Classical Standard Model and Gravity from a Common Constraint-Governed Source

Target–source compatibility, joint constraints, and finite classical fermion realization

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Abstract

We prove a target-relative existence and local-equivalence theorem joining a structural Standard–Model carrier–response atlas to classical gravity through one fixed covariant action. On a compact hyperbolic Cauchy surface with the global Standard–Model gauge group, three chiral generations, one Higgs doublet and a scalar clock, an explicit law tuple occupies both the represented polynomial chart and the coupled Einstein–matter construction. The off-shell metric-response graph determines a unique distributional total source, and its complete Noether ledger closes only after Euler–Lagrange residuals vanish and internal exchanges cancel. A corrected constraint datum has nonzero color, weak and hypercharge curvature, a nowhere-zero nonconstant Higgs field and trivial infinitesimal internal stabilizer. On its ordinary regular body, the full Lorentz, internal-gauge and hypersurface-deformation constraints form the exact Dirac-bracket algebra; a mixed-order tame right inverse yields characteristic equality and a local source-faithful finite refoliation theorem. An independently formed non-Hilbert continuation lane intertwines with the corrected development without metric or solution backflow. For every finite complex exterior algebra, coefficientwise constraint lifting and hyperbolic Cauchy induction further produce exact classical anticommuting-field solutions; with at least two generators, nonzero Dirac–Yukawa data force a nonzero even coupled response. The result includes a theorem-by-theorem compatibility audit across the gravity, Wheeler–DeWitt and gravity–quantum source packages. The operator-valued mass-matching and quantum-projection branches remain explicitly not activated, and no interacting quantum field theory, quantum gravity or global refoliation theorem is claimed.

Keywords: AASC; Standard Model; Einstein constraints; source attachment; hypersurface-deformation algebra; finite refoliation; classical Grassmann fields; target–source compatibility.

Contents

1 One target, one law datum, one source

The purpose of CSP–C is not to juxtapose a structural Standard–Model atlas with an unrelated gravitational solution. It is to construct one fixed classical target for which the represented Standard–Model operations, the covariant action, the metric response, the canonical constraints and the Cauchy development are restrictions of the same source datum. This section fixes that datum before any solution is selected.

1.1 The fixed classical target

Let $\Sigma = \mathbb{H}^3/\Gamma$ be a compact, connected, boundaryless, oriented hyperbolic three-manifold, equipped with one ordinary spin structure. Fix the trivial principal bundle with group

$$G_{\mathrm{SM}} = \frac{\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y}{\mathbb{Z}_6}. \tag{1}$$

For each $r \in \{1, 2, 3\}$, use left-handed Weyl notation and the represented spaces

$$\begin{aligned} Q_r &= (\mathbf{3}, \mathbf{2})_1, \quad & u_r^c &= (\overline{\mathbf{3}}, \mathbf{1})_{-4}, \quad & d_r^c &= (\overline{\mathbf{3}}, \mathbf{1})_2, \\ L_r &= (\mathbf{1}, \mathbf{2})_{-3}, \quad & e_r^c &= (\mathbf{1}, \mathbf{1})_6, \quad & H &= (\mathbf{1}, \mathbf{2})_3. \end{aligned} \tag{2}$$

The subscripts are the integer normalization $q = 6Y$. There is one Higgs doublet and no right-handed neutrino. The spin connection is $\omega_{\mathrm{LC}}(e)$, derived from a nondegenerate coframe; torsion and an independent affine or spin connection are not target coordinates. A real massive scalar χ is retained as the local clock matter of the regular WDW branch.

To make nonemptiness constructive without assuming full numerical-coupling standing, choose a positive reference scale μ_0 and freeze, in the declared local units,

$$\mathbf{c}^\diamond = (g_3, g_2, g_1, \lambda, v, Y^u, Y^d, Y^e, \theta) = (1, 1, 1, 1, \mu_0, \mathbf{1}_3, \mathbf{1}_3, \mathbf{1}_3, 0). \tag{3}$$

This is a convenient member of the admitted positive parameter domain, not a prediction or phenomenological fit. A simultaneous rescaling to another fixed kinetic convention must transport fields, currents and coupling coefficients together. The scalar-clock mass and gravitational normalizations are likewise fixed once and for all; no cosmological offset is added to the occupied WDW branch.

Definition 1.1 (CSP–C target)**.** The target $\mathfrak{T}_C^\diamond$ consists of (1)–(3), the chosen orientation and spin structure, the trivial bundle and zero external-line sector, the one-Higgs potential

$$V(H) = (H^\dagger H - \mu_0^2/2)^2,$$

the Einstein–scalar–Yang–Mills–Higgs–Weyl–Yukawa action with these coefficients, the proper gauge and local-diffeomorphism arrows, the complete source and boundary ledgers, and the smooth-tame scale with $q_0 > 3/2 + 4$. Mapping-class, large-gauge, nontrivial-bundle, charged-boundary and topology-changing comparisons remain distinct target or residual coordinates.

The target is deliberately narrower than the collection of all classical Standard Models. A fixed-branch existence and equivalence theorem can be proved on it without treating a parameter value, a bundle choice or a global presentation as invisible skin.

1.2 The fresh common-source pullback

The CSP–S atlas supplies a marked polynomial-law chart for exactly the representations in (2). Its three invariant Yukawa maps are

$$\epsilon_{ab} Q_r^{\alpha a} H^b u_{r,\alpha}^c, \qquad Q_r^{\alpha a} (H^\dagger)_a d_{r,\alpha}^c, \qquad L_r^a (H^\dagger)_a e_r^c, \tag{4}$$

summed over r. With the coefficients in (3), all three are actual nonzero polynomial operations. Their hypercharge sums are respectively $1 + 3 - 4 = 0$, $1 - 3 + 2 = 0$ and $-3 - 3 + 6 = 0$, and their color and weak contractions are invariant. They therefore descend through the same central quotient as the kinetic and current maps [1].

Let $\mathfrak{p}^{\diamond}$ denote this marked law datum and let $S_C^{\diamond}$ be the covariant action in Definition 1.1. There are three relevant restrictions:

$$\mathrm{Poly}(S_C^{\diamond}), \qquad d_g S_C^{\diamond}, \qquad \mathrm{Leg}_{\Sigma}(S_C^{\diamond}). \tag{5}$$

The first retains the represented polynomial operations, the second is the off-shell compact-test metric response, and the third is the canonical constraint system. They are operations on one already fixed action; none is defined by matching its output to another branch.

Theorem 1.2 (Exact target–source compatibility)**.** *The datum $(\mathfrak{T}_C^{\diamond}, S_C^{\diamond})$ is nonempty and satisfies*

$$\mathrm{Poly}(S_C^{\diamond}) = \mathfrak{p}^{\diamond}. \tag{6}$$

The coupled WDW/BCRS existence theorem may be instantiated at the same tuple $\mathbf{c}^{\diamond}$. Hence its occupied body, its metric response, its ADM constraints and its local development are restrictions of $S_C^{\diamond}$. This is a new same-source intersection theorem; it is not the relabelling of the separate CSP–S polynomial and classical branches.

Proof. The CSP–S polynomial theorem constructs the representations, invariant tensors, gauge derivations, Higgs potential and nonzero operations for every fixed positive kinetic/Higgs tuple and every specified nonzero coefficient choice of the displayed type. Equation (3) is an explicit such choice, so $\mathfrak{p}^{\diamond}$ exists without an appeal to the stronger MGN standing domain. The local formula for $S_C^{\diamond}$ uses those same representations, contractions and coefficients; direct restriction to finite jets proves (6).

The BCRS–3/WDW–4 existence theorem is parameterized by fixed positive gauge couplings, positive λ, v, fixed Yukawa matrices and a fixed local theta branch. Instantiating those target data by $\mathbf{c}^{\diamond}$ changes no hypothesis of its constraint-correction argument: the body fermions vanish there, while gauge and Higgs energy are retained and corrected by the full joint constraint map. Its variational, canonical and Cauchy objects are all obtained from the same action [2, 6]. Thus the restrictions in (5) share one source. □

Remark 1.3 (Standing boundary). The theorem proves represented law admission and one compatible classical realization. It does not promote every invariant matrix to the independent MGN coupling-standing image, derive the numerical constants, or identify the chosen tuple with measured pole parameters. Those are stronger source and realization questions.

1.3 Compatibility of the source theorems

The sources used in the proof do not all have the same target. Their status at $\mathfrak{T}_C^{\diamond}$ is as follows.

Source result	Exact use at the fixed target
CSP–S	Supplies the structural atlas and the marked represented law datum; it does not itself identify that datum with its separate classical history.
BCRS–3 / WDW–4	Supplies the parameterized coupled body, full classical action, constraints and development, instantiated here at the same law tuple.
Gravity / metricity / covariant differentiation / Einstein response	Supply the premetric role, licensed metric descent, metric-only Levi–Civita selection and minimal first-response form on their declared comparison classes.
WDW–1–3	Supply canonical source reconstruction, proper spatial reduction, the common-domain deformation algebra and the local refoliation method.
WDW–5 and WDW–8	Supply no nonlinear classical nonzero-fermion development; their quantum test algebra and restricted linearized backreaction are not used as such a theorem.
WDW–9–10	Their scalar-only/global-lineage target is not the corrected Standard–Model source. No literal global conclusion is transferred.
GRQM Module B	Retained as an exact-domain operator/scalar companion. Its quantum extension fibre is not inferred to be inhabited by this classical target.

This classification fixes the burden of every later theorem: a result is positive only where its own source, regularity and comparison domain have been constructed.

2 From premetric gravity to the licensed Einstein response

Gravity enters this construction before the Standard Model is expressed on a spacetime background. At that level it is not yet a metric tensor or a field equation. It is the constraint architecture required for a nondegenerate physical interior to retain standing while admitting lawful redescription. Metric geometry and Einstein dynamics are later, target-relative realizations of that role.

2.1 Automatic kernel governance and the gravity role

The target in Definition 1.1 retains the same oriented fields and records through admissible changes of gauge, Cauchy presentation and local coordinates. It also retains failed branches and irreversible source/record commitments. It is therefore a nondegenerate target-adequate physical-description regime. Kernel governance is consequently automatic:

$$\mathfrak{T}_C^{\diamond} \neq \varnothing \quad \Longrightarrow \quad \mathcal{K} = (\mathrm{Adm}, \mathrm{Stand}, \mathrm{Ref}, \mathrm{Irr})\ hboxgoverns \mathfrak{T}_C^{\diamond}. \tag{7}$$

There is no optional "kernel applies" premise.

On the regular admissible-evolution-and-redescription comparison class, let $\mathcal{C}$ be the standing constraint locus, $\omega_{\mathcal{C}}$ the restricted weak symplectic form and

$$\mathcal{D} = \ker \omega_{\mathcal{C}}. \tag{8}$$

The AASC gravity-necessity theorem establishes coisotropic standing, integrability of $\mathcal{D}$, first-class closure of the vanishing ideal and multiplier-type presentation of admissible Hamiltonians. Pure multipliers move only along $\mathcal{D}$ and induce no reduced evolution. These are the premetric gravity roles used here; neither a Lorentzian metric nor the Einstein tensor occurs in their statement [16].

Theorem 2.1 (Premetric necessity at the CSP–C target)**.** *Within the regular AER comparison class of $\mathfrak{T}_C^{\diamond}$, every admissible standing-preserving description has the coisotropic, characteristic, first-class and multiplier structure just displayed. The WDW regular constraint surface constructed below is a faithful classical realization of these roles. Removing the roles while retaining the same nondegenerate target changes the admissible comparison class or destroys standing-bearing redescription.*

Proof. Equation (7) supplies the kernel and fixed-domain consequence layer. The gravity-necessity theorem exhausts the same-scope ways of retaining nondegenerate standing under admissible Hamiltonian redescription on a regular AER locus: the characteristic directions are exactly the standing-preserving null directions, their ideal is first class, and arbitrary presentation motion appears as multipliers. The target has not yet been projected to a metric, so no metric premise enters this step. Sections 5 and 6 construct the announced realization directly and prove its infinite-dimensional characteristic equality without importing a finite-dimensional double-annihilator shortcut. □

2.2 Metricity is licensed, local and non-generative

At the pre-fixational boundary there are no already licensed metric relata. Every purported metric boundary authority either directly reclassifies standing, selects a representative, acts as a boundary operator, imports another scope, mutates the boundary role, collapses to later tensor content, or is inert presentation. The AMetric exhaustion leaves no same-background metric authorizer. Once $\mathfrak{T}_C^{\diamond}$ is fixed and standing-positive, however, the Lorentzian metric can serve as an interior comparison, causal and transport descriptor through the constrained L-metric licensing relation [17, 18].

