes both color and the electromagnetic circle; its global lift remains the one specified by the common bundle.

Theorem 6.3 (Joint reducing-support decomposition)**.** *On the branch of Definition 6.1, the gauge response has rank three and*

$$\ker K_\phi = F_c \oplus E_\gamma, \qquad \dim \ker K_\phi = 9.$$

Every real gauge subspace invariant under the full residual action and M_ϕ is a sum of a subset of

$$E_c = F_c, \qquad E_\gamma, \qquad E_W, \qquad E_Z, \qquad \dim(E_c, E_\gamma, E_W, E_Z) = (8, 1, 2, 1).$$

These are reducing subspaces for the retained kinetic form and response. With the scalar normal quotient separately typed, the graded reducing-support lattice is Boolean on five atoms, the fifth being the one-real-dimensional radial support. All projectors and inclusions commute with lawful joint presentation transport and glue through the retained transitions wherever the nonzero fixation branch exists.

Proof. The color action on the Higgs vanishes identically. The electroweak map has a one-dimensional kernel and rank three by the accepted coupled-annihilator theorem [2, Thms. 4.1–4.2]. Kinetic block separation therefore gives the displayed joint rank and kernel.

Average the residual color action on F. It is trivial on F_{EW}, and its fixed subspace in the color adjoint is zero: a fixed element commutes with the entire simple color algebra. The average is consequently the orthogonal projector onto F_{EW}. It preserves every residual-invariant finite-dimensional subspace S, as does its complementary projector onto F_c. Thus $S = (S \cap F_c) \oplus (S \cap F_{\mathrm{EW}})$. Irreducibility of the real color adjoint makes the first intersection either zero or all of F_c [3, Thm. 2.1].

On the second intersection, average the residual circle to obtain its fixed-space projector P_0. The spectral projector P_γ onto the kernel of the electroweak response is a polynomial in its self-adjoint response operator, and therefore preserves $S \cap F_{\mathrm{EW}}$. These projectors commute, with $P_0 P_\gamma = P_\gamma$. The remaining projectors are

$$P_W = I - P_0, \qquad P_Z = P_0 - P_\gamma.$$

Their one-dimensional neutral images have only zero and full subspaces. The charged image is the irreducible real rotation plane of the residual circle, with the same property for invariant subspaces. This proves the gauge classification, including the converse for every subset sum. Orthogonal complements are also invariant because the residual actions are isometries and the response is self-adjoint. Scalar/gauge grading then adjoins the independent zero/full choice in the radial quotient [2, Thms. 5.4–5.6].

The constructions use the full transported action, metric, coupling map and fixation class. Their kernels, averages, spectral projectors and complements therefore commute with the actual comparison maps. On overlaps their restrictions obey the original cocycle, proving the asserted gluing. □

The classification uses residual action and Higgs response together. Color and the photon have zero Higgs response but different residual representations. The photon and neutral massive-response line, by contrast, are both residual-circle trivial; their distinct response eigenvalues separate the mixed lines that the residual action alone would allow. The charged plane remains distinct from the neutral line even when their positive response eigenvalues coincide. The five atoms therefore classify local reducing supports, not source laws, primitive carriers or physical particles.

6.3 One covariant functional and its mixed derivatives

Use an anti-Hermitian color connection A_c, with $F_c = dA_c + A_c \wedge A_c$, and canonical weak and hypercharge fields. The color curvature symbol here denotes a two-form, rather than the coefficient space used above. In local components put

$$\begin{aligned}
W^a_{\mu\nu} &= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu,\\
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu,\\
D_\mu H &= \partial_\mu H - i g_2 T^a W^a_\mu H - i g_1 Y B_\mu H.
\end{aligned}$$

For an admitted matter representation the derivative also contains $\rho_{c*}(A_c)$, together with its independently specified spacetime or spin connection where required. The different color and electroweak field normalizations are explicit: the color coupling is in its kinetic coefficient, while the weak and hypercharge couplings are in the canonical field's representation-valued connection. Rescaling a color field must transport its kinetic and current maps together.

The local density of the variable bosonic functional is

$$\begin{aligned}
\mathcal{L}_{\min} = &- \frac{1}{4g_c^2} \kappa_c(F_{c,\mu\nu}, F_c^{\mu\nu}) - \frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}\\
&- (D_\mu H)^\dagger D^\mu H - (a_0 + bH^\dagger H + c(H^\dagger H)^2).
\end{aligned}$$

The complete joint local functional also contains the retained independent terms $\mathcal{L}_{\text{fixed}}$ and any separate boundary functional. Here "fixed" means a fixed law with fixed independent coefficients and incidences, evaluated on the same variable fields and reconstructed parameters. It does not mean a fixed numerical output from another parameter point.

Proposition 6.4 (Global local-law reconstruction)**.** *The retained joint bundle, connection and independent coefficient laws, together with g_c, g_2, g_1, v, c, a_0, reconstruct the complete stipulated local functional and every admitted finite field variation, current insertion and contraction. They reconstruct its metric variations when the metric dependence and metric protocols of all terms are also retained. The construction commutes with actual source-preserving presentation comparisons, including comparisons fixing specified siblings and their incident maps.*

Proof. On every local frame the representation-valued connections obey

$$\mathcal{A}^u = u\mathcal{A}u^{-1} - (du)u^{-1}, \qquad F_{\mathcal{A}^u} = uF_{\mathcal{A}}u^{-1}, \qquad D_{\mathcal{A}^u}(u\psi) = uD_{\mathcal{A}}\psi.$$

The inhomogeneous derivative terms cancel by differentiating $u^{-1}u = I$. The same calculation applies to the honest joint associated representations; it does not require separate global fundamental factor bundles. Invariant pairings and the Higgs norm remove the remaining frame actions. Thus the displayed densities agree on overlaps under the retained transition maps. The independently attached terms and boundary functional use those same maps and their stated source laws, so they glue as well. The underlying derivative grammar is the connection grammar of admitted smooth finite transport, not a new choice of coupling operation [6, Thm. 19.1].

Substitution of the coefficients reconstructs each term once. Differentiating this same functional gives all mixed multilinear vertices with their common coefficients; the color cubic and quartic relations are precisely those of its retained curvature law [3, Thm. 2.3]. Successive differentiation and contraction are therefore operations on one reconstructed law. On a nontrivial bundle use an affine connection path about an actual reference connection; the separate bare expressions dA_c and $A_c \wedge A_c$ are not independently asserted to be global tensors.

A lawful comparison transports the complete local law, current maps and their prolonged jet-coordinate changes. The chain rule gives commuting vertex and incidence squares. At every partial operation its actual source domain is transported and reflected. Induction over a finite composition preserves these domains and outputs. Fixing a sibling means using its identity comparison and the corresponding certified incident squares, not choosing a new relative map after comparing marginals [2, Thm. 9.7]. The same argument applies to metric variations when their complete dependence and source protocol domain are included. □

The doublet realizes the structural Higgs incidence through the displayed local law. Its color-singlet kinetic term contains no direct color connection, while matter currents, Yukawa insertions and their admitted continuations can connect Higgs and color responses. A physical scalar channel is a further realization of these interactions: the absence of one bare vertex does not establish that a physical Higgs-to-color channel vanishes.

6.4 An explicit integrated response chart

The electroweak quadratic, current and radial responses, together with a color quartic response, determine the variable interaction coefficients. When the potential value is observed, it supplies a sixth coordinate. With the independent geometry and probe attachments fixed, define these invariant local responses by

$$
\begin{aligned}
w &= \frac{g_2^2 v^2}{4}, \quad & z &= \frac{(g_1^2+g_2^2)v^2}{4}, \quad & q_e &= \frac{g_1^2 g_2^2}{g_1^2+g_2^2},\\
h_r &= 2cv^2, \quad & r_c &= \frac{N_a}{2g_c^2}, \quad & \ell &= a_0 - \frac{cv^4}{4}.
\end{aligned}
$$

The first, second and fourth quantities are quadratic response eigenvalues, not asserted physical pole masses. The third is the squared normalized electromagnetic current coefficient. The fifth is the admitted color quartic response, and the last is the potential value at fixation. All are measured in the retained units and source-attached normalization.

Theorem 6.5 (Constructive joint local-response chart)**.** *For the value-sensitive elementary coefficient family, the six responses give a bijection onto*

$$
\Omega_6^{\mathrm{loc}} = \{(w,z,q_e,h_r,r_c,\ell) : 0<w<z,\ q_e,h_r,r_c>0,\ \ell\in\mathbb{R}\}.
$$

Its inverse is

$$
\begin{aligned}
g_2^2 &= \frac{q_e z}{z-w}, \quad g_1^2 = \frac{q_e z}{w}, \quad v^2 = \frac{4w(z-w)}{q_e z}, \quad g_c^2 = \frac{N_a}{2r_c},\\
c &= \frac{h_r}{2v^2}, \qquad b = -cv^2, \quad a_0 = \ell + \frac{cv^4}{4}.
\end{aligned}
$$

Positive square roots fix the stipulated coupling convention. For the derivative-only local target, omitting ℓ gives the five-coordinate version Ω_5^{loc}, with the potential taken modulo its additive constant.

The following actual-image statement is fibrewise over fixed metric and bundle data, normalized fixation reduction, current and test-probe attachments, operation domains, and all independent sibling laws. If any such independent datum varies, it is an additional coordinate of the reconstruction datum, not an unrecorded part of the six-coordinate chart. On this actual source fibre, let

$$
\Omega_T^{\min} = \{\iota(x) : x\in X_T^{\min}\}, \qquad \iota(x) = (w,z,q_e,h_r,r_c,\ell)(x).
$$

With the fixed independent sibling data retained, equality of these coordinates is equivalent to equality of all admitted local response observations of this stipulated law. Every such observation factors uniquely through $\Omega_T^{\min}$. This is the local observation quotient, not an identification of broader source, marked-configuration, spectral or boundary distinctions.

Proof. The electroweak response gives $z - w = g_1^2 v^2/4 > 0$. Substitution into the definition of q_e gives the two squared gauge couplings in the inverse; the formula for w then gives v^2. The radial response fixes c, stationarity fixes b, and the value at fixation fixes a_0. These are the accepted constructive electroweak relations [2, Thm. 10.2].

For the color probe, curvature along the actual affine path is

$$F_{A_{\mathrm{ref}}+ta} = F_{A_{\mathrm{ref}}} + t d_{A_{\mathrm{ref}}} a + t^2 a \wedge a.$$

Its squared-curvature functional therefore has fourth coefficient $N_a/(2g_c^2)$, independently of the reference curvature. Solving gives the displayed color inverse [3, Prop. 2.4]. The source admits this subfunctional probe; a quartic coefficient of a different total functional is not substituted without its own separation argument.