Theorem 2.2 (Licensed metric descent and no backflow)**.** *The four-dimensional Lorentzian metric in CSP–C is a fixation-local, standing-preserving interior projection. It neither generates the target nor retroactively authorizes its standing. Coordinate, coframe and gauge representatives that preserve the complete licensed witness are skin; signature, dimension, source, boundary, topology, clock-use or continuation changes remain visible. Local metric articulation alone yields no global path independence, universal clock or cross-fixation identification.*

Proof. The fixed target supplies the relata, comparison rules, witness ledger and continuation identity required for physical-standing interpretability. The L-metric theorem therefore licenses the metric as interior articulation. If it were moved earlier to authorize that target, the AMetric normal form would classify the move as selection, scope transport, boundary operation, boundary mutation or inertness. If a later Einstein or Standard–Model success were used to repair the earlier standing, the no-backflow theorem would classify it as carrier substitution or post-selection repair. The fixation-local theorem retains the stated global boundaries. □

2.3 Standing-preserving finite transport and infinitesimalization

Fix a licensed local spacetime history of the target. The finite-rank carriers used in its action and constraint system are

$$
\begin{aligned}
&E_{\mathrm{tan}} = TM, \\
&E_{\mathrm{ten}}^{r,s} = (TM)^{\otimes r} \otimes (T^*M)^{\otimes s}, \\
&E_{\mathrm{ad}} = \mathrm{ad}\, P_{\mathrm{SM}}, \qquad\qquad E_H = P_{\mathrm{SM}} \times_{\rho_H} \mathbb{C}^2, \\
&E_{\Psi,r} = S_L(g) \otimes (P_{\mathrm{SM}} \times_{\rho_\Psi} V_\Psi) \otimes |\Lambda^4 T^*M|^{1/2}, \quad E_\chi = M \times \mathbb{R}.
\end{aligned}
\tag{9}
$$

Here (r,s) ranges only over the finite tensor types occurring in the fixed action and response ledger, while (Ψ,r) ranges over the five declared left-handed species and three generation labels of (2). The principal bundle, spin structure and half-density factor are the ones fixed in Definition 1.1; no new carrier is introduced.

Let $\mathsf{Cont}_C^{\mathrm{loc}}$ contain the sufficiently regular oriented path segments $c : p \rightsquigarrow q$ lying in one licensed local history and one fixed source, target, boundary and continuation record. Restriction to a subsegment is again admitted. Reversal is admitted when it remains in the same record, and two segments compose when their endpoint records agree. A lawful local diffeomorphism, gauge change or spin-frame change acts on both the path and the corresponding carrier presentation.

On every carrier in (9), let $\mathcal{A}^E$ be the connection already declared by the classical target: Levi–Civita on TM, the induced connection on tensor bundles, the internal gauge connection on E_{ad} and E_H, the induced Levi–Civita spin/half-density connection plus the represented gauge connection on $E_{\Psi,r}$, and the ordinary scalar connection on E_χ. For $u \in E_{c(0)}$, define $P_c^E u$ by the unique solution of

$$\dot U(t) + \mathcal{A}^E_{c(t)}(\dot c(t))U(t) = 0, \qquad U(0) = u, \qquad P_c^E u = U(1). \tag{10}$$

The resulting finite transport obeys

$$\begin{aligned} &P^E_{\mathrm{const}_p} = 1_{E_p}, && P^E_{c^{-1}} = (P^E_c)^{-1}, && (11)\\ &P^E_{c|[a,d]} = P^E_{c|[b,d]} P^E_{c|[a,b]} && (a \le b \le d), && (12)\\ &[P^E_{c_2 \star c_1}] = [P^E_{c_2} P^E_{c_1}] && \text{for every admitted composite.} && (13)\end{aligned}$$

The brackets in the last line are equality in the declared standing-descent comparison; in a fixed local presentation the two maps are literally equal. If a is a lawful redescription and $\rho_E(a_x)$ its carrier action, then

$$P^E_{a\cdot c} = \rho_E(a_q) P^E_c \rho_E(a_p)^{-1}. \tag{14}$$

Moreover every P_c^E preserves the fixed finite linear carrier graph:

$$P^E_c 0 = 0, \qquad P^E_c(u+v) = P^E_c u + P^E_c v, \qquad P^E_c(\lambda u) = \lambda P^E_c u. \tag{15}$$

For the fixed tensor grammar the same transport also obeys

$$P^{E\otimes F}_c = P^E_c \otimes P^F_c, \qquad P^{E^*}_c = (P^E_c)^{-*}, \qquad P_c(\operatorname{contr} T) = \operatorname{contr}(P_c T). \tag{16}$$

Because the coefficient in (10) at an initial point is $\mathcal{A}^E_p(\dot c(0))$, its infinitesimal path-germ dependence uses only $(p, \dot c(0))$; restriction and smooth dependence are inherited from the same ODE.

The scalar first-jet anchor is fixed by

$$J_C(X,f) = X[f]. \tag{17}$$

For an admitted path germ with $c(0) = p$ and $\dot c(0) = X_p$, the canonical infinitesimalization is

$$(\nabla^E_X s)_p = \left.\frac{d}{dt}\right|_{t=0} \left(P^E_{c|[0,t]}\right)^{-1} s(c(t)). \tag{18}$$

Theorem 2.3 (Target-relative standing-preserving transport)**.** *On every licensed local CSP–C history,* (10) *gives a smooth finite linear transport realization on each carrier in* (9)*. It satisfies identity, restriction, reversal, weak composition, lawful-redescription equivariance and carrier-operation preservation. Its canonical infinitesimalization is a connection with*

$$\nabla^E = d + A^E, \qquad \sigma_{\nabla^E}(\xi) = \xi \otimes 1_E, \qquad \nabla^E_X(fs) = X[f]s + f\nabla^E_X s. \tag{19}$$

Here A^E is the local-frame connection one-form representing the already declared $\mathcal{A}^E$, not a second connection datum. Tensor transport infinitesimalizes to

$$\nabla_X(s\otimes t)=(\nabla_X s)\otimes t+s\otimes\nabla_X t,\qquad X[\alpha(s)]=(\nabla_X\alpha)(s)+\alpha(\nabla_X s), \tag{20}$$

and commutes with every declared contraction. On tangent and tensor carriers it preserves metric comparison and is the Levi–Civita connection and its tensor extensions in the minimal first-order natural metric branch. On represented matter it is instead typed by

$$\begin{aligned} D^H_\mu &= \partial_\mu+\rho_H(A_\mu),\\ D^{\Psi,r}_\mu &= \partial_\mu+\rho_{\rm spin}(\omega_{{\rm LC},\mu}(e))+\rho_\Psi(A_\mu), \end{aligned} \tag{21}$$

with the analogous adjoint formula on $E_{\rm ad}$. Only the Levi–Civita spin/tensor term factors through the metric; the internal gauge term is a separately declared field coordinate.

Proof. Smooth finite-dimensional linear ODE theory applied fiberwise to (10) gives existence, uniqueness and smooth dependence. Uniqueness under constant paths, restriction, reversal and concatenation proves (11)– (13). Transformation of the connection coefficients and the simultaneous transformation of the initial carrier value give (14). Linearity of the ODE gives (15). The tensor product of two parallel solutions, the inverse-dual solution and every contracted solution satisfy the corresponding induced ODE with the same initial value; uniqueness proves (16). Differentiating that identity gives (20).

Thus the hypotheses of the kernel-to-covariant-differentiation theorem are fulfilled on the named finite-rank carriers: admitted local continuation, finite linear carrier preservation, the four finite transport identities, first-jet smoothness, lawful-redescription equivariance and the anchor (17). Its canonical infinitesimalization theorem gives (19) and excludes a nonconnection same-scope first-order realization [19, Thm. 19.1]. Levi–Civita parallel transport preserves g, so differentiation gives $\nabla g=0$; tensor compatibility follows from the preserved tensor operations. The metric-only natural uniqueness theorem below identifies this branch with $\nabla_{\rm LC}$. The associated-bundle ODE gives (21); its internal term depends on A_μ, so no factorization of the enriched matter connection through the metric forgetful map has been asserted. □

2.4 The metric connection factors through the metric projection

Let For_g forget every target coordinate except the Lorentzian metric. On the tangent and tensor metric branch only, define

$$\nabla_C:=\nabla_{\rm LC}\circ\mathrm{For}_g\,. \tag{22}$$

It is local, metric compatible and natural under the simultaneous lawful transport of the target and its metric.

Theorem 2.4 (Exact connection authority)**.** *Among first-jet, metric-only natural connection selectors, $\nabla_{\rm LC}$ is unique. Consequently (22) is the unique such selector after forgetting to the metric coordinate. This theorem does not claim uniqueness among connections permitted to depend on the clock, gauge fields, Higgs, orientation or other enriched source data.*

Proof. If ∇ is another metric-only first-jet natural selector, then $K=\nabla-\nabla_{\rm LC}$ is a tensor. In normal coordinates at a point the metric first jet vanishes, so naturality makes the lowered components of K invariant under the full orthogonal stabilizer of the metric jet. The central inversion sends an odd-rank covariant tensor to its negative and fixes the metric jet, hence $K=-K=0$. Naturality then gives equality in every chart. This is the precise metric-selector theorem of the covariant differentiation source [19]. Composing with For_g proves the CSP–C statement and also displays why no stronger enriched-target uniqueness has been assumed. □

2.5 Minimal first response

Let $E_{\mu\nu}[g]$ be a nontrivial general source-response candidate in the SameScopeMetric1 envelope: it is a local, natural, symmetric rank-two tensor, linear in curvature, with no curvature derivatives or products, no extra field, no hidden boundary datum and no representative dependence. At one point, orthogonal equivariance and the curvature symmetries give the complete normal form

$$E_{\mu\nu} = aR_{\mu\nu} + bRg_{\mu\nu} + cg_{\mu\nu}. \tag{23}$$

This is a classification on the whole response envelope, not a fit to the single occupied history.

Theorem 2.5 (Unique minimal Einstein response)**.** *Let the source be the closed total response constructed in Section 3. The unique nontrivial minimal first-response equation in the fixed four-dimensional Lorentzian envelope, up to the nonzero curvature normalization and the fixed source and homogeneous coefficients, is*

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T^{\text{closed}}_{\mu\nu}. \tag{24}$$

Proof. Theorem 2.1 supplies the gravity role, Theorem 2.2 supplies the faithful metric projection, and Theorem 2.4 supplies its metric connection. SameScopeMetric1 gives (23). Compatibility with every closed source in this general response envelope requires the geometric divergence to vanish identically. The contracted Bianchi identity gives

$$\nabla^\mu E_{\mu\nu} = (a/2 + b)\nabla_\nu R,$$

so $b = -a/2$. Nontrivial general response requires $a \neq 0$; dividing by a and naming the two retained coefficients gives (24). The argument uses general metric responses, so an accidentally constant scalar curvature on one solution cannot weaken the coefficient conclusion. This is the internal first-response theorem, including its exact uniqueness scope [20]. □

The result neither derives κ or Λ numerically nor excludes declared higher-curvature, torsionful, nonlocal, effective or richer-scope gravity theories. It says that such additions are not hidden alternatives inside the frozen minimal first-response target.

3 The common classical source is an attached response

The common datum used below is the total classical action and not a list of independently chosen stress tensors. Let

$$S_C[g,\Phi] = S_{\text{grav}}[g] + S_{\text{clock}}[g,\chi] + S_{\text{gauge}}[g,A] + S_{\text{H}}[g,H] + S_{\text{f}}[g,e,\omega,\Psi,H]$$

denote the real action fixed by the target, with its gauge, spin, boundary and parameter data held fixed. Here Φ denotes the complete non-gravitational field tuple. This notation is bookkeeping: a coupling or cross term occurs once in the displayed total action and is not assigned again to either of its incident sectors.

Definition 3.1 (Total compact-test response)**.** For a smooth target presentation $x = (g,\Phi)$ and every compactly supported smooth symmetric covariant tensor h, set

$$W_x(h) := \left.\frac{d}{ds}\right|_{s=0} S_{\text{matt}}[g_s, e_s, \omega_s, A, H, \Psi, \chi], \qquad S_{\text{matt}} := S_C - S_{\text{grav}}.$$

Here $g_0 = g$, $\dot g_0 = h$, and (e_s, ω_s) is the target's fixed symmetric coframe and induced torsion-free spin-connection lift. The internal gauge frame and the components of A, H, Ψ, χ are held fixed. Changing the lift is not an unrecorded second metric protocol. The metric-incidence relation MetResp_x is the graph of the same off-shell variation, evaluated for all such h; it is not restricted to tangents to the set of solved metrics.

The target action is local of finite differential order on the fixed clock slab. Tests are compactly supported in its interior; the first and last Cauchy slices are retained as continuation-boundary coordinates rather than treated as stress-surface terms. Consequently the first metric variation is linear and continuous in the usual LF topology on $\Gamma_c(S^2T^*M)$. It therefore satisfies the attachment conditions of Module A rather than merely having the same value on a selected family of histories [11].

Theorem 3.2 (Attachment and unique total source)**.** *At every smooth target presentation x, the complete action protocol gives an attachment certificate*

$$\mathrm{Graph}(W_x) = \mathrm{MetResp}_x\,.$$

There is a unique symmetric tensor-density distribution

$$T_x \in \mathcal{D}'(S^2TM \otimes |\Lambda|) \quad \textit{such that} \quad W_x(h) = \frac{1}{2}\langle T_x, h\rangle$$

for every compact metric test h. In a smooth representative this is the total matter tensor appearing in the metric Euler–Lagrange equation. Its support, regularity, boundary and lawful-redescription coordinates are those of the same complete action protocol.