Conversely, every point of Ω_6^{loc} gives positive squared couplings, $v^2 > 0$ and $c > 0$. Inserting these recovered values into all six forward expressions returns the original point. The recovered potential has its minimum at $s = v^2/2$. Thus both compositions are identities on the elementary coefficient family. Omitting the constant changes only its declared scalar-value quotient.

Equal response coordinates at two actual source points reconstruct the same coefficient law. The fixed independent sibling laws, attachments and exact local operation domains then give all equal local responses by Proposition 6.4. Conversely, each retained coordinate is one of this target's admitted observations, so complete local observational equality implies coordinate equality. Define the factored value at $\iota(x)$ to be the original observation of x. The proved implication makes it well defined, and surjectivity onto the actual image makes it unique. No positive tuple was thereby promoted to a native source instance. □

The inverse reconstructs the whole variable local interaction, including its linked vertices, while allowing its response coordinates to vary. It does not select a numerical point. A topological term, Yukawa tensor, relative current map or boundary law that varies in a broader target remains an additional argument of that reconstruction.

Proposition 6.6 (Irredundancy and the actual image)**.** *On the elementary six-coordinate family, no one coordinate is recoverable from the other five. The analogous statement holds for the five-coordinate derivative-only family. On an arbitrary admitted image Ω_T^{min}, deletion of a coordinate is lossless exactly when that coordinate is a function of the retained ones on that image, provided no duplicate pointed datum already retains it.*

Proof. At any point of Ω_6^{loc}, independently replace one coordinate by

$$w/2, \quad z + w, \quad 2q_e, \quad 2h_r, \quad 2r_c, \quad \ell + 1,$$

respectively, leaving the others fixed. Each resulting point satisfies the same strict inequalities. The inverse gives two elementary coefficient laws with the same proposed retained data and a different deleted observation. No function of the retained data can recover both values. The first five pairs also prove the derivative-only claim.

For the actual image, a recovery function exists only if the deleted coordinate is constant on every fibre of the retained-coordinate map. If it is constant, that common value defines the recovery function; the full inverse then reconstructs all local observations. This argument uses only pairs in the actual image. It does not assert that the displayed elementary mutations preserve every source or physical admission condition. □

6.5 Vacuum, metric and global information

Proposition 6.7 (The metric-visible vacuum combination)**.** *Suppose the target admits the compact metric first variations of the joint local functional, with its independent background* *terms fixed. Then an additive potential shift $\delta\ell$ is detected by those responses: its Hilbert source shift is*

$$\delta T_{\mu\nu} = -\delta\ell\, g_{\mu\nu}.$$

Thus the value-sensitive chart, rather than the derivative-only chart, applies to this local metric response. If instead the target observes only a total vacuum contribution and admits an independent compensating constant λ, the observable coordinate is $\ell + \lambda$; separate ℓ is not fixed by that total response alone.

Proof. The constant contribution is $S_\ell = -\ell \int d\mathrm{vol}_g$. Its compact metric variation, with $k_{\mu\nu} = \delta g_{\mu\nu}$, is

$$\delta S_\ell(k) = -\frac{\ell}{2} \int g^{\mu\nu} k_{\mu\nu}\, d\mathrm{vol}_g.$$

This is the source convention $\delta S = \frac{1}{2} \int T^{\mu\nu} k_{\mu\nu} d\mathrm{vol}_g$, giving the displayed tensor. Choose $k = fg$, with compactly supported smooth f of nonzero integral. In four dimensions the response difference is $-2\delta\ell \int f\, d\mathrm{vol}_g$, which is nonzero when $\delta\ell \neq 0$. Small such variations preserve the Lorentzian signature on their compact support. Admitted complete metric protocols make this an actual separating response [3, Thms. 11.2 and 11.4].

If the only observed constant contribution is $-\int(\ell + \lambda) d\mathrm{vol}_g$, every one of these responses depends on the sum. Whenever both are actual admitted points, the replacements $(\ell, \lambda) \mapsto (\ell + t, \lambda - t)$ preserve that entire contribution. Conversely its compact trace probe determines the sum. Additional separate observations could distinguish the two terms, but total vacuum response alone cannot. ☐

A gauge topological term likewise can have zero compact bulk Euler derivative while retaining global, line or boundary consequences. Its shared-group periods and phase comparisons remain part of the global color construction, and boundary functionals remain attached wherever they are observed. Such information neither introduces a new primitive carrier nor disappears with the bulk Euler derivative. The complete local and boundary response target therefore retains more than a field-equation quotient.

6.6 Mixed operators beyond the minimal branch

The minimal chart reconstructs the stipulated variable law with its independent siblings fixed. Gauge covariance alone allows a larger operator family, as the electroweak local-law definition already recognizes for scalar–curvature couplings [2, Def. 10.1].

Theorem 6.8 (Complete mixed polynomial reconstruction)**.** *Fix a source-admitted differential-polynomial local law of finite jet order and field degree at most d, on the actual joint gauge, Higgs and any additional admitted field-jet bundle. At a marked joint jet j_0, retain every mixed coefficient tensor $D^k P(j_0)$, $0 \le k \le d$, as a section over the full base, with its source maps, transition functions and exact operation domains. These tensors reconstruct the complete law and all its admitted finite local operations. Separate sector-marginal jets need not do so. If metric responses are included, the full stipulated metric dependence of the coefficient laws must also be retained or independently reconstructed; their values at one metric alone are not sufficient.*

Proof. In an affine joint jet chart, finite Taylor reconstruction gives exactly

$$P(j_0 + \eta) = \sum_{k=0}^{d} \frac{1}{k!} D^k P(j_0)[\eta, \ldots, \eta].$$

The tensors contain every choice of gauge, Higgs and other field arguments, including repeated and mixed choices. They therefore recover every monomial coefficient of the entire polynomial, not merely its restrictions to sector axes. Coefficient sections retained over the base also determine their admitted base derivatives. On overlaps the recovered expressions transform by the actual prolonged jet-coordinate maps and hence glue to the original law. Differentiation, polarization, contraction and composition then recover its finite operations, with the original source domains. This is the full joint-jet application of the accepted polynomial theorem [2, Thm. 9.4].

For the marginal limitation, the polynomial $P(x, y) = \alpha xy$ vanishes on each coordinate axis but has mixed second derivative α. Thus even exact laws on both marginal axes fail to recover the joint law. Finally, the Taylor identity concerns the specified polynomial variables. A coefficient's unrecorded metric dependence cannot be differentiated from its single fixed-metric value. Retaining its complete prescribed dependence makes that further operation reconstructible as well. □

Two explicit tests show why these scope requirements matter. Enlarge the local operator family by

$$\alpha(s - v^2/2)\kappa_c(F_{c,\mu\nu}, F_c^{\mu\nu}).$$

At zero color background and the original fixation, this term leaves the quadratic response and pure color quartic probe at fixed $H = \phi$ unchanged. Nevertheless its mixed radial–two-color derivative is nonzero for suitable inputs when $\alpha \neq 0$: the coefficient of $t\tau^2$, for $H = \phi + t\eta$ and $A_c = \tau a$, is

$$\alpha\, ds_\phi(\eta)\, \kappa_c(da, da).$$

The exact mixed derivative is twice this coefficient. It is additional local information in the enlarged family, not evidence for a new primitive color or Higgs carrier. The comparison becomes a separating pair in the source domain only when the target admits both laws and the indicated probe.

A scalar–curvature term $\xi R(g)s$ gives a second test. It vanishes for all field inputs at a fixed flat metric, yet need not have zero metric response there. For a compact conformal variation $k = fg$ of a four-dimensional flat metric, linearizing scalar curvature gives $\delta R = \partial_\mu\partial_\nu k^{\mu\nu} - \Box \operatorname{tr}_g k = -3\Box f$. Consequently

$$\delta \int \xi R s\, d\mathrm{vol}_g = -3\xi \int s \Box f\, d\mathrm{vol}_g = -3\xi \int f \Box s\, d\mathrm{vol}_g,$$

which separates appropriate nonconstant scalar profiles. Compact support justifies the integrations by parts. Connection covariance does not force $\xi = 0$; an enlarged metric-responsive target must retain its actual coefficient law.

The local reconstruction is complete on its declared source image and extends constructively to larger admitted polynomial families. It determines the generating local input for analytic responses, renormalized operators, physical poles and asymptotic or composite channels; those constructions also retain their state, continuation, boundary and existence data. Before turning to those projections, we establish a simultaneous classical configuration on which the coupled gauge, Higgs and gravitational laws act.

7 A simultaneous classical gauge–Higgs realization

The fixed-background local inverse determines how the coupled fields respond. A simultaneous classical realization additionally requires one configuration satisfying their common gravitational constraints. The classical occupancy theorem of the Wheeler–DeWitt matter construction, WDW–4, provides such a history with color, weak, hypercharge, and Higgs fields present together [4, Theorems W4B.9–W4B.12 and W4.I.1]. Its massive scalar clock remains part of the construction. We exhibit the nonzero gauge–Higgs seed, the exact correction of the coupled constraints, and the covariant development of that same datum, then identify the simultaneous sector restrictions that enter the bosonic atlas.

7.1 The fixed classical branch and its common action

Definition 7.1 (Classical source branch)**.** Fix the WDW–4 branch consisting of a compact, connected, boundaryless, oriented hyperbolic three-manifold Σ, one ordinary spin structure, and the nonvacuum real massive scalar-clock subsystem of WDW–4A. Gravity is second-order metric gravity in a nondegenerate coframe presentation; the spin connection is the derived Levi–Civita connection $\omega_{\mathrm{LC}}(e)$, with no independent torsion or connection variable. Fix

$$G_{\mathrm{SM}}^{(6)} = (\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y)/\mathbb{Z}_6$$

on the trivial principal bundle, with no external line insertion. The matter carrier has three minimal chiral generations and no right-handed neutrino. Its Higgs field is one doublet $H \in (\mathbf{1}, \mathbf{2})_{1/2}$, with

$$V(H) = \lambda\big(H^\dagger H - v^2/2\big)^2, \qquad \lambda > 0, \quad v > 0.$$

The positive gauge couplings, Yukawa matrices, and local theta branch are fixed target data. Analytic occupation is on the even body of the graded carrier, with zero fermion fields. The Sobolev base and compatible smooth-tame scales are those of WDW–4, with $q_0 > 3/2 + 4$ and coefficient loss two.

The action and target values are specified physical data; the construction does not derive their numerical values. Its global form, bundle, spin structure, and clock are fixed parts of source identity. Other global forms or topological sectors are outside this chosen branch, not excluded by its existence theorem.

The hypercharge label in this branch follows the source's conventional notation. The global circle retains the integer normalization and central identification specified above. The action below also uses the source's canonical gauge fields, whereas the preceding local inverse places the color coupling in its kinetic coefficient. Comparison of the two conventions transports the field, kinetic and current normalizations together.

Write $A = (A_3, A_2, B)$ for the internal connections and Ψ for the graded fermion carrier. Their common covariant action is

$$S_{\mathrm{joint}} = S_{g\phi} - \frac{1}{4}\sum_{s=3,2,Y}\int F_s^{\mu\nu}\cdot F_{s\mu\nu}\, d\mu_g - \int \big((D_\mu H)^\dagger D^\mu H + V(H)\big)\, d\mu_g + S_{\mathrm{Weyl}} + S_{\mathrm{Yukawa}}, \tag{8}$$

where $S_{g\phi}$ is the fixed gravity–scalar action of the occupied WDW–4A branch. In particular, no new cosmological offset is inserted into that branch. Each derivative, current, stress tensor, and canonical constraint used below comes from this same action and its actual history [4, Theorems W4.2 and W4B.4–W4B.7].

Proposition 7.2 (Covariant and canonical source agreement)**.** *Use signature $(-+++)$, a future unit normal n, and spatial tangent vectors e_i. For the total matter Hilbert tensor set*

$$\rho = T_{\mu\nu}n^\mu n^\nu, \qquad j_i = -T_{\mu\nu}n^\mu e_i^\nu.$$

If the canonical action uses $-N\mathcal{H}_\perp - N^i\mathcal{H}_i$, then its covariant source projections satisfy

$$\mathcal{H}_\perp^{\mathrm{matter}} = \sqrt{h}\,\rho, \qquad \mathcal{H}_i^{\mathrm{matter}} = -\sqrt{h}\,j_i. \tag{9}$$

All sector currents and stress exchanges are restrictions of one variational identity. On the coupled field equations their sum obeys $\nabla_\mu T^{\mu\nu} = 0$.

Proof. With $T^{\mu\nu}$ defined by $\delta S_{\text{matter}} = \frac{1}{2}\int \sqrt{-g}\, T^{\mu\nu}\delta g_{\mu\nu}$, vary lapse and shift at fixed spatial metric and matter variables. The normal decomposition gives

$$\frac{\delta S_{\text{matter}}}{\delta N} = -\sqrt{h}\,\rho, \qquad \frac{\delta S_{\text{matter}}}{\delta N^i} = \sqrt{h}\, j_i.$$

Comparison with the stated canonical convention proves (9). For example the scalar momentum $p_\phi = \sqrt{h}\, n(\phi)$ gives $\mathcal{H}_i^\phi = p_\phi D_i\phi$ and $j_i^\phi = -p_\phi D_i\phi/\sqrt{h}$, fixing the sign explicitly. The same Legendre splitting of (8) gives the gauge and Higgs momentum densities.

Diffeomorphism invariance of the complete matter action yields its Noether identity. Terms proportional to the scalar, Higgs, gauge, and fermion Euler–Lagrange expressions vanish on a coupled history. The gauge-force and Yukawa exchange terms cancel between their respective sectors because the currents and interaction terms are variations of the same action. The remaining identity is the stated total conservation law. No independent sector stress assignment is needed. □

Equation (9) uses the displayed definition of j_i and the canonical shift convention throughout this section. The Higgs field is color neutral. At $\Psi = 0$ it has no direct color current, yet color and Higgs contribute to the same metric source and gravitational constraints. Their common dynamical realization therefore does not require a direct color–Higgs vertex.

7.2 A nonzero gauge–Higgs seed

The construction begins with an occupied gravity–scalar datum. Nonzero gauge and Higgs fields are then added, and the shared constraints determine a correction before the history is developed. Choose the sectional-curvature -1 metric h_0 on Σ, fix $\epsilon > 0$, and put

$$c_\epsilon = \sqrt{1+\kappa\epsilon^2/6}, \quad \phi_0 = 0, \quad p_{\phi,0} = \epsilon\sqrt{h_0}, \quad K_{0,ij} = c_\epsilon h_{0,ij}, \quad \pi_0^{ij} = -\frac{c_\epsilon\sqrt{h_0}}{\kappa} h_0^{ij}. \tag{10}$$

The scalar density is $\rho_\phi = \epsilon^2/2$ and its momentum density vanishes. Consequently

$$R(h_0) + K_0^2 - K_{0,ij}K_0^{ij} = -6 + 6c_\epsilon^2 = \kappa\epsilon^2 = 2\kappa\rho_\phi,$$

and the momentum constraint also vanishes. This is a nonvacuum datum although ϕ_0 itself vanishes on the initial slice. WDW–4 proves the compatible smooth-tame constraint split needed for the exact correction [4, Theorems W4.3–W4.4]. The absence of a generalized Killing initial datum has a particularly transparent source: preservation of ϕ_0 requires $N\epsilon = 0$, so the lapse parameter vanishes; the remaining metric preservation requires a Killing vector of h_0. Since $\mathrm{Ric}(h_0) = -2h_0$, the integrated Killing-field identity forces that vector to vanish. The nonzero clock is essential to this argument.

Lemma 7.3 (Simultaneously nonzero gauge and Higgs data)**.** *There are smooth connection one-forms $a^{(3)}, a^{(2)}, a_Y$ and a smooth nonconstant real function f such that, for all sufficiently small nonzero δ,*

$$A_\delta = (\delta a^{(3)}, \delta a^{(2)}, \delta a_Y), \qquad H_\delta = (1+\delta f)H_{\text{vac}}, \qquad H_{\text{vac}} = (0, v/\sqrt{2})^{\mathsf{T}}, \tag{11}$$

has nonzero curvature in each gauge factor, nowhere-zero Higgs amplitude of nonconstant norm, and trivial joint infinitesimal internal stabilizer.

Proof. For $\mathfrak{su}(n)$, $n = 2, 3$, choose a basis X_k and disjoint coordinate balls. In the kth ball choose a compactly supported one-form b_k with $b_k(x_k) = 0$ and $db_k(x_k) \neq 0$, and set $a^{(n)} = \sum_k X_k b_k$. At x_k,

$$F_{\delta a^{(n)}}(x_k) = \delta X_k\, db_k(x_k).$$

If $D_{A_\delta}\alpha = 0$, integrability implies $[F_{A_\delta}, \alpha] = 0$. Transport the finitely many displayed curvature values along fixed paths to one base point. Divided by δ, suitable nonzero components converge to a basis as $\delta \to 0$. The linear map $X \mapsto ([X_1, X], \ldots, [X_{n^2-1}, X])$ is injective because $\mathfrak{su}(n)$ has zero center. Injectivity is open in finite dimensions, so the transported values have zero common centralizer for small nonzero δ. Thus both semisimple components of α vanish.

Choose a_Y with $da_Y \not\equiv 0$. The remaining parallel Abelian parameter is constant on connected Σ. Its action on the Higgs field is $ig_1 Y_H \alpha_Y H_\delta$. Since $g_1 > 0$, $Y_H = 1/2$, and H_δ is nowhere zero, that parameter also vanishes. Finally compactness permits $1 + \delta f > 0$ for all sufficiently small $|\delta|$. Its nonconstant norm follows from nonconstancy of f. This is a specific nonzero choice of the Higgs direction in the finite curvature-centralizer construction of [4, Lemmas 30.1–30.2]. □

7.3 Exact coupled constraint correction

Set the gauge electric momenta, Higgs momenta, and fermions to zero:

$$E_\delta = 0, \qquad \Pi_{H,\delta} = \Pi_{H^\dagger,\delta} = 0, \qquad \Psi_\delta = 0. \tag{12}$$

The added gauge–Higgs and fermion fields then have zero internal charge and spatial-momentum densities. The scalar clock retains its nonzero momentum, and its corrected spatial contribution need not vanish separately. The fermionic second-class matrix remains the nondegenerate matrix of the full graded carrier; a zero solution coordinate does not remove the carrier. For any corrected metric momentum choose $P_I^i = 2\pi^{ij} e_{jI}$. Its Lorentz moment map vanishes and $P_I^i \delta e_i^I = \pi^{ij}\delta h_{ij}$, so this is the actual cotangent lift of the same metric datum.

Let $\Phi_{g\phi}$ be the gravity–scalar source's constraint map, z_0 the datum (10), and R_0 its compatible right inverse. On the chosen correction complement $X_0 = \operatorname{im} R_0$, form

$$\mathcal{F}(u, \delta) = \Phi_{g\phi}(z_0 + u) + S_{GH}(z_0 + u; A_\delta, H_\delta). \tag{13}$$

Here S_{GH} is the gauge–Higgs contribution to the normal and spatial constraints of (8) with (12).

Proposition 7.4 (Exact populated constraint lift)**.** *For sufficiently small $|\delta|$, there is a unique local smooth-tame correction curve $u_\delta \in X_0$, with $u_0 = 0$ and $u_\delta = O(\delta^2)$ in each fixed lower norm, satisfying $\mathcal{F}(u_\delta, \delta) = 0$. For nonzero δ the resulting datum w_δ is an exact zero of the full normal, spatial, internal Gauss, Lorentz, and primary constraints.*

Proof. At $\delta = 0$, the connection is zero, $DH_{\mathrm{vac}} = 0$, and $V(H_{\mathrm{vac}}) = 0$. Therefore

$$\mathcal{F}(0,0) = 0, \qquad D_u\mathcal{F}(0,0) = D\Phi_{g\phi}(z_0)|_{X_0},$$

and the latter has inverse R_0. Gauge curvature, covariant Higgs derivative, and Higgs displacement are first order in δ; their energy contribution starts at second order. The exact one-high-norm product estimates and the compatible inverse family with coefficient loss two are proved for this map in [4, §32, equations (109)–(110)]. Its parametric smooth-tame inverse theorem thus gives the stated curve and exact equation at every Sobolev level. Smooth data yield a smooth curve, and differentiation at zero gives $u_0' = 0$.

That equation solves the total normal and spatial constraints. The vanishing momenta in (12) solve all internal Gauss and matter primary constraints. The symmetric coframe lift solves the Lorentz constraint. These are components of one constraint map; hence w_δ solves them simultaneously. □

The uniqueness in Proposition 7.4 concerns the specified seed and the chosen complement X_0. It does not identify different choices of seed, parameters, or initial history. Nor is the corrected gravity–scalar projection a separate zero of $\Phi_{g\phi}$: its source is now $-S_{GH}(z_0 + u_\delta; A_\delta, H_\delta)$.

Corollary 7.5 (An explicit fixed-sibling obstruction)**.** *Appending the nonzero seed* (11) *to the unchanged gravity–scalar datum z_0, with* (12)*, does not solve the total constraint.*

Proof. The normal source added on the unchanged metric is

$$\sqrt{h_0}\left(\frac{1}{4}\sum_{s=3,2,Y}|F_{s,\delta}|^2_{h_0}+|DH_\delta|^2_{h_0}+V(H_\delta)\right).$$

Each summand is nonnegative and at least one gauge-curvature term is nonzero somewhere. This density is therefore not identically zero, whereas $\Phi_{g\phi}(z_0)=0$. The normal component of (13) at $u=0$ cannot vanish. □

The shared metric has a direct dynamical role: the added positive gauge–Higgs energy changes the normal constraint, and the coupled correction restores it. Compatibility of the sector labels alone cannot keep the original gravitational datum unchanged.