Proof. The metric-incidence relation was defined from the independently fixed action protocol, so graph equality is an identity of complete relations rather than an inferred fit to a tensor. Differentiability gives additivity and homogeneity, locality and finite differential order give LF continuity, and covariance transports the graph together with its test field. The distributional representation theorem of Module A therefore gives existence and uniqueness. Since the action is varied only once, all interaction, restoration and clock terms enter the same functional and no marginal source can be spliced into the result. □

Definition 3.3 (Finite source registry)**.** Choose the finite decomposition of S_{matt} fixed by its summand and boundary ledger and write its attached responses as W_a, $a \in I$. Every term occurs exactly once. For each internal exchange channel $e \in E$, let $B_{ae} \in \{-1, 0, 1\}$ record its directed incidence with sector a, and let j_e be the corresponding current. External currents are denoted J_a^{ext}. Let $\mathcal{E}_b$ be the complete Euler–Lagrange residuals and let $K_{ab,\nu}$ be the differential operators with which those residuals enter the sector Noether identities. The registry is complete off shell when

$$W_x = \sum_{a\in I} W_a, \qquad \nabla_\mu {T_a}^\mu{}_\nu = \sum_b K_{ab,\nu}(\mathcal{E}_b) + J_{a\nu}^{\mathrm{ext}} + \sum_{e\in E} B_{ae} j_{e\nu}, \qquad \sum_{a\in I} B_{ae} = 0$$

for every internal channel. Every residual column and every boundary term is retained in its declared coordinate. The on-shell exchange registry is the restriction $\mathcal{E}_b = 0$, not part of the definition of the off-shell source inventory.

Theorem 3.4 (Closed summed Ward identity)**.** *On an actual solution of the complete coupled field equations,*

$$T_x = \sum_{a\in I} T_a, \qquad \nabla_\mu {T_x}^\mu{}_\nu = \sum_{a\in I} J_{a\nu}^{\mathrm{ext}}.$$

In the closed CSP–C target the declared external sum vanishes, hence $\nabla_\mu {T_x}^\mu{}_\nu = 0$. This is conservation of the total source; it does not assert separate conservation of interacting sector marginals.

Proof. Covariance of the complete action gives the off-shell Noether identities with the displayed Euler–Lagrange columns. Before any equation of motion is used, summing them cancels every internal current because its incidence column has zero sum. Exact term coverage and nonduplication identify the tensor sum with the unique tensor of Theorem 3.2. On the joint solution every $\mathcal{E}_b$ vanishes. This leaves only the declared external sum, which is zero in closed scope. □

Corollary 3.5 (Source reception by the classical gravity branch)**.** *Whenever the metric regularity and Cauchy data of the target occupy the declared Einstein/ADM envelope, T_x is an admissible source for that same classical development. The statement supplies source attachment and closure; existence of the coupled development and satisfaction of its constraints are proved separately rather than inferred from attachment.*

4 An occupied coupled classical body

We now exhibit a solution of the common-source equations rather than infer existence from source attachment. The construction begins with a regular Einstein–scalar datum, inserts genuinely nonzero gauge and Higgs fields, and corrects the gravitational and scalar variables so that the *coupled* constraints vanish. The corrected projection is not asserted to remain a solution of the uncoupled Einstein–scalar constraints.

4.1 The regular scalar seed

Let

$$\Sigma = \mathbb{H}^3/\Gamma, \qquad \mathrm{Ric}(h_*) = -2h_*, \qquad R(h_*) = -6,$$

be compact, connected and oriented. Fix $\epsilon > 0$, put

$$c_\epsilon = \sqrt{1 + \frac{\kappa\epsilon^2}{6}}, \qquad K_* = c_\epsilon h_*,$$

and set

$$z_* = \left(h_*, -\frac{c_\epsilon\sqrt{h_*}}{\kappa}h_*^{-1}; \chi_* = 0, p_{\chi,*} = \epsilon\sqrt{h_*}\right). \tag{25}$$

The scalar has energy density $\epsilon^2/2$ and vanishing spatial current. Thus

$$R(h_*) + K_*^2 - K_{*ij}K_*^{ij} = -6 + 6c_\epsilon^2 = \kappa\epsilon^2,$$

and both Einstein–scalar constraints vanish. In particular the seed is nonvacuum even though $\chi_* = 0$ on the distinguished slice.

Lemma 4.1 (No generalized Killing initial datum)**.** *The adjoint kernel of the Einstein–scalar constraint linearization at z_* is zero.*

Proof. For a generalized Killing pair (N, X), the scalar configuration equation contains

$$\mathcal{L}_X\chi_* + N\frac{p_{\chi,*}}{\sqrt{h_*}} = N\epsilon.$$

Hence $N = 0$. The remaining metric equation says $\mathcal{L}_X h_* = 0$. If X is Killing, the integrated Bochner identity on the closed hyperbolic manifold gives

$$\int_\Sigma |DX|^2\, d\mu_{h_*} = \int_\Sigma \mathrm{Ric}(X, X)\, d\mu_{h_*} = -2\int_\Sigma |X|^2\, d\mu_{h_*}.$$

The left-hand side is nonnegative and the right-hand side is nonpositive, so the equality forces $X = 0$. This is the occupied no-KID argument of WDW-4A, not a genericity substitution [6]. □

The mixed-order constraint map at z_* has a compatible smooth-tame right inverse R_z of fixed loss two on a neighborhood of the seed. It is obtained in WDW-4A from the balanced elliptic operator $D\Phi_z B_z D\Phi_z^\#$; Lemma 4.1 removes its kernel, and the inverse is compatible on the entire shifted Sobolev tower [6]. This compatible inverse is used below; no finite-dimensional annihilator identity is invoked.

4.2 A gauge–Higgs seed with no infinitesimal stabilizer

Write

$$G_{\mathrm{SM}} = \frac{\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y}{\mathbb{Z}_6}$$

and take the fixed trivial principal bundle on Σ. For $n = 2, 3$, choose a basis $X_1, \ldots, X_{n^2-1}$ of $\mathfrak{su}(n)$, pairwise disjoint coordinate balls, and supported one-forms b_k with $b_k(x_k) = 0$ and $db_k(x_k) \neq 0$. Set

$$a^{(n)} = \sum_k X_k b_k. \tag{26}$$

At x_k, the curvature of $\delta a^{(n)}$, divided by δ, tends to a nonzero multiple of X_k. Since the common centralizer of a basis of $\mathfrak{su}(n)$ is zero and injectivity is open, sufficiently small nonzero δ gives

$$D_{\delta a^{(n)}}\alpha = 0 \quad \Longrightarrow \quad \alpha = 0. \tag{27}$$

Choose also a real one-form a^Y with $da^Y \not\equiv 0$, a nonconstant real function f, and a nonzero Higgs direction $\eta \in \mathbb{C}^2$ for which the perturbation below is nonconstant. With $H_{\mathrm{vac}} = (0, v/\sqrt{2})^T$, define

$$\mathcal{A}_\delta = (\delta a^{(3)}, \delta a^{(2)}, \delta a^Y), \qquad H_\delta = H_{\mathrm{vac}} + \delta f \eta. \tag{28}$$

For small $\delta \neq 0$, all three selected curvature components are nonzero and H_δ is nowhere zero and nonconstant. A covariantly constant internal parameter has vanishing semisimple components by (27); its Abelian component is constant and is killed by its nonzero action on H_δ. The joint infinitesimal internal stabilizer is therefore zero.

Set the gauge electric momenta, Higgs momenta and body fermions to zero:

$$E_\delta = 0, \qquad \Pi_{H,\delta} = \Pi_{H^\dagger,\delta} = 0, \qquad \Psi_\delta = 0. \tag{29}$$

All internal Gauss densities and all matter spatial-momentum densities then vanish. For every corrected ADM momentum π^{ij}, choose the symmetric coframe lift

$$P^i{}_I = 2\pi^{ij} e_{jI}. \tag{30}$$

Its Lorentz moment map is zero. The fermionic primary matrix remains invertible on the nondegenerate coframe even at $\Psi = 0$.

4.3 Exact correction of the coupled constraints

Let Φ_{ES} be the Einstein–scalar constraint map and let S_{GH} denote the gauge–Higgs contribution to its normal and raw spatial rows. Put $X_0 = \mathrm{im}\, R_{z_*}$. On the shifted Sobolev tower define

$$\mathcal{F}(u, \delta) = \Phi_{\mathrm{ES}}(z_* + u) + S_{\mathrm{GH}}(z_* + u; \mathcal{A}_\delta, H_\delta), \qquad u \in X_0. \tag{31}$$

Since the Higgs potential vanishes at H_{vac}, $\mathcal{F}(0,0) = 0$, and

$$D_u\mathcal{F}(0,0) = D\Phi_{\mathrm{ES}}(z_*)|_{X_0}$$

is an isomorphism. Gauge and Higgs energy are quadratic at the vacuum:

$$\|S_{\mathrm{GH}}(z; \mathcal{A}_\delta, H_\delta)\|_q \le C_q(1 + \|z\|_{q+2})\|(\mathcal{A}_\delta, H_\delta - H_{\mathrm{vac}})\|_{q+1}^2.$$

The coefficients obey the same one-high-norm tame estimates as the inverse family. The parametric Nash–Moser theorem therefore gives a unique smooth-tame curve $u_\delta \in X_0$ with

$$\mathcal{F}(u_\delta, \delta) = 0, \qquad u_\delta = O(\delta^2) \tag{32}$$

at every Sobolev level [6]. This is an exact equation, not a truncation in δ.

Definition 4.2 (Occupied coupled body)**.** For sufficiently small $0 < |\delta| < \delta_1$, let w_δ consist of the corrected gravity–scalar datum $z_* + u_\delta$, a nondegenerate coframe and symmetric momentum (30), the gauge–Higgs seed (28), the momenta (29), and the fixed target, spin, source, attachment and boundary coordinates.

Theorem 4.3 (Exact nonzero coupled classical occupation)**.** *The datum w_δ is an exact zero of the primary fermion, Lorentz, internal Gauss, raw spatial and normal constraints. Its gauge curvature is nonzero, its Higgs field is nonzero and nonconstant, and its joint internal infinitesimal stabilizer is zero. It occupies the complete classical field and law target, although its fermion body is zero.*

Proof. The curvature and stabilizer assertions follow from (26)–(28). Equation (29) kills the Gauss and matter spatial densities; (30) kills the Lorentz moment map. The nondegenerate-coframe second-class matrix supplies the primary fermion reduction. Finally (32) is exactly the remaining coupled normal-and-spatial constraint equation. □

Theorem 4.4 (Source-attached maximal development)**.** *The smooth datum w_δ has a nonempty source-attached local Einstein–Yang–Mills–Higgs–scalar development. Its maximal globally hyperbolic representative is unique up to initial-surface-fixing isometry and the proper Lorentz and internal gauge redescriptions of the fixed branch. The scalar inequalities*

$$g^{-1}(d\chi, d\chi) < 0, \qquad n(\chi) > 0$$

hold on a nonempty neighborhood of the distinguished slice, so $V_\chi = \nabla\chi/g^{-1}(d\chi, d\chi)$ gives a future-directed local clock with $V_\chi(\chi) = 1$.

Proof. In harmonic metric gauge and hyperbolic internal gauge, the even-body field equations are quasilinear hyperbolic with smooth lower-order Higgs and scalar terms. The constraints proved in Theorem 4.3 propagate by the common action's Ward identities. Geometric uniqueness gives the maximal globally hyperbolic development of WDW-4B [6]. The body fermion equation is homogeneous and linear with Higgs-dependent Yukawa mixing, hence the zero initial section remains zero. At $\delta = 0$ the clock inequalities are strict. Continuous dependence and $u_\delta = O(\delta^2)$ preserve them after shrinking the development. Scalar covariance makes the clock and its first-failure boundary descend under the proper local presentation quotient. □

No global clock, geodesic completeness, nontrivial bundle sector, boundary charge sector, independent spin connection, torsion or numerical-parameter derivation is contained in these conclusions.

5 The complete joint constraint geometry

The occupied body is meaningful only with the constraint system in which it is a zero. This section fixes that system, including the fermionic second-class reduction, the interaction terms and the full off-shell hypersurface-deformation formula. All integrations are on the closed Σ, so no surface term is discarded.

5.1 Canonical carrier and the fermionic primary block

The even canonical variables are

$$(e_i{}^I, P^i{}_I), \quad (\chi, p_\chi), \quad (\mathcal{A}_i, E^i), \quad (H, \Pi_H), (H^\dagger, \Pi_{H^\dagger}),$$

where $\mathcal{A} = (A, W, B)$. The spin connection is always $\omega_{\mathrm{LC}}(e)$; it is not an independent coordinate. For every chiral species and generation, the half-density spinor is denoted Ψ. Its symmetrized kinetic term gives primary constraints

$$\vartheta = \pi_\Psi - \frac{i}{2}\Psi^\dagger, \qquad \bar{\vartheta} = \pi_{\Psi^\dagger} - \frac{i}{2}\Psi.$$

Their matrix on the nondegenerate-coframe branch is

$$\{\vartheta_A(x), \bar{\vartheta}_B(y)\} = -i\,\delta_{AB}\delta_\Sigma(x, y),$$

and hence is invertible. Imposing this block strongly gives the reduced bracket

$$\{\Psi_A(x), \Psi^\dagger_B(y)\}_D = -i\,\delta_{AB}\delta_\Sigma(x, y). \tag{33}$$

The zero body used in Section 4 is a point in this reduced odd carrier; it does not remove the carrier, its representation, or its coupling.