7.4 The mixed inverse and local covariant history

The regular constraint structure also depends on a mixed response. The inverse below compensates the gauge–Higgs momentum variation in the gravitational rows, so the same constraint system is solved throughout the local neighborhood.

Proposition 7.6 (Joint constraint split)**.** *At each sufficiently small nonzero occupied datum w_δ, the full even constraint derivative has a compatible smooth-tame right inverse with coefficient loss two. Its local regular quotient is nonempty.*

Proof. In the gauge-electric and Higgs-momentum columns write

$$K_\delta(\dot{E},\dot{\Pi})=D_{A_\delta,i}\dot{E}^i+\mu_H(H_\delta,\dot{\Pi}).$$

With the fixed invariant inner products its formal adjoint obeys

$$\langle\alpha,K_\delta K_\delta^{\#}\alpha\rangle=\|D_{A_\delta}\alpha\|^2_{L^2}+\|\rho_H(\alpha)H_\delta\|^2_{L^2}.$$

Lemma 7.3 makes the kernel zero. On compact Σ this positive self-adjoint elliptic normal operator is invertible, giving $Q_G=K_\delta^{\#}(K_\delta K_\delta^{\#})^{-1}$ and $K_\delta Q_G=1$.

Let A_δ^{lin} be the derivative of the total normal–spatial constraint in the gravity–scalar columns, and J_e^{lin} the Lorentz derivative in vertical coframe momentum. Their right inverses are R_δ and S_e. The remaining momentum column is

$$B_\delta(\dot{E},\dot{\Pi})=\left(0,\ \sum_s\dot{E}_s^j\cdot F_{s,ij}+\dot{\Pi}_HD_iH_\delta+(D_iH_\delta)^\dagger\dot{\Pi}_{H^\dagger}\right). \tag{14}$$

The normal row is zero because the energy is quadratic in these momenta at (12); the spatial row is not discarded. The selected derivative and its inverse are

$$D\Phi_{\mathrm{joint}}=\begin{pmatrix}A_\delta^{\mathrm{lin}} & B_\delta & 0\\ 0 & K_\delta & 0\\ 0 & 0 & J_e^{\mathrm{lin}}\end{pmatrix},\qquad Q_\delta=\begin{pmatrix}R_\delta & -R_\delta B_\delta Q_G & 0\\ 0 & Q_G & 0\\ 0 & 0 & S_e\end{pmatrix}. \tag{15}$$

Multiplication gives the identity. In particular its upper off-diagonal block is $-A_\delta^{\mathrm{lin}}R_\delta B_\delta Q_G+B_\delta Q_G=0$. The product estimates and compatible neighborhood inverse proved in [4, §§35–38, Theorem W4B.10] retain precisely this mixed term and loss two. They give a regular constraint chart with vanishing joint adjoint kernel and the proper local slice quotient. □

The exact common-domain Dirac algebra is an independent part of the same source result. If $V[\xi]$ denotes the raw covariant spatial generator and $D[\xi] = V[\xi] - G[\iota_\xi A] - J[\iota_\xi \omega_{\mathrm{LC}}]$, then [4, Theorem W4B.7] gives

$$\{H[N], H[M]\}_D = V[s] = D[s] + G[\iota_s A] + J[\iota_s \omega_{\mathrm{LC}}], \qquad s^i = h^{ij}(N\partial_j M - M\partial_j N). \tag{16}$$

The improved spatial expression alone is obtained only after the Gauss and Lorentz constraints are imposed. This structure-function identity is not replaced by an assertion that sector brackets vanish independently.

Theorem 7.7 (Occupied simultaneous classical history)**.** *On the branch of Definition 7.1, sufficiently small nonzero δ gives a nonempty pointed covariant-history class*

$$a_\delta = [\mathrm{Dev}_{\mathrm{EYMH}\phi}(w_\delta; \Psi = 0)]_{\cong_{\iota_\delta}}. \tag{17}$$

Its gauge and Higgs restrictions are simultaneous restrictions of one source-attached solution. It has nonzero curvature in all three gauge factors, a nonconstant Higgs norm on the initial slice, the regular joint constraint structure, and a nonempty local scalar-clock patch. For fixed corrected initial datum the development is unique in the stated pointed local equivalence.

Proof. Proposition 7.4 supplies smooth exact constraint data. The specialized hyperbolic Einstein–Yang–Mills– Higgs–scalar Cauchy theorem used in [4, Theorems W4B.9 and W4.I.1], with constraint propagation, provides the covariant development and its geometric uniqueness. The fermion equations are homogeneous in Ψ, including Yukawa mixing, so their zero initial data remain zero. This is an actual solution of the coupled equations, not an independent choice of sector solutions.

The seed has $g^{-1}(d\phi, d\phi) = -\epsilon^2 < 0$ and $n(\phi) = \epsilon > 0$. The correction is $O(\delta^2)$ and the development depends continuously on the smooth data. These strict inequalities therefore persist on a nonempty local neighborhood. There

$$V_\phi = \frac{\nabla\phi}{g^{-1}(d\phi, d\phi)}, \qquad V_\phi(\phi) = 1$$

is future-directed. Covariance makes these conditions invariant under the proper local redescriptions. The regular quotient of Proposition 7.6 thus retains the clock patch and its failure boundary, as proved in [4, Theorem W4B.11]. The inherited equivalence in (17) fixes the initial embedding and uses the proper Lorentz and internal gauge transformations of the fixed branch. □

Corollary 7.8 (Occupied pointwise bosonic response geometry)**.** *On a sufficiently small neighborhood of the occupied initial slice, the actual nowhere-zero Higgs section supplies a smooth reduction to its color-correlated electromagnetic stabilizer. Pointwise the Higgs gauge-action map has rank three and kernel $E_c \oplus E_\gamma$, of dimension nine. Its full residual action and algebraic kinetic Gram response give the smooth reducing gauge subbundles of ranks* $(8, 1, 2, 1)$*.*

Proof. Compactness and continuous evolution preserve the positive lower bound on the initial Higgs norm on a sufficiently small time neighborhood. Normalize this nonzero section locally and apply the coupled-annihilator calculation with its actual pointwise norm in place of the constant fixation scale. Color singletness gives the eight-dimensional color kernel and the nonzero doublet gives the remaining one-dimensional electromagnetic kernel. The residual color and circle averages and the spectral projectors used in Theorem 6.3 then give the gauge subspaces. They vary smoothly and are equivariant under the actual bundle transitions, so glue to subbundles. This calculation uses the algebraic Higgs action and not stationarity of its potential. At each point, the stabilizer of that Higgs value in the full gauge group is a conjugate of the stabilizer in Theorem 5.3 at $n = 6$. This fibre stabilizer is distinct from the stabilizer of the entire connection–Higgs history. □

The classical history thus realizes the pointwise bosonic response geometry. Each individual Higgs value has a nontrivial stabilizer, even though the chosen connection–Higgs configuration has no nonzero infinitesimal gauge automorphism. These are different stabilizer questions. The nonconstant Higgs history is not thereby a stationary vacuum, and its instantaneous algebraic subbundles are not asserted to be separately preserved by the full differential evolution. A pole requires its own construction, while the fixed-stationary-scale coordinate chart retains its specified probe and fixation domain.

7.5 Classical restrictions and their joint image

Let $\mathcal{A}^{\mathrm{occ}}_{\mathrm{cl}}$ be the pointed histories just constructed, with their target data, source attachments, local clock patches, and proper isomorphisms retained. For a in this domain define $R_{EW/H}(a)$ and $R_C(a)$ by restricting its fields and variational source to the indicated bosonic sectors. These are classical records on the same gravitational history, not autonomous Einstein solutions. Let $R_{g\phi}(a)$ retain the common metric, spin connection, scalar-clock subsystem, and initial embedding. Write $J_{\mathrm{cl}}(a)$ for the joint action, total source, constraints, exchange identities, and their mixed evaluations on the fields and covariant derivatives just defined. The relevant image is

$$\mathcal{I}_{\mathrm{cl}} = \mathrm{im}\left[a \longmapsto (R_{EW/H}(a), R_C(a), R_{g\phi}(a), J_{\mathrm{cl}}(a))\right]. \tag{18}$$

Theorem 7.9 (Classical common-source reception)**.** *The image $\mathcal{I}_{\mathrm{cl}}$ is nonempty. Its sector records share one actual metric, source attachment, global branch, and coupled constraint history. Proper source isomorphisms transport all restrictions and these mixed evaluations simultaneously. Quotienting by equality of these displayed classical records gives precisely the image* (18)*. No converse surjectivity from a formal fiber product of marginal records is asserted.*

Proof. Theorem 7.7 supplies a_δ and therefore an element of the image. All restriction maps evaluate the same pointed history, so common arguments are identical before quotienting. Source agreement follows from Proposition 7.2; constraint and mixed compatibility follow from the exact correction, (15), and (16).

If $f : a \to a'$ is a proper source isomorphism, its pullback intertwines the common action, fields, covariant derivatives, currents, and initial embedding. Naturality of these constructions therefore induces the component maps by restriction of f. Every mixed operation in this record is evaluated on those jointly transported arguments, so composition with f intertwines its domain and value. This is one joint transport, not separate covariance statements with an untracked sibling.

Finally define $a \sim_{\mathrm{cl}} a'$ when the entire displayed record agrees, using its stated transport identifications. The map $[a] \mapsto (R_{EW/H}, R_C, R_{g\phi}, J_{\mathrm{cl}})(a)$ is well-defined and injective by this equivalence, and surjective onto its actual image by construction. Its nonemptiness was proved independently above. □

A fixed-sibling comparison here relates actual histories with the same specified sibling restriction. Equality of some boundary entries, or a redescription of one marginal, is insufficient to establish such a joint history. Corollary 7.5 exhibits the obstruction: even compatible nonzero gauge fields cannot be appended while the gravity–scalar seed is held unchanged.

7.6 From classical histories to physical channels

The occupied branch retains the proper local quotient, all singular-stabilizer data outside its chart, its fixed global form and trivial bundle, spin structure, theta branch, compact boundaryless scope, and local clock failure boundary. The ordinary-spin chiral anomaly sums and Witten parity checks proved in [4, Theorem W4B.8] are also retained in their actual scope. A different bundle, line insertion, asymptotic boundary, torsion theory, or fermion content is a different source branch.