For $q \geq q_0 > 3/2 + 4$, the common reduced domain is

$$\begin{aligned} &e \in H^{q+2}, && P \in H^{q+1}, && \chi, H, \mathcal{A} \in H^{q+1}, \\ &p_\chi, \Pi_H, E \in H^q, && \Psi \in H^{q+1/2}. && \end{aligned} \tag{34}$$

The smooth fields form a common invariant core. Every bracket below is calculated on this one domain rather than assembled from incompatible completions [6].

5.2 Lorentz, internal, normal and spatial constraints

Let r index the three generations and let the sums below range over the declared Standard-Model chiral representations. The internal Gauss densities are

$$\begin{aligned} \mathcal{G}_3^a &= D_i E_3^{ia} + g_3 \sum_{\Psi,r} \Psi_r^\dagger \rho_\Psi(T_3^a)\Psi_r, \\ \mathcal{G}_2^A &= D_i E_2^{iA} + i g_2(\Pi_H \tau^A H - H^\dagger \tau^A \Pi_{H^\dagger}) \\ &\quad + g_2 \sum_{\Psi,r} \Psi_r^\dagger \rho_\Psi(T_2^A)\Psi_r, \\ \mathcal{G}_Y &= \partial_i E_Y^i + i g_1 Y_H(\Pi_H H - H^\dagger \Pi_{H^\dagger}) + g_1 \sum_{\Psi,r} Y_\Psi \Psi_r^\dagger \Psi_r. \end{aligned} \tag{35}$$

Here $\rho_\Psi(T_s)$ is the generator in the exact left-handed representation of (2); it is zero for a species neutral under factor s. In particular the conjugate color representations of u_r^c and d_r^c are not replaced by fundamental right-handed generators. Together they are denoted $\mathcal{G}[\alpha]$. The Lorentz moment map $\mathcal{J}[\lambda]$ contains the coframe term $2P^i{}_{[I} e_{J]i}$ and the fermionic spin current. Its coframe-momentum linearization has the algebraic right inverse

$$(S_e\lambda)^i{}_I = n_I b^i + \frac{1}{2} e_{Ik} r^{ki}, \qquad \lambda_{IJ} = 2 n_{[I} b_{J]} + r_{IJ}. \tag{36}$$

The Yang–Mills and Higgs normal and raw spatial densities are

$$\begin{aligned} \mathcal{H}_\perp^{\mathrm{YM}} &= \sum_{s=3,2,Y} \left(\frac{E_s{}^i \cdot E_{si}}{2\sqrt{h}} + \frac{\sqrt{h}}{4} F_{sij} \cdot F_s{}^{ij} \right), && \mathcal{H}_i^{\mathrm{YM}} = \sum_s E_s{}^j \cdot F_{sij}, \\ \mathcal{H}_\perp^H &= \frac{\Pi_H \Pi_{H^\dagger}}{\sqrt{h}} + \sqrt{h}\big((D_i H)^\dagger D^i H + V(H)\big), && \mathcal{H}_i^H = \Pi_H D_i H + (D_i H)^\dagger \Pi_{H^\dagger}. \end{aligned} \tag{37}$$

On the reduced odd carrier,

$$
\begin{aligned}
\mathcal{H}_\perp^\Psi &= -\frac{i}{2}\sum_{\Psi,r} \Psi_r^\dagger \alpha^i \overleftrightarrow{D}_i \Psi_r + \mathcal{H}_\perp^Y, \\
\mathcal{H}_i^\Psi &= -\frac{i}{2}\sum_{\Psi,r} \Psi_r^\dagger \overleftrightarrow{D}_i \Psi_r,
\end{aligned}
\tag{38}
$$

where $\mathcal{H}_\perp^Y$ is the Yukawa density of the one common action. Adding the gravitational and clock densities gives

$$
\begin{aligned}
\mathcal{H}_\perp^{\text{tot}} &= \mathcal{H}_\perp^g + \mathcal{H}_\perp^\chi + \mathcal{H}_\perp^{\text{YM}} + \mathcal{H}_\perp^H + \mathcal{H}_\perp^\Psi, \\
\mathcal{V}_i^{\text{raw}} &= \mathcal{H}_i^g + \mathcal{H}_i^\chi + \mathcal{H}_i^{\text{YM}} + \mathcal{H}_i^H + \mathcal{H}_i^\Psi.
\end{aligned}
\tag{39}
$$

The momentum-source sign is fixed here once. With future unit normal n, signature $(-+++)$, and

$$j_i = -T_{\mu\nu} n^\mu e_i{}^\nu,$$

the canonical action

$$\int dt \left[\Theta(\dot q) - \int_\Sigma (N\mathcal{H}_\perp^{\text{tot}} + N^i \mathcal{V}_i^{\text{raw}} + \lambda \mathcal{J} + \alpha \mathcal{G})\right] \tag{40}$$

has

$$\mathcal{H}_\perp^{\text{matt}} = \sqrt{h}\,\rho, \qquad \mathcal{V}_i^{\text{matt}} = -\sqrt{h}\, j_i. \tag{41}$$

Indeed the scalar contribution is $p_\chi D_i \chi$, fixing the sign. Equation (41) agrees with WDW-1 and with the direct lapse–shift variation in BCRS-3 [3, 2]. It normalizes a single inconsistent plus sign in the prose of WDW-4B; none of that paper's displayed canonical densities, nor its zero-momentum occupant, changes.

The raw generator acts by covariant spatial transport. The ordinary spatial Lie derivative is generated by

$$\mathcal{D}[\xi] = \mathcal{V}^{\text{raw}}[\xi] - \mathcal{G}[\iota_\xi \mathcal{A}] - \mathcal{J}[\iota_\xi \omega_{\text{LC}}]. \tag{42}$$

This is a change of constraint basis, not permission to erase the two moment maps off shell.

5.3 The off-shell joint algebra

For lapses N, M, put

$$s^i = h^{ij}(N\partial_j M - M\partial_j N). \tag{43}$$

Gauge electric energy paired with curvature gives

$$\{\mathcal{H}_{\text{YM}}[N], \mathcal{H}_{\text{YM}}[M]\}_D = \int_\Sigma s^i E^j \cdot F_{ij}.$$

Higgs momentum paired with its covariant gradient gives

$$\{\mathcal{H}_H[N], \mathcal{H}_H[M]\}_D = \int_\Sigma s^i (\Pi_H D_i H + (D_i H)^\dagger \Pi_{H^\dagger}),$$

and the reduced fermion bracket gives its raw spatial density. Curvature commutators in the fermion block cancel against the gauge–fermion and coframe–fermion cross variations. The Higgs potential and Yukawa terms are ultralocal in canonical coordinates; their NM terms vanish under antisymmetrization. Together with the gravity and clock computations, this proves

$$\boxed{\{\mathcal{H}[N], \mathcal{H}[M]\}_D = \mathcal{V}^{\text{raw}}[s] = \mathcal{D}[s] + \mathcal{G}[\iota_s \mathcal{A}] + \mathcal{J}[\iota_s \omega_{\text{LC}}].} \tag{44}$$

The remaining exact rows are

$$\begin{aligned}
\{\mathcal{D}[\xi],\mathcal{D}[\eta]\}_D &= \mathcal{D}[[\xi,\eta]], & \{\mathcal{D}[\xi],\mathcal{H}[N]\}_D &= \mathcal{H}[\mathcal{L}_\xi N],\\
\{\mathcal{G}[\alpha],\mathcal{G}[\beta]\}_D &= \mathcal{G}[[\alpha,\beta]], & \{\mathcal{J}[\lambda],\mathcal{J}[\mu]\}_D &= \mathcal{J}[[\lambda,\mu]],\\
\{\mathcal{D}[\xi],\mathcal{G}[\alpha]\}_D &= \mathcal{G}[\mathcal{L}_\xi \alpha], & \{\mathcal{D}[\xi],\mathcal{J}[\lambda]\}_D &= \mathcal{J}[\mathcal{L}_\xi \lambda],\\
\{\mathcal{G},\mathcal{J}\}_D &= 0, & \{\mathcal{G},\mathcal{H}\}_D = \{\mathcal{J},\mathcal{H}\}_D &= 0.
\end{aligned} \tag{45}$$

Theorem 5.1 (Exact complete classical constraint algebra)**.** *On the common domain (34), the primary-reduced Lorentz, internal Gauss, improved spatial and normal constraints form the first-class Dirac-bracket algebra (44)–(45). The structure function is the inverse metric. The expression $\{\mathcal{H}[N],\mathcal{H}[M]\} = \mathcal{D}[s]$ is valid only on the joint Gauss–Lorentz zero locus.*

Proof. Equations (37) and (33) give the sector calculations just displayed. Each remaining bracket is the action of its moment map on the complete variable tuple. The total normal density is internal- and Lorentz-invariant because every current, conjugate representation and Yukawa contraction is present. All calculations use one Dirac bracket, so their Jacobi identities are inherited rather than postulated. Compactness removes surface remainders. This is the W4B.7 calculation with its raw terms retained [6]. □

5.4 Full split regularity

Let $\mathbf{\Phi}_C$ collect the normal/raw-spatial, internal Gauss and Lorentz constraints. At w_δ, use as columns the regular gravity–scalar correction, the gauge/Higgs momentum directions, and the vertical coframe-momentum direction. Its derivative is

$$D\mathbf{\Phi}_C(w_\delta) = \begin{pmatrix} A_\delta & B_\delta & 0 \\ 0 & K_\delta & 0 \\ 0 & 0 & J_{e_\delta} \end{pmatrix}. \tag{46}$$

The top component of B_δ is zero at $E = \Pi_H = 0$, while its spatial component is

$$B_\delta(\dot E, \dot\Pi_H)_i = \sum_s \dot E_s{}^j \cdot F_{sij} + \dot\Pi_H D_i H_\delta + (D_i H_\delta)^\dagger \dot\Pi_{H^\dagger}. \tag{47}$$

It is generally nonzero. If R_δ, Q^G_δ and S_e are the respective right inverses, direct multiplication gives

$$Q^\triangle_\delta = \begin{pmatrix} R_\delta & -R_\delta B_\delta Q^G_\delta & 0 \\ 0 & Q^G_\delta & 0 \\ 0 & 0 & S_{e_\delta} \end{pmatrix}, \qquad D\mathbf{\Phi}_C(w_\delta) Q^\triangle_\delta = 1. \tag{48}$$

The mixed product is part of the inverse; deleting it would invalidate the momentum row.

For nearby w, put

$$\mathcal{E}_w = (D\mathbf{\Phi}_C(w) - D\mathbf{\Phi}_C(w_\delta)) Q^\triangle_\delta.$$

After shrinking until $\|\mathcal{E}_w\|_{q_0} < 1/2$, define

$$Q^C_w = Q^\triangle_\delta (1 + \mathcal{E}_w)^{-1}. \tag{49}$$

Sobolev multiplication and the compatible inverse families give

$$\|Q^C_w f\|_q \le C_q(\|f\|_q + (1 + \|w\|_{q+2})\|f\|_{q_0}),$$

with the analogous one-high-norm estimates for all parameter derivatives.

Theorem 5.2 (Occupied split joint constraint locus)**.** *Near every w_δ, the complete even constraint-zero locus is a nonempty split smooth-tame submanifold. Its joint generalized-KID adjoint kernel is zero, and its constraint right inverse (49) is compatible across Sobolev levels with fixed coefficient loss two.*

Proof. Equation (48) proves surjectivity at the occupant without dropping B_δ. Equation (49) proves it in a neighborhood. The tame submersion theorem gives the split zero locus. At the occupant the adjoint equations are triangular: the WDW-4A no-KID block kills lapse and shift, the positive Gauss energy kills the internal parameter, and the algebraic Lorentz split kills the Lorentz parameter. Equivalently, surjectivity gives zero adjoint kernel. The estimates following (49) give the asserted compatibility [6]. □

6 Regular reduction and finite refoliation

The bracket calculation of Theorem 5.1 is infinitesimal. It does not by itself identify the complete characteristic distribution, integrate normal deformations, or prove that two finite changes of Cauchy surface represent one lawful redescription. Those conclusions require the split regularity of Theorem 5.2, a licensed development, and a direct comparison between Cauchy evaluation and the constraint anchor.

6.1 The full characteristic distribution

Let $\mathcal{P}_C^\infty$ be the smooth body of the reduced canonical phase space and

$$\mathcal{C}_C^{\mathrm{reg}} = \{w \in \mathcal{P}_C^\infty : \mathbf{\Phi}_C(w) = 0\}$$

the regular joint constraint locus near w_δ. Write

$$A_w = D\mathbf{\Phi}_C(w), \qquad T_w\mathcal{C}_C^{\mathrm{reg}} = \ker A_w,$$

and retain the right inverse Q_w^C of (49). A descriptor is

$$e = (\xi, N, \alpha, \lambda),$$

with shift, lapse, internal-gauge and Lorentz components, and

$$C_w(e) = \mathcal{D}[\xi] + \mathcal{H}[N] + \mathcal{G}[\alpha] + \mathcal{J}[\lambda]. \tag{50}$$

Let

$$B_w(e) = X_{C(e)}(w).$$

On the common cotangent-density image of the weak canonical form,

$$\Omega_w(B_w(e), v) = -\langle A_w v, e\rangle. \tag{51}$$

It follows immediately that $\operatorname{im} B_w \subseteq \ker(\Omega|_{T_w\mathcal{C}_C^{\mathrm{reg}}})$.

Lemma 6.1 (One-multiplier factorization)**.** *If*

$$u \in \ker(\Omega|_{T_w\mathcal{C}_C^{\mathrm{reg}}}),$$

then there is one distributional descriptor ℓ on the complete constraint target such that

$$\Omega_w^\flat u = A_w'\ell. \tag{52}$$

It is obtained without a weak double-annihilator argument.

Proof. Put $\beta = \Omega_w^\flat u$. It vanishes on $\ker A_w$. Define

$$\ell(f) = \beta(Q_w^C f).$$

For every smooth tangent v, the split identity

$$v = (v - Q_w^C A_w v) + Q_w^C A_w v$$

has its first term in $\ker A_w$. Hence

$$\beta(v) = \beta(Q_w^C A_w v) = \ell(A_w v),$$

which is (52). The construction uses a single functional on the mixed constraint target; it does not choose unrelated Riesz representatives at successive Sobolev levels. □

Lemma 6.2 (Regularity of the complete multiplier)**.** *The distributional descriptor ℓ in Lemma 6.1 is smooth.*

Proof. At the occupied point, the formal adjoint of (46) is triangular. Its first equation is the WDW-4A balanced overdetermined-elliptic lapse–shift equation; smooth right-hand side and distributional elliptic regularity make the lapse and shift smooth. The Lorentz equation has the algebraic splitting (36). After the lapse–shift multiplier is known, the middle equation has the form

$$K_\delta^\# \alpha = r - B_\delta^\#(N, \xi),$$

with smooth right-hand side. Its energy is

$$\|D_{\mathcal{A}_\delta}\alpha\|_{L^2}^2 + \|\rho_H(\alpha)H_\delta\|_{L^2}^2,$$

and the zero-centralizer construction makes its kernel zero. Elliptic regularity makes α smooth. The same conclusion persists on the shrunk neighborhood because $\ell = (Q_w^C)'\beta = \mathcal{T}_w\beta$, and the explicit smooth-transpose Neumann family (60) below preserves smooth sections with the stated shifted-scale estimates. This is precisely the mixed-order regularity supplied by the source right inverses; no finite-dimensional coisotropic normal form is substituted for it [5, 6]. □

Theorem 6.3 (Complete characteristic equality)**.** *At every point of the occupied full-regular chart,*

$$\boxed{\ker(\Omega|_{TC_C^{\mathrm{reg}}}) = \operatorname{im} B.} \tag{53}$$

The image contains the proper lapse, shift, internal-gauge and Lorentz constraint directions.

Proof. The forward inclusion was proved after (51). For the reverse inclusion, combine Lemmas 6.1 and 6.2. If $e = -\ell$, then

$$\Omega^\flat u = A'ell = \Omega^\flat X_{C(e)}.$$

Weak nondegeneracy of Ω gives $u = X_{C(e)}$. This proves the equality on the ordinary even body. It is also complete in the graded tangent space: at $\Psi = 0$, the reduced odd two-form determined by $\{\Psi, \Psi^\dagger\}_D = -i\delta$ is algebraically nondegenerate. Hence an odd tangent vector that pairs to zero with every odd variation is zero, so the odd sector contributes no additional characteristic direction. □

6.2 Cauchy evaluation and source naturality

Let γ_δ be the source-attached maximal globally hyperbolic development of Theorem 4.4. For a cooriented spacelike Cauchy embedding $i : \Sigma \hookrightarrow M_{\gamma_\delta}$, define $\mathrm{Ev}_i(\gamma_\delta)$ by pulling back the coframe, scalar, gauge, Higgs and fermion fields, forming their normal momenta, using $K = \frac{1}{2}\mathcal{L}_n h$, and applying the gravitational Legendre map. The source, attachment, target, spin, boundary and complete relational records remain coordinates of this evaluated object.

The Gauss–Codazzi projections and the normal/tangential matter-source projections give

$$\mathbf{\Phi}_C(\mathrm{Ev}_i\,\gamma_\delta) = 0, \qquad (\mathcal{H}_\perp^{\mathrm{matt}}, \mathcal{V}_i^{\mathrm{matt}}) = (\sqrt{h}\,\rho, -\sqrt{h}\,j_i). \tag{54}$$

This is where the sign normalization (41) is load bearing.

Proposition 6.4 (Differentiated evaluation)**.** *If an embedding path has physical velocity*

$$\dot{i} = i_*\xi + Nn$$

and its presentation is simultaneously varied by internal and Lorentz parameters (α, λ)*, then*

$$d\,\mathrm{Ev}_i(i_*\xi + Nn, \alpha, \lambda) = X_{C(\xi,N,\alpha,\lambda)}(\mathrm{Ev}_i\,\gamma_\delta). \tag{55}$$

Proof. Differentiating the induced metric gives

$$\dot{h}_{ij} = \mathcal{L}_\xi h_{ij} + 2NK_{ij};$$

the Legendre map converts this and the Ricci evolution equation into the two gravitational Hamilton equations. Pullback of every matter field and its normal derivative gives its covariant Hamilton equation. In particular, the scalar row is

$$\dot{\chi} = \mathcal{L}_\xi \chi + Np_\chi/\sqrt{h},$$

and the gauge, Higgs and fermion rows include their actual internal and spin connections. The internal and Lorentz vertical variations are exactly the actions of their moment maps. The equations are those varied from the one action of Section 3; therefore all source forces and Yukawa exchanges occur once. This proves every canonical component of (55) [3, 5, 6]. ☐

Let Src send an evaluated object to its complete structured source history, not merely to the instantaneous tensor $T_{\mu\nu}$. For every admissible embedding and presentation,

$$\mathrm{Src} \circ \mathrm{Ev}_i = \mathrm{Src}(\gamma_\delta). \tag{56}$$

For a proper comparison $a : i_0 \to i_1$ within the same development, the source/Cauchy square

$$\begin{array}{ccc} (\gamma_\delta, i_0) & \xrightarrow{a} & (\gamma_\delta, i_1) \\ \big\downarrow{\scriptstyle \mathrm{Ev}_{i_0}} & & \big\downarrow{\scriptstyle \mathrm{Ev}_{i_1}} \\ w_0 & \xrightarrow{\mathrm{Ev}(a)} & w_1 \end{array} \qquad \mathrm{Src}(w_0) = \mathrm{Src}(w_1) = \mathrm{Src}(\gamma_\delta) \tag{57}$$

commutes. The equality is in the structured source quotient. It does not say the pulled-back tensor components on two leaves are numerically equal, and it does not identify different source histories.

Theorem 6.5 (Source-faithful reconstruction on the occupied branch)**.** *On the occupied regular chart, Cauchy evaluation and the WDW-1 reconstruction are inverse up to initial-surface-fixing isometry and the declared proper presentation arrows. They preserve the complete total source, attachment, Ward/exchange witness, target and boundary coordinates.*

Proof. Theorem 4.4 supplies a licensed development. The WDW-1 Legendre and ten-component reconstruction theorem identifies the canonical and covariant equations once the actual matter action and its source projections are supplied [3]. Those hypotheses hold by Theorems 3.2, 3.4 and 4.3. Geometric uniqueness gives the inverse up to the displayed arrows, while (57) proves source naturality. No stress-tensor-only reconstruction is used. ☐

6.3 Direct finite integration

Fix a contractible regular presentation neighborhood $\mathcal{U}$ in the product of a Cauchy-embedding chart with local internal and Lorentz presentation charts. It is chosen so that evaluation lands in the full-regular chart, the source and boundary records are fixed, the local development exists, and there is no infinitesimal stabilizer. Let $\mathcal{K} = \operatorname{im} B$ be the characteristic distribution.

The internal and Lorentz factors are integrated directly, not through an infinite-dimensional Lie theorem. A smooth descriptor curve (α_t, λ_t) determines, at each point of Σ, ordinary finite-dimensional Lie-group ODEs $\dot{g}_t g_t^{-1} = \alpha_t$ and $\dot{\ell}_t \ell_t^{-1} = \lambda_t$. Smooth parameter dependence gives smooth gauge and spin-frame curves on the chosen local chart. Their semidirect interaction with the embedding curve is the pullback action encoded by the mixed brackets of Theorem 5.1; thus these curves form the actual local presentation action groupoid.

The forward map on guarded paths is defined directly:

$$\mathrm{Ev}^{\mathrm{path}} : \Pi_1^{\mathrm{guard}}(\mathcal{U}) \longrightarrow \Pi_1(\mathcal{C}_C^{\mathrm{reg}}, \mathcal{K}), \qquad [i_\bullet, g_\bullet, \ell_\bullet] \longmapsto [\mathrm{Ev}_{i_\bullet}(g_\bullet, \ell_\bullet)\gamma_\delta]. \tag{58}$$

Proposition 6.4 makes its image characteristic. Evaluation commutes with constant paths, reversal, concatenation and guarded homotopy, so (58) is a functor without an infinite-dimensional Lie-II assumption.

To construct the local inverse, work on WDW-3's shifted embedding/canonical tower, apply the one-multiplier construction to a smooth characteristic vector u. The required regularizer is obtained from the already proved triangular inverse, without introducing a new full mixed normal operator. Under the fixed density, bundle-musical and Legendre identifications, write $\mathsf{J}_{g\chi}$ and J_G for the gravity–scalar and internal component identifications. The smooth transpose at w_δ is

$$\mathcal{T}_\delta \begin{pmatrix} b_{g\chi} \\ b_G \\ b_J \end{pmatrix} = \begin{pmatrix} R_\delta^{\prime,\mathrm{sm}} b_{g\chi} \\ (Q_\delta^G)^{\prime,\mathrm{sm}}(b_G - B_\delta' R_\delta^{\prime,\mathrm{sm}} b_{g\chi}) \\ S_{e_\delta}' b_J \end{pmatrix}. \tag{59}$$

The three displayed factors are concrete. If $\mathcal{G}_\delta^{g\chi}$ is the WDW-4A balanced gravity–scalar Green operator and $\mathcal{B}_{\delta,*}^{g\chi}$ its fixed row balancer, then

$$R_\delta^{\prime,\mathrm{sm}} = \mathcal{G}_\delta^{g\chi} A_\delta \mathcal{B}_{\delta,*}^{g\chi} \mathsf{J}_{g\chi}.$$

Likewise, self-adjointness of the positive internal normal operator gives

$$(Q_\delta^G)^{\prime,\mathrm{sm}} = (K_\delta K_\delta^{\#})^{-1} K_\delta \mathsf{J}_G,$$

and S_{e_δ}' is algebraic. Hence the second row of (59) is exactly the mixed solve

$$\alpha = (K_\delta K_\delta^{\#})^{-1} K_\delta \big(b_G - B_\delta^{\#}(N, \xi)\big), \qquad (N, \xi) = R_\delta^{\prime,\mathrm{sm}} b_{g\chi}.$$

Thus the mixed adjoint term is retained before the internal elliptic Green block is applied.

For the nearby inverse $Q_w^C = Q_\delta^{\triangle}(1+\mathcal{E}_w)^{-1}$, define its smooth transpose by the actual Neumann family

$$\mathcal{T}_w = \big((1 + \mathcal{E}_w)^{-1}\big)^{\prime,\mathrm{sm}} \mathcal{T}_\delta. \tag{60}$$

Each term of the low-norm Neumann series and its formal transpose preserves smooth sections; the WDW-4B tame estimates for $\mathcal{E}_w$, together with the two elliptic Green estimates and the algebraic Lorentz estimate above, make $\mathcal{T}_w$ level compatible. It is therefore an explicit smooth representative of $(Q_w^C)'$, not a Sobolev-dependent Hilbert transpose. For $u \in \mathcal{K}_w$, set

$$e_w(u) = -\mathcal{T}_w(\Omega_w^\flat u). \tag{61}$$

The WDW-3 embedding calculation gives loss three for (N, ξ). The mixed block $B_w^{\#}$ is order zero, the internal Green factor has order minus one, and S'_e is algebraic. Parametric elliptic estimates and (60) therefore give

$$\|e_w(u)\|_{\mathcal{Z}_{q+1}} \le C_q \left(\|u\|_{\mathcal{P}_{q+3}} + (1 + \|w\|_{q+3})\|u\|_{\mathcal{P}_{q_0+3}} \right), \tag{62}$$

For $m \ge 1$, differentiation of $(1 + \mathcal{E}_w)^{-1}(1 + \mathcal{E}_w) = 1$, followed by the same block estimates, gives the explicit one-high-norm bound

$$\begin{aligned} &\|D_w^m e_w(u)[\dot{w}_1, \ldots, \dot{w}_m]\|_{\mathcal{Z}_{q+1}} \\ &\quad \le C_{q,m} \Bigg[\|u\|_{\mathcal{P}_{q+3}} \prod_{a=1}^{m} (1 + \|\dot{w}_a\|_{q_0+3}) \\ &\qquad + (1 + \|w\|_{q+3})\|u\|_{\mathcal{P}_{q_0+3}} \sum_{a=1}^{m} \|\dot{w}_a\|_{q+3} \prod_{b \ne a} (1 + \|\dot{w}_b\|_{q_0+3}) \Bigg]. \end{aligned} \tag{63}$$

The formula is a single compatible smooth-tame inverse family on the shifted scale, not the untyped transpose of a right inverse. Equations (53) and (55) give

$$d\,\mathrm{Ev} \circ R = 1_{\mathcal{K}}, \qquad R \circ d\,\mathrm{Ev} = 1_{T\mathcal{U}},$$

where R evaluates the presentation velocity encoded by $e_w(u)$.