The resulting histories establish simultaneous classical coexistence, exact coupled constraints, joint transport, and a local physical clock. Existence and local uniqueness do not collapse the family of histories to a single member. Projection to bosonic records does not erase their clock subsystem.

Particle and channel projections extend this realization through their represented and analytic constructions. A shared classical action alone does not specify a joint quantum representation, state, and dynamics. Particle poles, resonance widths, scattering channels, confinement data, and a common quantum realization require their corresponding source constructions, with an actual identification of the arguments shared with the classical branch. The next section develops these additional projections while retaining the common-source relations established here.

8 Polarization, physical poles, and neutral-composite channels

The coupled history establishes classical coexistence on its fixed branch. Particle and channel projections add three distinct constructions: kinematical polarization, analytic continuation to a physical pole, and a composite map into a physical boundary. The familiar designations $\gamma, W^{\pm}, Z, h$ and the color-field designation g therefore refer to different realization maps. An unstable channel differs from a stable Wigner particle; an adjoint color field likewise differs from a freely propagating gluon in a confined physical sector. Each projection must be attached to the same source as the responses it interprets.

8.1 Projections on a common source

Definition 8.1 (Authority-bearing projection)**.** A projection family at T consists of a source-defined domain $D_\rho \subseteq X_T$, a typed output family Y_ρ, and an actual source construction $\rho : D_\rho \to Y_\rho$. The domain retains every state, spectrum, boundary, continuation, stability, and operator assumption used to produce that output. Its missing-domain diagnostics are received observations. For the full projection atlas the intrinsic output of ρ itself is a received probe; if a coarser target observes only some output tests, the corresponding conclusion concerns that output observational class instead. Source comparison acts on the entire construction, including the source attachment and physical ports.

A projection constructed for one sector applies to the joint source on the inverse image of its original domain under source restriction, with all required joint incidences retained. If $D_1, \dots, D_r$ are several such domains, simultaneous projection requires one $x \in \bigcap_i D_i$. Separate nonemptiness of every D_i does not guarantee that common configuration.

Theorem 8.2 (Authority-exact integrated projection)**.** *Every received projection of Definition 8.1 descends to the complete mixed observational quotient, retaining its exact definedness and intrinsic output. Several projections can be evaluated simultaneously precisely on their actual common-source intersection. Lawful transport of the full source and ports commutes with this evaluation; separate isomorphisms of the output spaces are insufficient.*

Proof. The domain and output probes of the projection are among the full contextual observations. Equivalent inputs therefore have equal definedness and output values. The quotient map is well-defined on the image of its exact domain. Simultaneous evaluation is evaluation of each partial map at the same source, hence has the stated intersection domain. Proposition 1.2 transports the actual construction and its ports. An isomorphism of output spaces alone supplies no equation relating either port to the source, so cannot prove that commutation statement. □

8.2 Vector polarization and a physical photon channel

On the licensed oriented and time-oriented Minkowski branch, use $(+---)$ for this paragraph's polarization calculation; a metric convention change elsewhere transports all formulas together. For nonzero future null p and future timelike p, respectively,

$$\mathcal{P}_0(p) = p^\perp/\mathbb{C}p, \qquad \mathcal{P}_m(p) = p^\perp, \quad p^2 = m^2 > 0.$$

The positive Hermitian form is induced by $-\eta(\bar{\epsilon}, \zeta)$. The scalar polarization is a separate complex line. BCRS–IIEW/H constructs the complete fibres and their Lorentz transport [2, Sec. 6].

Proposition 8.3 (Polarization does not change the carrier source)**.** *The null and massive vector polarization fibres have complex dimensions two and three. The null gauge line is removed exactly by $[\epsilon] \mapsto p \wedge \epsilon$. The null little-group translations act trivially on the quotient; its two circular directions have helicities ± 1. These are response fibres over real momentum orbits, not new carrier identities or unstable-pole witnesses.*

Proof. For $p = (\omega, 0, 0, \omega)$, orthogonality gives $\epsilon^0 = \epsilon^3$. Removing the gauge line leaves the unique transverse representative $(0, \epsilon^1, \epsilon^2, 0)$, with positive norm $|\epsilon^1|^2 + |\epsilon^2|^2$. The wedge vanishes exactly on that gauge line. Null translations add a gauge direction, while transverse rotations have circular weights ± 1. For $p = (m, 0, 0, 0)$, orthogonality sets $\epsilon^0 = 0$ and leaves $\mathbb{C}^3$. Lorentz covariance transports the calculation to the entire orbit. The construction uses neither a source identity generator nor an analytic continuation to a complex pole. □

The free Hilbert space $L^2(\mathcal{N}_+, \mathcal{P}_0, d^3p/(2|p|))$ realizes the two helicities. Its identification with an interacting electromagnetic channel requires a positive physical representation, its vacuum and field strength F, the massless spectral projection E_0, and the dense-range Gram identification

$$\langle E_0 F(f)\Omega, E_0 F(g)\Omega\rangle = \langle If, Ig\rangle_{\mathrm{hel}}.$$

The rule $If \mapsto E_0 F(f)\Omega$ then extends uniquely to a unitary identifying the helicity space with the represented source channel. Incoming and outgoing scattering maps have their own independently established domains. The null direction of the classical response matrix specifies kinematics, not these additional spectral data.

8.3 Unstable channels and physical residues

The invariant channels $H^\dagger H$ and $H^\dagger i\overleftrightarrow{D}_\mu H$ are scalar and neutral vector candidates. A charged W channel instead requires its actual charged-sector or physical scattering attachment. An unstable-state projection consists of a continued channel on a specified sheet and neighborhood, an existing pole, and a nonzero physical residue. The continuation must be established for that channel; a propagator coordinate alone does not define it.

Proposition 8.4 (Joint transport of an existing physical pole)**.** *On a fixed authorized continuation domain, suppose a finite matrix $D(s, \xi)$ is holomorphic in s, continuously differentiable in ξ, and satisfies $\partial_\xi D = AD + DB$, with coefficients holomorphic in s, jointly continuous in (s, ξ), and regular throughout the common domain. Require the physical channel maps J_L, J_R to be holomorphic at the pole and on the comparison neighborhood. Then its determinant zeros and multiplicities are unchanged along the connected ξ-interval, provided the determinant is not identically zero. A simple zero s_* is a genuine pole of a physical channel $J_L D^{-1} J_R$ exactly when its simple-pole numerator $J_L\,\mathrm{adj}(D) J_R$ is nonzero there. Jointly transported channel maps preserve the pole term; determinant-zero invariance alone does not prove a physical residue.*

Proof. The adjugate identity gives $\partial_\xi \det D = (\operatorname{Tr} A + \operatorname{Tr} B) \det D$, including at a singular matrix. Integration multiplies the determinant by a nonvanishing holomorphic exponential, preserving its zero divisor. Near a simple zero the inverse is its adjugate divided by that determinant. Its physical-channel residue is

$$\frac{J_L(s_*) \operatorname{adj} D(s_*) J_R(s_*)}{\partial_s \det D(s_*)}.$$

The denominator is nonzero by simplicity. A zero numerator removes that simple pole from this channel. To check full transport, solve $L' = AL$, $R' = RB$ with identity initial values, so $D(\xi) = LD(\xi_0)R$. With $J_L(\xi) = J_L(\xi_0)R$ and $J_R(\xi) = LJ_R(\xi_0)$, the entire meromorphic channel equals its initial value. This is exactly the source-qualified pole transport of BCRS–IIEW/H; it assumes neither a missing simple zero nor an absent physical port [2, Sec. 6]. □

The comparison remains within a domain without singular continuation coefficients, a changed sheet, an entering threshold, or untransported ports. The real quadratic mass coordinates w, z, h describe the local response; the final symbol here denotes the radial Higgs mass-response coordinate. These coordinates and a complex physical pole can coexist in the same atlas without being identified.

8.4 Color fields, neutral channels, and exact gap inheritance

The Yang–Mills realization includes local and nonlocal structure over its fixed source. Its color realization theorem relates the represented local net, vacuum and dynamics to the completions satisfying its same-domain conditions [3, Sec. 9]. Applying this result to the integrated atlas requires matching the state and dynamics, not only the color law.

On the matched local cyclic space $\mathcal{H}_{\text{loc}}$, the Gelfand–Naimark–Segal (GNS) comparison transports the full Hamiltonian and its spectral projections. The source's gapped branch has

$$E_H([0, m_*))\mathcal{H}_{\text{loc}} = \mathbb{C}\Omega.$$

On the same-domain completions covered by the source theorem, exclusion of additional orthogonal subgap occupants extends this gap to the full completion Hilbert space. This is a statement about those completions, not only their local cyclic subspaces. The following result determines whether that full-space conclusion is compatible with a photon channel.

Theorem 8.5 (No full-gap inheritance to a photon-populated apex)**.** *Suppose a proposed integrated Hilbert-space realization has $E_H((0, m_*)) = 0$, $m_* > 0$. It cannot contain an isometric time-translation-intertwining copy of a nontrivial standard massless one-particle photon space. Consequently a photon-populated bosonic apex cannot inherit the pure color completion's full-space gap by merely identifying its color-law restriction. A gap on an actually matched reducing color subspace is a different, compatible assertion.*

Proof. The massless one-particle Hamiltonian h has a nonzero spectral subspace on $(0, m_*)$, represented by wave packets with $0 < |p| < m_*$. A bounded intertwiner J of the unitary time translation groups intertwines their spectral measures. Therefore

$$JE_h((0, m_*)) = E_H((0, m_*))J = 0,$$

contradicting isometry on that nonzero subspace. This obstruction uses the full-space gap, not a gap on another reducing subspace. Equality of structural color laws proves neither equality of the full Hilbert spaces nor scope-faithfulness of their dynamics. Hence it cannot license the forbidden transfer. □

The obstruction separates two compatible uses of spectral information: a gap on the matched color subspace and a massless photon channel in the joint representation. It excludes transferring the pure color completion's full-space gap to the matter-coupled, photon-containing theory merely from agreement of their color laws.

The source also proves color neutrality of its observable-generated vacuum sector when the actual color action fixes both its local observable algebra and vacuum. If $\mathcal{K}^G = 0$, every bounded equivariant map $J : \mathcal{K} \to \mathcal{H}_{\text{loc}}$ then vanishes: invariance gives $JR(g) = J$, and Haar averaging yields $J = JP_{\mathcal{K}^G} = 0$. This excludes a nonzero colored port into that sector. It is not a statement about every possible boundary or charged representation, and a gap alone is not a proof of all-sector confinement.

8.5 Productive composite boundary maps

For an actual joint configuration let P_C be its color-invariant projector, j_C its admitted composite-operator preparation into the physical source representation, U its actual continuation, and a_D an independently justified isometric physical boundary attachment. The channel is

$$J_{D,C} = a_D^* U j_C P_C. \tag{19}$$

The maps must share their actual weak, hypercharge, Higgs, source, and boundary incidences. Color neutralization retains those data by Proposition 5.2.