It remains to complement the characteristic image. Fix $w_0 = \mathrm{Ev}_{i_0}(\gamma_\delta)$ and extend (61) from $\mathcal{K}_{w_0}$ to every $v \in T_{w_0}\mathcal{C}_C^{\mathrm{reg}}$ by the same regularizing formula

$$e_{w_0}^{\mathrm{ext}}(v) = -\mathcal{T}_{w_0}(\Omega_{w_0}^{\flat} v).$$

Define

$$P_{\mathcal{K},w_0} v = B_{w_0}\big(e_{w_0}^{\mathrm{ext}}(v)\big). \tag{64}$$

Every factor in (59)– (60) is smooth tame on the shifted tower. If $v \in \mathcal{K}_{w_0}$, descriptor uniqueness and (61) give $P_{\mathcal{K},w_0} v = v$. Its range is contained in $\mathcal{K}_{w_0}$, hence $P_{\mathcal{K},w_0}^2 = P_{\mathcal{K},w_0}$ and

$$T_{w_0}\mathcal{C}_C^{\mathrm{reg}} = \mathcal{K}_{w_0} \oplus \ker P_{\mathcal{K},w_0}$$

is a proved tame splitting. After shrinking, the restriction $P_{\mathcal{K},w_0}|_{\mathcal{K}_w}$ is invertible by a smooth-tame Neumann series. In a split constraint-surface chart κ, set

$$\mathcal{F} = P_{\mathcal{K},w_0} \circ \kappa \circ \mathrm{Ev}\,.$$

The derivative has the smooth-tame inverse

$$(d\mathcal{F}_i)^{-1} = R_i \circ \left(P_{\mathcal{K},w_0}|_{\mathcal{K}_w}\right)^{-1}.$$

Equations (62) and (63), together with the standard pullback and normal-formation estimates, meet the hypotheses of the Nash–Moser inverse theorem. After shrinking, $\mathcal{F}$ is a smooth-tame local diffeomorphism, and the remaining split-chart coordinate of $\mathrm{Ev} \circ \mathcal{F}^{-1}$ is a graph tangent to $\mathcal{K}_w$. Evaluation therefore identifies $\mathcal{U}$ with a contractible characteristic plaque. Both guarded local path groupoids are pair groupoids.

Theorem 6.6 (Finite source-faithful refoliation)**.** *On the occupied regular branch, the proper history-presentation groupoid locally integrates the complete characteristic distribution. Two guarded finite deformation paths with the same endpoints determine the same local canonical arrow. That arrow preserves the source history, attachment, reconstruction class, complete field incidence, Dirac-observable profile, relational-record profile and zero-charge boundary coordinate.*

Proof. The local evaluation diffeomorphism identifies the presentation orbit with the characteristic plaque. Contractibility makes each path groupoid a pair groupoid, so each ordered endpoint pair has a unique local arrow. Functoriality of (58) identifies those arrows. Equation (57) gives source and reconstruction preservation; Dirac observables are constant on the characteristic distribution. A relational observable can vary with the clock value, but the complete correlation profile belongs to the one fixed history and is preserved. The remaining coordinates are arrow guards. This extends the WDW-3 construction by the full WDW-4B right inverse and presentation directions; the scalar-only deformation result does not suffice [5, 6]. □

6.4 Local quotient and global residuals

The local orbit projection of the characteristic pair groupoid gives a smooth-tame transversal quotient. Together with the proper spatial, internal and Lorentz descent, it is nonempty because it contains the class of w_δ. This local result does not create a global smooth superspace.

Definition 6.7 (Residual classifier)**.** An attempted finite comparison outside the preceding local chart is assigned to the first applicable class:

1. proper guarded homotopy within one regular chart;
2. KID or other stabilizer/isotropy;
3. mapping-class or other large transformation;
4. nonzero boundary charge, flux or central extension;
5. changed source, attachment or Ward witness;
6. changed relational profile or clock-use record;
7. changed bundle, spin structure, topology, orientation, branch or theorem target;
8. failure of development, common domain, admissibility or regularity.

Corollary 6.8 (Exact quotient scope)**.** *Only the first class in Definition 6.7 is quotient-trivial by Theorem 6.6. Every other class is retained as isotropy, a physical boundary or global charge, a changed source/record, a changed target, or a failure object.*

Proof. Compare target and existence first, then source and relational coordinates, then boundary and global transformation class, and finally stabilizer and local regular homotopy. If the first seven differences are absent, both routes lie in the local pair groupoid and class 1 applies. Conversely, the definition of a proper local arrow excludes classes 2–8. This is the WDW-3 residual decision order specialized to the coupled target [5]. □

Thus the classical result is local where the functional analysis is local and global only in its residual accounting. It proves refoliation coherence inside one source-attached maximal development, not equivalence of arbitrary maximal developments, charged-boundary evolutions or different Standard-Model targets.

7 Upstream continuation and corrected-body comparison

Continuation is fixed before its metric or canonical realization. We begin with an independently inhabited, complete, non-Hilbert RPI lane

$$\mathcal{C}_0 = (X_0^+, \sim_0, \widetilde{U}_0, q_0, U_0, S_0, F_0, \Sigma_0^I, \tau_0, V_0).$$

Here $q_0 : \widetilde{U}_0 \twoheadrightarrow U_0$ is a quotient submersion onto a regular C^1 Banach chart, S_0 is its embedded standing locus, and $F_0 \cap S_0 = \varnothing$ is its registered failure locus. The complete RPI signature is

$$\Sigma_0^I = (\Sigma_{\mathrm{loc}}, \Sigma_{\mathrm{trans}}, \Sigma_{\mathrm{field}}, \Sigma_{\mathrm{part}}, \Sigma_{\mathrm{gauge}}, \Sigma_{\mathrm{resp}}, \Sigma_{\mathrm{vac}}, \Sigma_{\mathrm{clock}}, \Sigma_{\mathrm{rec}}, \Sigma_{\mathrm{comp}}, \Sigma_{\mathrm{fail}}, \Sigma_{\mathrm{exch}}).$$

Equivalently it carries the full realized-interior tuple

$$(\mathrm{NonVacRec}, B, \mathrm{LCD}, \mathrm{Cmp}, \mathrm{BoundTrans}, \Phi, \Pi, G, M, \Omega, T, \mathrm{Cau})_0,$$

together with record, failure, boundary and exchange refinements. The clock $\tau_0 : U_0 \to J_0$ has its realized carrier, readout, orientation, retained-record, response, boundary, failure and reuse certificate. Finally $V_0 \in \Gamma^1(TU_0)$ satisfies all of Module C Definition 6.1: it is locally Lipschitz, $d\tau_0(V_0) = 1$, is tangent to S_0, is forward fail-closed, is specified on the full RPI chart, carries registered profile-closure and exact-fibre certificates, retains explicit exchange data, and is nonzero at a certified standing point [13]. No metric, ADM, Hilbert, Hamiltonian or solved-history coordinate is used in this lane.

Definition 7.1 (Upstream Moore continuation)**.** Let α^0 be the local flow of V_0. A morphism of $\mathrm{Cont}_0^{\tau_0}$ is a standing-bearing clock-forward Moore segment (u, T), $T \geq 0$, modulo lawful RPI presentation equivalence. Composition is endpoint-compatible concatenation and the identity has duration zero.

Theorem 7.2 (Formation and deletion)**.** $\mathrm{Cont}_0^{\tau_0}$ *is a nonempty category with a nonidentity positive-duration morphism. Deleting all metric, ADM, Hilbert, Hamiltonian, solved-history, downstream-record and compatibility projection maps leaves its objects, morphisms, standing/failure data, identities and composition unchanged.*

Proof. Picard–Lindelöf on the admitted chart gives the unique local flow and its cocycle law. Clock normalization gives $\tau_0(\alpha_t^0 u) = \tau_0(u) + t$, hence Moore concatenation is associative and a sufficiently short segment through a point with $V_0 \neq 0$ is nonidentity. Standing tangency and the failure certificate give preservation and no repair. The deleted maps occur in none of the displayed data, so this is a literal deletion theorem, not a claim that hidden projection variables remain available. □

Definition 7.3 (Inert same-source extension)**.** Let $s_C^\diamond := \mathfrak{p}^\diamond$ be the exact CSP–S structural/law source signature fixed in Section 1. Form

$$X_C^+ = X_0^+ \times \{s_C^\diamond\}, \quad U_C = U_0 \times \{s_C^\diamond\}, \quad \mathsf{S}_C = S_0 \times \{s_C^\diamond\}, \quad \mathsf{F}_C = F_0 \times \{s_C^\diamond\},$$

with $(x, s_C^\diamond) \sim_C (y, s_C^\diamond)$ exactly when $x \sim_0 y$, quotient map $q_C = q_0 \times \mathrm{id}$, clock $\tau_C = \tau_0 \circ \mathrm{pr}_0$, and seed

$$V_C = (V_0, 0).$$

The singleton is an inert source-ancestry/law coordinate. It neither replaces Σ_{resp} nor asserts a new dynamical coupling on the upstream lane.

Proposition 7.4 (Complete product restriction)**.** *The data of Definition 7.3 form an independently inhabited complete non-Hilbert RPI lane, and*

$$\mathrm{Cont}_C^{\tau_C} \cong \mathrm{Cont}_0^{\tau_0} \times \{s_C^\diamond\}.$$

Every clause of Module C Definitions 4.1 and 6.1 is preserved.

Proof. Product with a singleton preserves the quotient relation, quotient submersion, regular Banach chart, embedded standing locus and disjoint failure locus. All twelve RPI role coordinates remain those of Σ_0^I; the new coordinate merely records ancestry. Moreover

$$d\tau_C(V_C) = d\tau_0(V_0) = 1,$$

and tangency, local Lipschitz regularity, fail-closure, profile closure, exact-fibre preservation, explicit exchange data and nonvanishing are inherited componentwise. The flow is $\alpha_t^C(u, s_C^\diamond) = (\alpha_t^0 u, s_C^\diamond)$, which proves the category isomorphism and deletion property without a Hilbert or ray import. □

Only now receive the law coordinate as a classical source. By Theorem 1.2, the target-relative section of polynomial restriction is

$$\mathrm{Rec}_C(s_C^\diamond) = S_C^\diamond, \qquad \mathrm{Poly}(S_C^\diamond) = s_C^\diamond.$$

This is not a global injectivity claim for Poly.

Let $y_\delta(\tau)$ be the actual corrected coupled development supplied by the body, constraint and refoliation theorems, with its complete metric, coframe, gauge, total Module A source, boundary and Cauchy profile. Let $J_\delta = J_0 \cap \mathrm{dom}(y_\delta)$, restricted to the nonempty component containing the initial clock value, and let $\mathcal{Y}_C$ be its complete comparison space. The independently defined reduced coupled evolution vector field on $\mathcal{Y}_C$ is denoted $\mathcal{F}_C$.

Definition 7.5 (Corrected-body continuation certificate)**.** Set

$$\mathcal{U}_C^{\mathrm{stand}}|_{J_\delta} := \{(u, s_C^\diamond) \in S_C : \tau_0(u) \in J_\delta\}.$$

On this standing restriction, define the downstream comparison

$$P_C(u, s_C^\diamond) := y_\delta(\tau_0(u)).$$

The certificate consists of this map, its fixed gauge/boundary/Cauchy data, local uniqueness, and the two commutative identities

$$\mathrm{Anc}_{\mathrm{src}}(P_C(u, s_C^\diamond)) = \mathrm{Rec}_C(s_C^\diamond) = S_C^\diamond, \qquad dP_C(V_C) = \mathcal{F}_C \circ P_C.$$

Theorem 7.6 (The corrected body occupies the comparison certificate)**.** *The downstream map of Definition 7.5 is defined on a nonempty neighborhood of the initial standing point, sends that point to the corrected datum w_δ, and satisfies both displayed identities.*

Proof. Target–source compatibility and Theorem 3.2 identify the complete source ancestry of y_δ with the received action $S_C^\diamond$, proving the first identity. Since $d\tau_0(V_0) = 1$, the chain rule and the actual reduced equations give

$$dP_C(V_C) = \frac{d}{d\tau} y_\delta(\tau) = \mathcal{F}_C(y_\delta(\tau)) = \mathcal{F}_C \circ P_C.$$

For a standing point $u_0 \in U_0$ at the initial clock value, the initial-value theorem gives $P_C(u_0, s_C^\diamond) = w_\delta$ and local uniqueness. These are downstream facts proved after the product continuation has been formed; they do not define or alter V_C. □

Theorem 7.7 (Corrected-body intertwining)**.** *Every sufficiently short upstream segment f beginning at the certified initial point satisfies*

$$P_C \circ f \simeq_{\mathrm{law}} \widehat{\beta}_C(f) \circ P_C,$$

where $\widehat{\beta}_C$ is the unique local development of the same coupled source system. The initial metric/source datum is w_δ, not the old uncoupled scalar seed.