Proposition 8.6 (Exact productive boundary criterion)**.** *When the maps in* (19) *are admitted on their complete domains, an input x gives a nonzero boundary response precisely when*

$$\langle U j_C P_C x, a_D a_D^* U j_C P_C x \rangle > 0.$$

Matched unitary transport of all four maps preserves this test and its authority. Existence of $P_C x$, of an interpolating operator, or of a neutral boundary by itself does not prove the test.

Proof. Since a_D is isometric, $a_D a_D^*$ is the orthogonal projection onto its image. The left side is $\|a_D^* U j_C P_C x\|^2$. Full joint intertwining moves each factor of the composition to its transported factor, so it preserves that norm and the source-domain conditions. None of the three isolated existence statements determines the overlap of the prepared vector with the physical boundary image. □

The norm test applies to stable and bound-state boundaries with an independently established Hilbert attachment. It applies to persistence boundaries under the same attachment condition. A resonance instead uses its continued physical channel and nonzero Laurent coefficient; it need not define an invariant Hilbert subspace. A general persistence record uses its history map. Jet or detector reconstruction is a subsequent observation of these channels, not a replacement for their physical attachments.

The neutral-channel construction thus gives a precise criterion for a nonzero boundary response. Extending it to unrestricted confinement requires complete upstream configuration coverage, phase-appropriate confinement and sector exclusion, actual physical boundaries, spectral and asymptotic constructions, and common-source continuation for every claimed sector [3, Def. 10.8]. These are additional properties of the physical realization, distinct from the neutral projection and boundary-overlap criterion.

9 Process composition and coupled metric exchange

A process can proceed from one operation to the next only when the produced intermediate configuration lies in the next operation's domain. This requirement applies equally to field interactions, state evolution, physical boundaries and measurement records. The integrated calculus retains those intermediate data, so that composition describes operations on one common source rather than an agreement of endpoint names.

9.1 Composition with exact intermediate domains

Definition 9.1 (Bosonic process diagram)**.** A process profile fixes a complete target–parameter state, an ordered input and output word in the admitted interface sorts, a boundary, and a process-type word. Primitive process types distinguish propagation, mode conversion, self-interaction, production, decay, splitting, confinement continuation, hadronization, restoration, and record formation. A diagram is a finite ordered source network of actual vertices and admitted continuation maps with that profile. All intermediate source, boundary, operator-domain and authority data are retained.

An algebraic vertex diagram has field-response endpoints. A physical diagram additionally has the exact state, pole, persistence or scattering authority required by its endpoints and internal maps. A record diagram also has an admitted measurement or reconstruction operation. Composition retains the ordered process-type word and intermediate boundary records.

Definedness and a nonzero response are separate questions. An admitted linear operation may vanish on a particular input, whereas an absent operation has no value there. Even a nonzero local vertex can yield a vanishing complete amplitude after contraction, a selection rule or a boundary projection.

Theorem 9.2 (Exact-domain process composition)**.** *The admitted diagrams form a source-evaluated typed composition system. For partial operations $f : D_f \subseteq X \to Y$ and $g : D_g \subseteq Y \to Z$,*

$$\mathrm{dom}(gf) = \{x \in D_f : f(x) \in D_g\}.$$

Finite composition is associative in value, domain, and received first-failure record. Lawful full-profile source comparison acts functorially and preserves physical authority. The calculus is universal as a typed grammar, with exactly the process population supplied by its source operations.

Proof. Expanding a triple composite gives, in either parenthesization,

$$\{x \in D_f : f(x) \in D_g,\ g(f(x)) \in D_h\},$$

with value $h(g(f(x)))$. Outside it, the first failed primitive operation in the ordered diagram is independent of parentheses. Identities have full interface domains and preserve that record. Admitted source equalities generate a composition congruence; they preserve the same values, domains and authority. For higher-arity vertices, graft matching actual ports and apply this argument to their evaluation order, with every joint admission predicate retained. Theorem 3.2 then permits observational descent. Full-profile comparisons intertwine each primitive vertex and continuation and reflect its domain. Induction gives functoriality for the diagram. None of these operations supplies a missing primitive state or boundary map, which proves the population qualification. □

Exact-domain composition preserves the analytic conditions establishing each physical operation. Any required closure of an unbounded operator product or convergence of an integral must be established on that operation's specified domain; neither follows from composition alone. This joint construction extends the electroweak process diagrams and the two-level color channel interface [2, Sec. 7][3, Sec. 10].

9.2 Selection rules preserved by contraction

Theorem 9.3 (Joint selection under contraction)**.** *Let every port be an honest representation of the same G_n, with its actual tensor and dual incidences. Decompose reducible ports into central-character and unbroken Abelian-weight components, or restrict the assertion to the corresponding isotypic inputs. An invariant multilinear scalar vertex on such inputs can be nonzero only if the product of its central characters is trivial and every unbroken continuous Abelian character has total weight zero. Contraction of a port with its admitted dual preserves these conditions on the external ports. A color-singlet output retains its weak and hypercharge action; color neutrality alone does not satisfy all selection conditions.*

Proof. Act with a central group element on all arguments. Equivariance multiplies a scalar value by the product character; a nonzero value requires that factor to be one for every such element. For an actual unbroken circle, the factor is the product of its characters, or $e^{ia\sum q_j}$ in a compatible local normalization. It is identically one exactly when the total character is trivial. A dual leg carries the inverse character. Its evaluation pairing cancels the two internal factors, leaving the same condition on the external legs. Honest joint representation descent uses Proposition 5.1; the color-singlet qualification is Proposition 5.2. □

These selection rules preserve the joint charge constraints through internal contractions. Nonzero amplitude also depends on the full non-Abelian representation, Lorentz type, derivative order, source coefficient and physical continuation. Coupling two kinds in an interaction accordingly relates their responses without identifying the kinds themselves.

9.3 Balanced boundary composition

Let $\mathcal{C}$ be actual prepared configurations, $\mathcal{D}$ admitted intermediate boundaries and $\mathcal{E}$ final boundaries in one source diagram. Suppose its interfaces have channels $K(C, D)$ and $L(D, E)$, whose left and right boundary actions are actual composition. Define

$$(L \otimes_{\mathcal{D}} K)(C, E) = \left(\coprod_{D} K(C, D) \times L(D, E) \right) / ((vk, l) \sim (k, lv)).$$

Proposition 9.4 (Boundary factorization image)**.** *The map $[k, l] \mapsto lk$ is well-defined and has exactly the image of channels factored through an actually supplied intermediate boundary using both supplied interfaces. It need not be injective or cover channels without such a factorization. It preserves the ordered process profile and inherited authority.*

Proof. Both sides of each generating balanced relation evaluate to lvk, with the same exact domain by Theorem 9.2. This proves well-definedness. Every represented pair gives the stated factorization, and every such factorization supplies a pair. Additional source equalities may identify images of distinct pairs, so injectivity does not follow. No pair has been supplied for a channel whose second leg or intermediate boundary lies outside the given interfaces. Retaining the primitive diagrams retains their type and authority. □

Balanced composition identifies alternative descriptions of the same intermediate action. In hadronization or measurement, the boundary itself must still be independently admitted and both channel maps must compose on their exact domains. Taking a quotient cannot supply an absent physical boundary.

9.4 Conservation of the coupled metric response

Internal exchange allows the total coupled metric source to be conserved even when its sector contributions are not conserved separately. Work in one licensed smooth metric envelope.

Let W_a be the full linear continuous compact metric first response of each term of a finite source decomposition, counted exactly once. This includes interactions, vacuum and boundary contributions whenever they are part of the target. Its representing symmetric distribution T_a is determined by

$$W_a(k) = \tfrac{1}{2}\langle \mathsf{T}_a, k\rangle.$$

Here $\mathsf{T}_a = T_a^{\mu\nu} d\mathrm{vol}_g$, with indices lowered by the retained metric when writing $T_{a,\mu\nu}$. The displayed Ward tests have compact support in the interior; any physical boundary flux is separately retained in its source boundary response and is not discarded by this convention. Uniqueness uses the source's full linear test domain. A selected finite probe set determines its own readout quotient, not the entire metric response.

Write the exchange identities of this source as

$$W_a(\mathcal{L}_X g) = -\left\langle J_a^{\mathrm{ext}} + \sum_e B_{ae} j_e, X \right\rangle, \qquad X \in \Gamma_c(TM),$$

where every internal exchange column satisfies $\sum_a B_{ae} = 0$.

Theorem 9.5 (Integrated Ward closure)**.** *The joint metric first response and its source are*

$$W_{\mathrm{tot}} = \sum_a W_a, \qquad \mathsf{T}_{\mathrm{tot}} = \sum_a \mathsf{T}_a,$$

and satisfy

$$\nabla^\mu T^{\mathrm{tot}}_{\mu\nu} = \sum_a J^{\mathrm{ext}}_{a,\nu}.$$

The total source is closed exactly when this external sum vanishes. Actual source comparison preserving metric protocols and mixed incidences transports the identity, including on fixed-sibling slices. Separate conservation of each term is not required.

Proof. Finite sums preserve continuity and distributional representation. For a symmetric source, duality and integration by parts give

$$\tfrac{1}{2}\langle \mathsf{T}, \mathcal{L}_X g\rangle = -\langle \operatorname{div} \mathsf{T}, X\rangle.$$

Sum the actual exchange identities. Column balance cancels every internal current. Testing against every compact vector field proves the distributional divergence equation and both directions of the closure criterion. Full source comparison intertwines the response laws, their protocols and exchange incidences, hence the finite sum. If a sibling is held fixed, use its identity comparison and the unchanged incident maps. This is the common-source extension of the color metric theorem [3, Thm. 11.6]. □

For a covariant joint local action, first variation produces these responses. On a joint solution, the remaining Euler–Lagrange terms vanish in a compactly supported diffeomorphism variation, and the total metric source is closed. An open subsystem retains its exchange terms: a nonzero color Lorentz force, for example, is balanced by the coupled matter response. Conservation belongs to the coupled total rather than to the color contribution in isolation.

Corollary 9.6 (No process-to-carrier promotion)**.** *Neither a process incidence, its contraction multiplicity, its polarization count, nor a boundary record creates a new premetric carrier. A productive process can alter a response or record while retaining the upstream carrier architecture.*

Proof. Each process map takes already typed source ports as operands. Its output type is specified before evaluation. No carrier-formation constructor occurs among the process operations, and the actual source reception preserves those types. A new terminal carrier would require its independent source identity and occupancy theorem; the process alone supplies neither. The conclusion is consistent with nontrivial changes of fields, states and records because those are distinct response types. □

10 Complete response reconstruction and minimal identity

What information determines every response at the chosen target? The process and metric constructions make this question precise. Local-law coefficients determine the interaction on their stated branch, while state-level responses also depend on a joint represented source. Even complete states on the separate marginal algebras leave mixed correlations undetermined. We first reconstruct a represented source from its complete joint kernel, then combine the realization levels and characterize exactly which information a description may omit.

10.1 Joint state kernels and operator domains

Let $\mathcal{W}_T$ be the source-admitted finite words formed from all the joint field, color, Higgs, current and boundary-preparation operators used by a target, with their smearing functions and exact operand domains. The vacuum word is included. Suppose an actual represented source supplies vectors Ψ_w with dense linear span $\mathcal{D}_T$ in its physical Hilbert space and complete mixed Gram kernel

$$K(w,v) = \langle \Psi_w, \Psi_v \rangle.$$

Mixed words retain correlations between sectors as well as within them. The operator action on word vectors and the source relations are also part of the data. Each closed operator to be reconstructed has $\mathcal{D}_T$ as a specified common core; when that condition is unavailable, its complete graph is retained independently.

Theorem 10.1 (Joint-kernel reconstruction)**.** *For two actual represented sources with the same admitted word relations, complete mixed Gram kernel, and word-action rules, the map $\Psi_w \mapsto \Psi'_w$ extends uniquely to a unitary between their cyclic Hilbert spaces. It intertwines each common-core operator including its closed domain, every supplied bounded boundary map whose values intertwine on the word span, and their admitted finite composites. If the actual time evolutions preserve the word structure with the same action, it also intertwines their self-adjoint generators and spectral measures. The theorem constructs no joint state from two separate marginal kernels.*

Proof. For a finite word combination $\sum_w c_w \Psi_w$, its squared norm is $\sum_{w,v} \bar{c}_w c_v K(w,v)$. Equality of the complete mixed kernel makes the proposed map isometric and makes every zero-norm combination map to zero. It is therefore well-defined on the span. Density in both actual cyclic spaces gives a unique unitary extension. The common word-action rules imply $UA\psi = A'U\psi$ for $\psi \in \mathcal{D}_T$. If that domain is a core on both sides, the unitary carries the closure of the first graph onto the closure of the second, proving equality of the full closed operators and their domains. Bounded boundary maps extend their equality from the dense span by continuity. For separately supplied complete graphs, their source comparison supplies the corresponding graph equality directly; it is not inferred from mere dense-domain agreement.

The same dense-span argument intertwines the actual strongly continuous unitary time evolution at every time. Uniqueness of its self-adjoint generator and spectral calculus give the remaining claims. Finite composite domains are the exact inverse-image domains of Theorem 9.2; graph intertwining reflects them. The proof starts with two actual complete joint kernels, so does not prove their existence from marginal restrictions. ☐

Example 10.2 (Equal marginal states, different mixed response)**.** On two qubits let

$$\rho_1 = |\Phi^+\rangle\langle\Phi^+|, \qquad |\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}, \qquad \rho_2 = I_4/4.$$

Both marginal states are $I_2/2$, but $\mathrm{Tr}(\rho_1 \sigma_z \otimes \sigma_z) = 1$ and $\mathrm{Tr}(\rho_2 \sigma_z \otimes \sigma_z) = 0$. Thus complete knowledge of the two marginal states cannot reconstruct the joint kernel. This finite-dimensional example diagnoses the missing mixed information; it is not proposed as a bosonic Standard Model realization.

Theorem 10.1 extends the source-attached Gram and operator reconstructions of the electroweak and color sectors to their complete mixed data [2, Sec. 11][3, Sec. 9]. Equality of those data constructs the joint unitary comparison, including the operator domains and dynamics under the stated hypotheses. The mixed kernel itself remains information about the represented source, distinct from its local interaction law.

10.2 A complete description across realization levels

Definition 10.3 (Integrated reconstruction datum)**.** At a specified target a reconstruction datum retains:

(i) the source-law families and their actual relative charts, kind and role incidences, boundary and continuation laws;

(ii) the shared global group, bundle, representation, charge and anomaly structure, one Higgs bridge, and actual fixation/residual branch;

(iii) the local invariant chart of Theorem 6.5, or the complete mixed coefficient tensors of Theorem 6.8, with independent Yukawa, topological, boundary and background laws retained;

(iv) each actual marked configuration and preparation used by the target, distinct from the law that evaluates it;

(v) when requested and supplied, the complete joint state kernel, operator cores or full graphs, dynamics, spectral/pole/confinement and physical boundary attachments; and

(vi) the source-defined partial domains, failure tests, and joint incidence maps of all the admitted operations.

Only applicable, independently admitted entries are included. In particular neither a future physical signature nor the desired role count is an input. A datum is a reduced description only where one of the preceding reconstruction theorems actually reduces its source information; otherwise the corresponding native source record remains.

Theorem 10.4 (Integrated finite-use reconstruction)**.** *On the actual domain of an integrated reconstruction datum, every received finite bosonic response, continuation, projection and process observation factors through that datum, with its exact domain and failure value. The induced observational quotient is its actual evaluation image. There is no further choice of a response value at a fixed complete datum. Different marked arguments or independent physical attachments are not identified merely because their local law coordinates agree.*

Proof. For a source-law vertex use the complete source graph and its retained arguments. For a local-law vertex the explicit inverse reconstructs its common coefficients, and the full mixed-jet theorem reconstructs any admitted polynomial extension. The global and relative source data retain all additional incidence and admission conditions. For an actual represented vertex use Theorem 10.1 on its precise core/graph scope and the independent supplied physical port. For a classical source vertex use the full received classical record, not an isolated sector solution. Each primitive value, domain and failure diagnostic is therefore determined. Induct over the finite context and use exact-domain composition. This proves the factorization and its uniqueness on the actual datum image. The quotient by its full evaluation kernel is precisely the observational image. The datum still includes marked arguments and physical attachments whenever they are independent, which prevents the final false identification. □

The atlas combines explicit reductions with the independent data they do not replace. The finite local chart reconstructs its interaction law; complete mixed data determine the represented comparisons on their stated domains. Consequently completeness across realization levels means preservation of every admitted response, not reduction of every physical observation to one short numerical tuple.

10.3 When information can be omitted

Let P_T be an actual domain of reconstructed source descriptions, and let $R_T : P_T \to \mathcal{V}_T$ be their full retained response signature, including every admitted domain and status observation. A proposed deletion or compression is a map $d : P_T \to D_T$.

Theorem 10.5 (Exact identity-packet minimality)**.** *The compression d is observationally lossless if and only if*

$$\ker d \subseteq \ker R_T. \tag{20}$$

When this holds there is a unique reconstruction $\bar{R}_T : \operatorname{im} d \to \operatorname{im} R_T$ with $R_T = \bar{R}_T d$. A field is deletion-essential precisely when its removal admits two actual points with the same retained description and different full signatures. A duplicated field is not essential merely because its scientific meaning is important.

Proof. Any factorization makes equal d-values have equal R_T-values, giving (20). Conversely that inclusion makes $\bar{R}_T(d(p)) = R_T(p)$ independent of the chosen representative. Surjectivity onto $\operatorname{im} d$ proves uniqueness. Failure of the inclusion is exactly the stated separating pair. If another retained entry already determines the deleted field, the full signature still factors, so mere duplicate storage cannot establish essentiality. □

On the elementary minimal local-law family, the five or six invariant coordinates are individually irredundant: the variations in Proposition 6.6 change an observed response while leaving the other coordinates fixed. Relations on a smaller actual image may instead make a coordinate redundant. The distinction between a field-only and a metric-visible vacuum value is measured by the trace-response separator of Proposition 6.7.

Global form carries information absent from the local Lie algebra when the target includes the relevant line-admission tests. The source family has screened electric quotients $\mathbb{Z}/(6/n)$, and a pure hypercharge probe $(0, 0, n)$ descends to G_n but not to G_m for $n \mid m$, $n < m$. The incomparable choices $n = 2, 3$ are separated by their corresponding probes [3, Prop. 6.4]. A purely local target omits these probes and therefore asks a coarser question; that omission does not select one global group.

Relative attachment is essential whenever an actual mixed observation separates configurations with the same retained marginal description, as in Theorem 3.4. The nonphysical example 3.6 isolates this mechanism. Complete mixed state kernels are likewise essential when joint correlations distinguish equal marginal states; Example 10.2 exhibits the elementary algebraic distinction. To establish either essentiality claim for the physical source, the separating pair must belong to its actual domain.

Every integrated description therefore retains its source scope, role and kind incidences, full observation and operation domains, and the comparisons that identify its responses. Which entries are deletion-essential depends on the target and actual source image. Gauge transport, Higgs fixation, global lines, resonance sheets and confinement boundaries add the information required by their respective mechanisms; they are not interchangeable entries.

10.4 Rigidity of the law and diversity of its responses

One complete law can evaluate many distinct configurations. The whole-law inventory identifies the source families supporting those evaluations; finer response identity also retains their marked inputs and physical attachments.

Theorem 10.6 (Whole-law role closure and terminal nonpromotion)**.** *Over a nonempty fixed source-realized extension fibre, the integrated bosonic architecture retains the unique complete gauge-law reduct and its charge, weak and color role restrictions, together with the one independently fixed canonical Higgs family. Its whole-law inventory remains* $(1, 1, 1; 1)$*. Every received finite higher-arity response network evaluates over these source families with its actual marks and auxiliary outputs; no response-support, polarization, generator, pole or process multiplicity creates an additional terminal source carrier in this target.*

This statement does not assert singletonhood of the finer marked response quotient. At a fixed complete reconstruction datum, however, every admitted full response is uniquely evaluated by Theorem 10.4; any further claimed response choice must exhibit a missing source input or a different target.

Proof. The source comparison of BCRS–I's complete law reduct fixes all operation graphs over its fixed SR7 class; the fixed color and Higgs families give its whole-law inventory [1, Thm. 13.3 and Cor. 13.4]. Its all-extension network coverage retains every actual marked use, rather than reducing those marks to one preferred value. Proposition 1.2 and Theorem 10.4 extend the evaluation to the admitted successor operations without inserting a new source identity constructor. Full kind and output sorts are retained by Theorem 2.4; process incidences are not carriers by Corollary 9.6. Thus all received uses belong to the retained source architecture. A proposal outside this covered source scope requires a new source reception and identity proof, not a count of downstream labels. Finally fixed-datum uniqueness is the explicit reconstruction theorem, whereas different marks can give different observations of the same law. The two claims are therefore compatible. □

For full terminal role-occupancy closure at a fixed target, let $\mathcal{E}^{+}_{T,b}$ be its actual complete compatible assignments and let $\mathcal{G}_{T,b}$ be the actual source comparison groupoid acting on them. These complete compatible assignments already satisfy the source, use, activity, quotient, continuation and failure completeness clauses, exact eligibility, and totality on the compatible live residue. These are the non-orbit requirements retained from the closure theorem [1, Thm. 12.35]. Once those requirements are discharged, closure requires a nonempty assignment space with exactly one source-comparison orbit:

$$\mathcal{E}^{+}_{T,b} \neq \varnothing, \qquad \left|\pi_0(\mathcal{E}^{+}_{T,b}//\mathcal{G}_{T,b})\right| = 1. \tag{21}$$

At a reconstruction-complete observational target, one complete datum determines its observational assignment uniquely. Uniqueness up to source comparison additionally requires the comparison reflection in Theorem 4.2. The latter condition connects observational equality to an actual source-comparison orbit.