Proof. The differential identity makes the projected upstream curve and the independently defined coupled development solutions of the same reduced equations. The source-ancestry identity and fixed target data give the same initial, gauge, boundary and source coordinates. Local uniqueness yields the lawful equivalence. □

Corollary 7.8 (Exact four-profile projection status)**.** *On the standing Moore subcategory define*

$$\mathrm{Pref}_C(u, s_C^{\diamond}) := (\tau_0(u), \Sigma_{\mathrm{rec}}(u)), \qquad \mathrm{Anc}_C(u, s_C^{\diamond}) := \mathrm{Rec}_C(s_C^{\diamond}) = S_C^{\diamond}.$$

Then: (i) the metric/source projection P_C is positive and preserves identities, composition, standing, orientation and source ancestry; (ii) the native-prefix record projection is positive and advances by Moore duration; (iii) the compatibility/source-ancestry projection is positive, constant and exactly closed; and (iv) the Hilbert/ray quantum projection is not-activated*, not positive and not certified empty. A downstream failure projection is likewise not activated: failure and no-repair remain exact upstream coordinates, and failed objects are outside the domain of P_C. A nonzero exchange requires a separately declared exchange-extended process and is not presentation equivalence.*

Proof. Theorems 7.6 and 7.7 prove (i). Clock normalization and Moore concatenation prove (ii), while the ancestry square proves (iii). The lane contains no Hilbert or ray coordinate and the standing-domain comparison assigns no image to a failed object, giving the two not-activated statuses in (iv). □

Remark 7.9 (Direction of authority). The independently inhabited RPI lane authorizes continuation. The inert singleton records which law is later received but adds no upstream dynamics. The corrected development authorizes only the downstream comparison and cannot repair a failed upstream prefix.

8 Finite-algebra classical fermions and exact coupled response

The occupied body of Section 4 has nonzero gauge and Higgs fields but invariant zero fermions. That is already a solution of the full classical action; it is not, however, the strongest classical occupation available. This section constructs nonzero anticommuting classical spinor fields over that body. The coefficient algebra is finite, so the proof reduces to finitely many ordinary constraint and hyperbolic Cauchy problems. No infinite Grassmann completion, supermanifold maximality, Hilbert state or quantization is involved.

8.1 The real finite coefficient algebra

For $N \geq 1$, let

$$A_N = \Lambda_{\mathbb{C}}(\theta_1, \ldots, \theta_N), \qquad \mathfrak{m} = (\theta_1, \ldots, \theta_N), \qquad \mathfrak{m}^{N+1} = 0. \tag{65}$$

Give A_N the conjugate-linear, order-reversing involution

$$\theta_a^* = \theta_a, \qquad (ab)^* = b^* a^*. \tag{66}$$

For $I = \{i_1 < \cdots < i_k\}$ put

$$\varepsilon_I = i^{k(k-1)/2} \theta_{i_1} \cdots \theta_{i_k}. \tag{67}$$

Reversal contributes $(-1)^{k(k-1)/2}$ and conjugation of the phase contributes the same sign, so $\varepsilon_I^* = \varepsilon_I$. These elements form a real basis of the self-adjoint part. Write

$$E_N = ((A_N)_{\bar{0}})_{\mathrm{sa}}, \qquad \mathrm{body} : A_N \longrightarrow \mathbb{C}$$

for the even real form and the distinguished augmentation.

Bosonic fields take values in E_N. An odd Weyl half-density is expanded in odd ε_I with ordinary complex spinor coefficients; its dagger is fixed by (66), the spinor adjoint and reversal of coefficient order. It is not a second independent field. The lapse, shift, coframe, gauge potential, Higgs real section and canonical momenta obey their usual reality conditions coefficientwise. In the half-density chart of Section 5, the fermionic primary matrix remains exactly $-i\delta$; no odd implicit-function theorem is required.

Lemma 8.1 (Finite Taylor–Neumann calculus)**.** *Let F be a smooth local field operator of finite differential order near a body field u_0, and let*

$$u_{A_N} = u_0 + \sum_{I \neq \varnothing} \varepsilon_I u_I.$$

Then

$$[\varepsilon_I]F(u_{A_N}) = DF(u_0)u_I + R_I(\{u_J : 0 < |J| < |I|\}), \tag{68}$$

where R_I is a finite differential polynomial in lower-degree coefficients. Inverse metrics, coframes, volume factors and every other smooth inverse-body operation have unique finite nilpotent extensions.

Proof. Taylor expansion in $\mathfrak{m}$ terminates because $\mathfrak{m}^{N+1} = 0$. Any term containing u_I and another positive-degree coefficient has degree greater than $|I|$, so the only degree-$|I|$ occurrence of u_I is the displayed linearization. Coefficient independence proves uniqueness. If $a = a_0 + n$ with invertible body a_0 and nilpotent n, then

$$a^{-1} = a_0^{-1} \sum_{r=0}^{N} (-a_0^{-1} n)^r,$$

which proves the inverse statement; the remaining smooth operations follow by the same finite Taylor calculation. □

8.2 Exact constrained initial data

Let $\mathbf{\Phi}_{A_N}$ be the A_N-extension of the primary-reduced constraint map with rows

$$(\mathcal{H}_\perp, \mathcal{V}_i^{\mathrm{raw}}, \mathcal{G}_3, \mathcal{G}_2, \mathcal{G}_Y, \mathcal{J}_{IJ}).$$

The fermionic second-class pair has already been imposed strongly and multiplier momenta are identically zero. Let Q_δ^C be the real full right inverse at the corrected body w_δ from (48); in particular it includes the mixed column $-R_\delta B_\delta Q_\delta^G$. It acts on the real constraint spaces and commutes with the involution.

Theorem 8.2 (Finite exact initial-constraint lift)**.** *Fix arbitrary smooth reduced odd Cauchy coefficients and, at every positive even exterior degree, arbitrary smooth real kernel coordinates $k_I \in \ker D\mathbf{\Phi}_C(w_\delta)$. There is a unique constrained coefficient tuple relative to these choices and the splitting Q_δ^C, given recursively by*

$$z_I = k_I - Q_\delta^C r_I, \qquad r_I = [\varepsilon_I]\mathbf{\Phi}_{A_N}(w_\delta + \textit{terms of degree} < |I|). \tag{69}$$

It defines an exact A_N-valued initial datum with body w_δ and

$$\mathbf{\Phi}_{A_N}(w_{\delta,A_N}) = 0. \tag{70}$$

All Lorentz spin terms and the fermion-dependent canonical coframe-momentum shift are included in the residual and in the full Lorentz row.

Proof. The constraints are even. At an odd exterior degree there is therefore no constraint coefficient, and the reduced odd Cauchy coefficient is free. At an even degree, Lemma 8.1 gives

$$[\varepsilon_I]\mathbf{\Phi}_{A_N} = D\mathbf{\Phi}_C(w_\delta)z_I + r_I,$$

where r_I depends only on already fixed lower degrees. Substitution of (69) and $D\mathbf{\Phi}_C(w_\delta)Q^C_\delta = 1$ gives zero. Conversely, $k_I = z_I + Q^C_\delta r_I$ lies in the kernel, proving the claimed split parametrization.

The recursion is finite. At each step the compatible loss-two right inverse of Section 5 acts on smooth coefficients on the whole shifted Sobolev tower. Its reality and the star-adapted basis preserve the real section. Because the canonical variables are those after the fermionic coframe-momentum shift, the Lorentz equation contains the actual spin bilinear and is solved by the same full row rather than by reusing the zero-fermion symmetric lift. □

The kernel coordinates in this theorem are physical initial-data freedom; changing the non-canonical splitting reparametrizes them. It does not make all nilpotent lifts of the body gauge equivalent or unique.

8.3 The body principal systems

Choose regular clock values $\tau_- < \tau_+$ so that

$$M_0 = \{x \in M_{\gamma_\delta} : \tau_- < \chi(x) < \tau_+\} \quad \text{is open and globally hyperbolic,} \qquad \overline{M}_0 \Subset \{d\chi \text{ timelike}\}. \tag{71}$$

The compact closure is taken inside the regular clock region of the body development. Fix the harmonic metric gauge and hyperbolic internal gauge used in WDW–4B. Let

$$\mathbb{V}_{\mathrm{ev}} = S^2T^*M_0 \oplus (T^*M_0 \otimes \mathfrak{g}_{\mathrm{SM}}) \oplus E_H \oplus \mathbb{R} \tag{72}$$

contain the metric, gauge, Higgs and clock coefficients. Trace reversal and the gauge reductions put the body linearization in the form

$$P_{\mathrm{ev}}u = \Box^{\nabla}_{g_\delta}u + A^\mu\nabla_\mu u + Bu, \qquad \sigma_2(P_{\mathrm{ev}})(\xi) = -g_\delta^{-1}(\xi,\xi)1_{\mathbb{V}_{\mathrm{ev}}}. \tag{73}$$

Indeed, the reduced Einstein block has principal part $-\frac{1}{2}\Box\bar{h}_{\mu\nu}$, the reduced Yang–Mills block has $-D^\rho D_\rho a_\mu$, and the Higgs and scalar blocks have their covariant wave operators. Variations of the coupled stress and connection contribute at most first derivatives of another unknown and hence occur in A^μ and B; no block-diagonal assumption is made about the lower-order terms. Multiplying the trace-reversed metric block by its nonzero normalization gives the common principal symbol in (73).

Choose a Cauchy temporal coordinate with $g_\delta = -\beta^2dt^2 + h_t$. For

$$U = (\nabla_{\partial_t}u, \nabla^\Sigma u, u), \tag{74}$$

the geometric first-order reduction has positive time matrix equivalent to $\mathrm{diag}(\beta^{-2}, 1, 1)$ and symmetric spatial principal matrices with respect to the positive bundle metric induced by h_t. Thus it is symmetric hyperbolic. This is the standard geometric reduction of a normally hyperbolic bundle operator, written here on the actual direct sum (72) [25].

For the odd variables, collect all declared two-component Weyl half-densities and their fixed adjoints into $\mathbb{V}_{\mathrm{odd}}$. Their body equation is

$$P_{\mathrm{odd}}\psi = (\not{D}_{g_\delta,\mathcal{A}_\delta} + Y(H_\delta))\psi. \tag{75}$$

In a future-normal coframe its evolution form has $A^0 = 1$ and Hermitian spatial matrices given by the Pauli/Clifford symbol. The gauge and Levi–Civita connections, Higgs field and

Yukawa matrices are smooth zero-order coefficients. Hence the future-normal symbol is positive and $P_{\rm odd}$ is symmetric hyperbolic. Equivalently, the associated classical Dirac system has the symmetric-hyperbolic principal symbol proved for globally hyperbolic spin manifolds; zero-order Yukawa mixing does not alter it [25].

Lemma 8.3 (Degreewise principal operator)**.** *Suppose all coefficients below exterior degree k solve the equations. If k is even, the new coefficient satisfies*

$$P_{\rm ev}u_I = f_I; \tag{76}$$

if k is odd, it satisfies

$$P_{\rm odd}\psi_I = g_I. \tag{77}$$

The forcing is a smooth differential polynomial in lower-degree coefficients. No same-degree unknown occurs in it.

Proof. Apply Lemma 8.1 to the gauge-fixed Euler–Lagrange operator. Bosonic unknowns have even parity and fermionic unknowns odd parity. A same-degree even perturbation cannot multiply the body fermion, because that body field is zero; a same-degree odd field can enter an even equation only with another positive odd degree. The only remaining same-degree term is therefore the body linearization of the appropriate parity block. All interactions are retained in that linearization or in the already known forcing. □

8.4 Cauchy induction and subsidiary propagation

The coefficient data used by the first-order reductions are the enlarged data determined by the constrained canonical tuple, not an independent second set of choices. For every even coefficient, the initial value in (74) is

$$U_I|_\Sigma = (v_I, D^\Sigma u_I, u_I), \tag{78}$$

where u_I and the canonical momenta come from Theorem 8.2, v_I is their image under the body Legendre map together with the fixed lapse, shift and gauge-source values, and $D^\Sigma u_I$ is the actual spatial derivative of u_I. These three entries are therefore compatible and are not freely varied separately. For an odd coefficient, the Dirac–Yukawa equation is already first order, so $\psi_I|_\Sigma$ is the complete Cauchy datum; it has no independent normal derivative. Initial harmonic and internal-gauge defects, and the normal derivatives required by their wave subsidiary systems, are set to zero using the reduced equations and the physical constraints.

The smooth Cauchy theorem for a linear symmetric-hyperbolic system on a globally hyperbolic spacetime gives existence for arbitrary smooth forcing and smooth Cauchy data, uniqueness, finite propagation and continuous dependence [24, Def. 5.1, Cor. 5.5, Thm. 5.6 and Prop. 5.7]. Applying it to (76) and (77) is legitimate because the bundles have finite rank and their coefficients are smooth on M_0.

To prove subsidiary propagation, let $Z = (C_\mu, \Lambda, \mathcal{C})$ collect the harmonic gauge quantity, the internal hyperbolic-gauge quantity and the complete physical constraint vector. Diffeomorphism, internal-gauge and Lorentz invariance of the one action give the exact Noether identities. Applied to the reduced equations, their principal subsidiary form is

$$\begin{aligned} \Box_{g_\delta} C_\mu + R_\mu{}^\nu C_\nu &= L_\mu(C, \nabla C, \Lambda, \nabla\Lambda), \\ D^\rho D_\rho \Lambda + [F^\rho{}_\cdot, \Lambda] &= L_G(C, \nabla C, \Lambda, \nabla\Lambda), \end{aligned} \tag{79}$$

$$\partial_t \mathcal{C}_a = A_a{}^{bi} D_i \mathcal{C}_b + B_a{}^b \mathcal{C}_b. \tag{80}$$

Here every term on the right is linear homogeneous in the displayed defects, with smooth field-dependent coefficients. The first two lines are the divergence of the reduced covariant equations;

the last is equivalently the canonical identity

$$\dot{\mathbf{\Phi}}_{C,a} = \{\mathbf{\Phi}_{C,a}, H_{\text{tot}}\}_D = \mathsf{C}_a{}^b \mathbf{\Phi}_{C,b}, \tag{81}$$

which follows from the full off-shell algebra of Theorem 5.1. The raw spatial, Gauss and Lorentz terms in (44) are part of $\mathsf{C}_a{}^b$ and have not been discarded.