A unique realization requires an occupied fibre. Moreover, uniqueness at each admitted b does not select one b among all couplings, histories, global branches or physical states. Thus the full marked-response criterion of BCRS–I and the whole-law inventory address different questions: identity within the specified complete data, and the source families over which those data are evaluated.

11 The integrated atlas and its realization levels

11.1 Simultaneous outcomes at one source

The integrated theory assigns distinct but simultaneous conclusions to one source. Its local law can be reconstructed, a classical history can exist, and a particular color boundary can be established while a requested physical pole still requires a spectral attachment. These outcomes concern different mathematical objects. The atlas retains their relations and the evidence for each, rather than reducing them to a single verdict.

Definition 11.1 (Branch-complete bosonic disposition)**.** For each requested source or realization interface, its disposition is one of the following, with its exact target and evidence retained:

(i) a positive constructed or source-proved response on its domain;

(ii) a negative result with the source-proved failed condition;

(iii) an inactive response under the fixed activity test;

(iv) a not-applicable mechanism at that kind or source branch; or

(v) NotReady, identifying the unsupplied premise needed for the requested stronger projection.

For several simultaneous requirements the disposition is a dependent family over one actual common source, not a product of verdicts obtained at unrelated states. Unresolved existence is never converted into a proved negative or an inactive zero.

Theorem 11.2 (Exact structural bifurcation)**.** *The integrated atlas distinguishes all of the following without changing their source identities:*

(i) one source carrier with several response supports;

(ii) a product network or higher-arity mixed incidence over several retained source roles;

(iii) gauge–vacuum and Higgs-mediated response at a nonzero fixation, and its distinct restored or singular-rank branch;

(iv) local law, marked configuration, and independently supplied physical realization;

(v) stable-particle, unstable-pole, confined-field, neutral-composite, partonic and detector-record descriptions at their distinct authorities; and

(vi) populated, excluded, inactive, not-applicable and unresolved joint domains.

The full disposition is invariant under lawful common-source redescription. An actual change of domain, mechanism, relative attachment, global branch, or authority can change it and is not identified as source skin.

Proof. The kind and role distinctions are preserved by Section 2. Full mixed congruence preserves the actual operation and failure types. The relative-attachment classification retains actual source comparison classes rather than unconstrained marginal orbits. Theorem 6.3 proves several reducing supports over the retained source roles, while Theorem 5.4 prevents bridge duplication. Fixation and restoration have different rank/domain records. The local inverse does not identify marked fields or physical states, and the projection theorems retain their independent physical hypotheses. Definition 11.1 distinguishes absence of authority from a proved exclusion. Each retained coordinate is invariant under the full source comparisons already proved; a changed standing coordinate is outside that comparison relation. □

11.2 The no-backflow theorem

Let $r : X_{T'} \to X_T$ be a lawful realization restriction that retains the source ancestry. A field component, a chosen polarization, a pole term or a detector response is a downstream evaluation on $X_{T'}$. It may help select or test a realized source state, but it is not the operation that formed the earlier carrier quotient.

Theorem 11.3 (Global bosonic no-backflow)**.** *Within the fixed source target, representation dimension, generator count, polarization count, vacuum display, pole multiplicity, scalarity, detector records, and empirical parameter fits cannot revise the premetric carrier or whole-law inventory. A stronger downstream observation can refine the realized response quotient only* *through a declared target extension whose source restriction remains explicit. If it distinguishes two previously identified objects, the earlier observation signature was coarser; its original theorem is not thereby a proof of the stronger identification.*

Proof. The source identities and whole-law roles were formed from their received source operations before downstream realization. None of the listed evaluations is a source-identity constructor in that target. Their actual codomains are support, response, projection, or record types; changing their numerical or descriptive multiplicities does not change the fixed source relation. For a declared extension, Proposition 3.7 gives the induced map from the finer realized quotient to the earlier quotient. It is not generally injective. Thus new distinctions may refine the description on the actual extended source without invalidating its coarser predecessor statement. Changing the source mechanism itself requires a new source target and reception, not a backward inference from a response label. □

11.3 Numerical prediction and source-image readiness

For a requested scalar physical quantity let $f : D_f \subseteq X_T \to \mathbb{R}$ be its actual source-defined readout with the units, renormalization, boundary and physical authority that give it that meaning. A parameter coordinate with no such physical readout is not silently a mass, width, lifetime or cross section.

Theorem 11.4 (Numerical determination criterion)**.** *A retained structural datum d determines the requested numerical readout, including its retained failure information, if and only if the domain predicate, the defined value of f, and every retained undefined diagnostic factor through d. If a single value is claimed at a fixed structural datum d_0, the admitted fibre must be nonempty and f must be constant on it. A proved range on that fibre is an exact restricted-image statement, not necessarily a singleton prediction.*

Proof. Apply Theorem 10.5 to the totalized readout. Its definedness coordinate gives the domain factorization, its defined value gives the value factorization, and its undefined coordinates give the retained failure-diagnostic factorization. A fixed datum has a unique numerical value exactly when its nonempty readout image is a singleton. More generally its set of actual values is the restricted image $f(d^{-1}(d_0) \cap D_f)$. No value outside that set follows, and no empty fibre supplies a physical prediction. □

The local chart yields positive structural inequalities $0 < w < z$, $q_e, h_r, r_c > 0$ on its declared branch and recovers its full local coefficients. It does not fix one point of that chart. Nor does it turn those response eigenvalues into physical poles. The classical occupied branch fixes its chosen parameters and proves a history exists; it does not derive their numerical values. These are positive mathematical results at distinct levels, not competing interpretations of the same number.

11.4 The integrated structure theorem

Theorem 11.5 (Bosonic carrier–response–projection atlas)**.** *The accepted bosonic source and realization theories, integrated on their exact actual source domains, have the following endpoint.*

(i) A mechanism-preserving kind atlas and common natural response grammar receive all admitted source operations, including mixed inputs, before quotienting.

(ii) The complete mixed observational quotient is the greatest domain-reflecting observational congruence. Its difference from the marginal quotient is exactly the mixed-signature kernel.

(iii) Actual relative attachments classify the remaining joint source data. A double-orbit normal form is valid on every specified branch satisfying the object- and comparison-completeness hypotheses. Formal compatibility is not used as an existence proof.

(iv) *The shared global-form family, honest associated bundles, one Higgs bridge and color-correlated electromagnetic stabilizer are retained. On the nonzero minimal local branch the joint reducing-support lattice has five atoms, with dimensions* $(8, 1, 2, 1; 1)$ *and the scalar/gauge grading preserved.*

(v) *Five local response coordinates, or six when the vacuum value is observed, reconstruct the complete stipulated joint local law on its actual source image. Complete mixed finite-jet data extend the result to independently admitted polynomial operator targets. The independent global, marked, metric, boundary and physical inputs remain explicit.*

(vi) *There is a populated simultaneous classical gauge–Higgs– gravity source on the fixed WDW branch, with its retained massive scalar clock, exact coupled constraints, proper local quotient, and pointed covariant development. Its scope is not a pure Standard Model vacuum or an unrestricted global gravitational or quantum realization.*

(vii) *Polarization, photon, pole and confinement-qualified projections, together with typed processes and joint metric exchange, form an authority-exact family on their actual common source intersections. Complete joint state kernels reconstruct their represented comparisons where independently supplied.*

(viii) *The whole-law inventory* $(1, 1, 1; 1)$ *and terminal nonpromotion are preserved. Complete marked-response and source-orbit closure remain distinguished; minimal identity is governed by the exact actual-image deletion theorem.*

(ix) *The final disposition retains every positive construction, proved obstruction and unresolved physical premise, with no backflow from particle labels or numerical fits.*

Proof. Items (i)–(iii) are Theorems 2.3, 2.4, 3.2, 3.4, 4.2 and 4.3. Item (iv) follows from the joint descent calculation, Theorems 5.3, 5.4 and 6.3. Item (v) is the constructive local inverse and mixed-polynomial reconstruction. Item (vi) is Theorem 7.7 with its exact reception theorem. For (vii), use Theorems 8.2, 9.2, 9.5 and 10.1; no intersection is presumed nonempty without its actual witness. Item (viii) is Theorems 10.5 and 10.6, with (21) retaining the additional source-orbit criterion. Finally Theorems 11.2, 11.3 and 11.4 establish (ix). Each cited result was proved without using the final inventory as a source constructor. □

11.5 Structural reconstruction and further realization

Source-law rigidity and response diversity coexist because the relative attachments and marked inputs remain part of the integrated theory. Its local inverse reconstructs one interaction law from complementary responses, while the complete mixed algebra determines which further distinctions matter at each target. The bosonic atlas and the fermionic atlas use the same canonical Higgs source through their distinct fermionic and electroweak incidences [7]. Their common-source maps preserve those incidences; the bosonic reconstruction does not revise the fermionic theorem chain.

The populated classical branch gives this integration an explicit simultaneous realization. With its retained scalar clock, global form, bundle, and proper local comparison scope, one corrected datum obeys the coupled constraints and admits covariant development. On a sufficiently small neighborhood of the initial slice, its nowhere vanishing Higgs field realizes the pointwise gauge-response geometry even though the history is not a stationary vacuum. This provides a concrete classical domain on which the previously separated response constructions belong to the same evolving configuration.

An unrestricted joint physical realization additionally requires one interacting state and dynamics, the matched photon and unstable physical channels, and the confinement, configuration, and asymptotic constructions appropriate to the claim, all on the same source and boundary attachments. The complete physical carrier-realization and confinement requirements, designated S59 and S60 in the source chain, therefore retain their independent content. Theorem 8.5 shows why their common domain matters: a full-space positive gap cannot be transferred unchanged to a representation containing the massless photon spectrum.

The resulting bosonic structure has an explicit local reconstruction, a nonempty simultaneous classical realization, and physical projection maps on their exact source domains. It supplies the laws, relative comparisons, and reconstruction criteria through which a stronger physical realization can extend the same structure. Carrier identity, response, and physical projection are thus connected stages of one construction, with the distinctions between them mathematically retained.

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