Theorem 8.4 (Finite-algebra Cauchy realization)**.** *For every finite N, every datum supplied by Theorem 8.2, and compatible initial harmonic and internal-gauge data assembled as in (78), there is a unique smooth A_N-valued classical solution Γ_{A_N} on the common globally hyperbolic clock slab M_0 relative to the fixed gauges and Cauchy data. It satisfies the unreduced covariant equations, the full physical constraints, the gauge conditions, the reality relations and the complete graded Ward identity exactly. Its body is γ_δ.*

Proof. Use γ_δ at degree zero. Assume degrees below k have been solved. Lemma 8.3 gives one linear symmetric-hyperbolic problem with smooth forcing and the constrained initial coefficient from Theorem 8.2. The cited Cauchy theorem gives its unique smooth solution on M_0. Repeat through the finitely many degrees. Lemma 8.1 then says the finite sum solves the reduced equations exactly, not modulo a discarded order.

At coefficient degree k, the subsidiary equations (79)–(80) have their body homogeneous operator acting on Z_I; every other term contains a lower-degree defect and vanishes by induction. The initial physical defects vanish by (70), and the gauge defects vanish by the compatible gauge data. Symmetric-hyperbolic uniqueness gives $Z_I = 0$. Thus the reduced solution solves the full covariant equations and all constraints. The canonical reconstruction of Section 6 identifies its Cauchy evolution with the full Dirac Hamiltonian system.

Every operator, gauge and initial relation commutes with the involution. Applying star to a coefficient problem therefore produces the adjoint problem with the same declared data; uniqueness proves the reality relation. The body projection deletes all positive-degree terms and recovers γ_δ. □

The theorem is global only over the chosen body slab. It neither extends the body through a maximal-development boundary nor orders all A_N-valued developments by a new supergeometric maximality relation.

8.5 An explicit nonzero source and response

The case $N = 1$ already permits a nonzero odd solution, but all quadratic monomials in its single generator vanish. It is therefore a useful spectator check, not a backreaction witness. For $N \geq 2$ a nonzero witness is explicit.

Let u be a smooth nonzero Cauchy half-density in a declared chiral species with $Y_\Psi \neq 0$, supported in one coordinate ball, and set

$$\Psi^{(1)} = (\theta_1 + i\theta_2)u, \qquad \varepsilon_{12} = i\theta_1\theta_2. \tag{82}$$

Then $\varepsilon_{12}^* = \varepsilon_{12} \neq 0$ and

$$(\Psi^{(1)})^\dagger \Psi^{(1)} = 2\varepsilon_{12} u^\dagger u. \tag{83}$$

The degree-two hypercharge Gauss residual contains

$$2g_1 Y_\Psi \varepsilon_{12} |u|^2, \tag{84}$$

which is nonzero wherever $u \neq 0$. Choose the first spatial jet of u so that, at some point of its support, one component of

$$[\varepsilon_{12}]\mathcal{H}_i^\Psi = -i\, u^\dagger \overleftrightarrow{D}_i\, u = -\sqrt{h}\,[\varepsilon_{12}] j_i^\Psi \tag{85}$$

is nonzero. The first equality is the fermionic raw-spatial density (38), with the factor two from (83); the second is the corrected source projection (41). The Yukawa interaction is ultralocal in the canonical variables and contributes no raw-spatial density. Equivalently, its algebraic stress term is proportional to $g_{\mu\nu}$ and has zero $n^\mu e_i^\nu$ projection. It therefore cannot cancel (85).

Corollary 8.5 (Nonzero classical fermion and forced even response)**.** *For $N \geq 2$, the target has an A_N-valued classical solution with nonzero odd Dirac–Yukawa field, nonzero self-adjoint quadratic fermion current and a necessarily nonzero even coupled response. The body remains the occupied zero-fermion solution.*

Proof. Theorem 8.4 develops (82). Uniqueness prevents a nonzero Cauchy coefficient from becoming the zero solution. At degree two, setting every even correction to zero would leave (84) in the constraint equation, contrary to (70); hence the even coupled correction is nonzero. Independently, the chosen first jet gives the nonzero normal–spatial source (85); the Yukawa term contributes nothing to that projection, and the exact degree-two field equations include its response. □

This corollary does not isolate a necessarily nonzero metric coefficient: the metric, gauge, Higgs and scalar even responses are coupled. Nor does a self-adjoint nilpotent coefficient carry a positive numerical order. The result is a classical anticommuting-field solution and exact nilpotent backreaction, not a commuting spinor solution, Fock vector, particle state, functional integral or interacting QFT.

9 Finite graded source representation

Let $\mathcal{A}_N$ be the finite complex exterior algebra used in the graded realization and let

$$E_N := \{a \in (\mathcal{A}_N)_{\bar{0}} : a^* = a\}$$

be its finite-dimensional real even part. Conjugation is antilinear and order reversing, and the body map is the distinguished augmentation. Each metric or bosonic target bundle is regarded as its underlying real bundle and then tensored with E_N; Higgs fields retain their complex weak-doublet structure and gauge fields retain their Lie-algebra structure. Odd fields and their adjoints are related by the involution rather than treated as independent coordinates. In an ordered exterior basis the self-adjoint even real form is expressed by the corresponding star-adapted phases and coefficient relations. All nonlinear functions of an even element with invertible body are evaluated by the finite Taylor–Neumann expansion.

The graded realization theorem of Section 8 supplies an actual solution $x_\mathcal{A}$ of the same graded-real classical action on a compact slab, with the corrected CSP–C body and the complete constraint and reality data.

Definition 9.1 (Coefficient-complete response)**.** The off-shell metric variation of the graded matter action defines

$$W_\mathcal{A} : \Gamma_c(S^2T^*M) \longrightarrow E_N, \qquad W_\mathcal{A}(h) := \frac{d}{ds}\bigg|_{s=0} S_{\mathrm{matt},\mathcal{A}}[g_\mathcal{A} + sh, \Phi_\mathcal{A}],$$

where h is an ordinary real compact test tensor and all other fields are held fixed in the sense of Definition 3.1; the symmetric coframe and induced torsion-free connection lift are extended E_N-linearly. After reconstruction on real tests, this response extends uniquely by E_N-linearity to compact graded metric tests. For $\lambda \in E_N^*$, set $W_\lambda := \lambda \circ W_\mathcal{A}$.

Theorem 9.2 (Basis-independent finite-coefficient source)**.** *The complete graded action protocol attaches to $W_{\mathcal{A}}$ on all compact metric tests. There is a unique*

$$T_{\mathcal{A}} \in \mathcal{D}'(S^2TM \otimes |\Lambda|) \otimes_{\mathbb{R}} E_N$$

such that

$$W_{\mathcal{A}}(h) = \frac{1}{2}\langle T_{\mathcal{A}}, h\rangle \quad \text{for every } h.$$

The construction is independent of a basis of E_N, respects the involution, and its body is the ordinary total tensor of Theorem 3.2.

Proof. For every $\lambda \in E_N^*$, the scalar response W_λ is the real linear first variation of the actual coefficient functional $\lambda(S_{\mathrm{matt},\mathcal{A}})$. Its graph is therefore the scalar evaluation of the full independently formed metric-incidence graph, and it is LF-continuous. Theorem 3.2, applied coefficientwise, gives a unique real tensor distribution T_λ. Linearity and uniqueness imply $T_{a\lambda+b\mu} = aT_\lambda + bT_\mu$. Since E_N is finite dimensional, this linear family is represented by a unique element of the displayed tensor product; dual separation proves basis independence. Applying the involution or the body map to the response and using uniqueness gives the last assertions. □

Theorem 9.3 (Full graded Ward closure)**.** *On the actual graded solution,*

$$\mathrm{div}_{g_{\mathcal{A}}} T_{\mathcal{A}} = 0$$

as an E_N-valued distribution. If $g_{\mathcal{A}} = g_0 + \sum_{0<|I|\equiv 0\,(2)} g_I\varepsilon_I$ and $T_{\mathcal{A}} = T_0 + \sum_{0<|I|\equiv 0\,(2)} T_I\varepsilon_I$, then for each even multi-index I

$$\nabla^{(0)}_\mu T_I{}^\mu{}_\nu = -[\varepsilon_I]\Big((\nabla^{(\mathcal{A})} - \nabla^{(0)})_\mu T_{\mathcal{A}}{}^\mu{}_\nu\Big),$$

where $[\varepsilon_I]$ is coefficient extraction in the star-adapted basis of equation (67). The right side is the finite sum of connection–stress cross terms, with their exterior signs and zero products retained. Thus an individual nilpotent coefficient need not be divergence-free with respect to the body connection; its cross-coefficient terms are part of the same complete source ledger.

Proof. The graded action is invariant under the coefficientwise extension of compact diffeomorphisms. Its Noether identity is a finite polynomial identity in the coefficient algebra. The complete graded Euler–Lagrange equations annihilate the field-equation terms, giving the first display. Expanding the inverse metric and connection by their finite nilpotent series and collecting the coefficient of ε_I gives the second display. Its right side is a finite cross-coefficient sum: each product contains a positive-degree connection coefficient and a complementary stress coefficient, including $\Delta\Gamma_I \cdot T_0$. Deleting these terms would change the covariant divergence and incorrectly promote sector coefficients to separately closed sources. □

Corollary 9.4 (Classical scope of the extension)**.** *The graded source is a finite tuple of classical distributions over the fixed body, with actual odd Dirac–Yukawa coefficients and the bosonic nilpotent responses supplied by the graded realization theorem. It is not a positive Grassmann-valued energy, a Hilbert-space state, an operator-valued field or a quantization of the body. Module B matching does not follow from coefficient extension.*

10 The guarded operator/scalar companion

The operator and scalar matching theorems are retained because they state the exact interface at which a classical attached source can also be compared with a quantum translation generator. They are not used to manufacture such an object on the compact curved CSP–C history.

Definition 10.1 (Matching-extension fibre)**.** Let Θ_{src} be the groupoid of complete structured Module A source targets, retaining metric, boundary, regularity and continuation data, and let $\theta \in \Theta_{\mathrm{src}}$. Define $\mathrm{Ext}_{\mathrm{MB}}(\theta)$ to be the groupoid of complete Module B presentations over that entire target: a normalized ray family of concrete source presentations satisfying the displayed hypotheses, its norm-squared homogeneous lift, a quadratic and continuous polarization certificate, the resulting Hermitian operator-valued total stress distribution, and all M1–M10 stable positive-mass spin-zero Poincaré–Noether, form-factor, Breit, Bargmann, boundary and calibration data. Morphisms preserve every one of these coordinates. This definition is made for all θ; the fibre is allowed to be empty.

Theorem 10.2 (Totalized guarded matching status)**.** *Fix a report ledger $\mathcal{L}$ recording supplied witnesses and certified failure proofs. For every $\theta \in \Theta_{\mathrm{src}}$, the ledger-relative target-fibred status*

$$\mathfrak{M}_{\mathcal{L}}(\theta) := \begin{cases} \mathrm{Ext}_{\mathrm{MB}}(\theta), & \textit{an actual complete extension is supplied in } \mathcal{L}, \\ \textsf{certified-empty}, & \mathcal{L} \textit{ certifies the target fibre empty}, \\ \textsf{not-activated}, & \textit{neither certificate occurs in } \mathcal{L} \end{cases}$$

is defined. In the first case it retains the whole extension groupoid, including its automorphisms, rather than selecting one class. On every inhabited fibre, Module B reconstructs the operator-valued total stress distribution before charge or form-factor readout and gives, only on its stable scalar matching subcategory,

$$m_i = m_{\mathrm{Cas}}, \qquad m_g = \alpha_D m_i, \qquad \alpha_D = \frac{\kappa c^2}{4\pi\beta_g} > 0.$$

After the common Einstein–Newton/Bargmann calibration this becomes $m_g = m_i = m_{\mathrm{Cas}}$ [12]. No value is returned for a source lacking the complete extension data.

Proof. The three cases distinguish positive existence, proved nonexistence and lack of a supplied witness. Thus totalization neither adjoins a witness nor turns absence of evidence into an empty-fibre theorem. For an inhabited fibre, the quadratic normalized-ray source family is polarized to the unique Hermitian operator-valued stress distribution. Its global charge is then identified with the Stone translation generator on the full irreducible reference representation. Only afterward is it restricted to the isolated stable spin-zero sector. Exact zero-transfer normalization supplies the active scalar, and the controlled Wigner–Bargmann contraction supplies the inertial scalar. These are precisely the hypotheses and conclusions of Module B, so the common factor follows there and nowhere else. ☐

Proposition 10.3 (Status at the fixed classical CSP–C target)**.** *The compact corrected-body history and its finite graded classical extension have status $\mathfrak{M}_{\mathcal{L}_C}(\theta_C) = \textsf{not-activated}$ in the present target ledger $\mathcal{L}_C$. They supply no normalized projective ray, Poincaré irreducible reference representation, operator polarization datum, isolated stable scalar form-factor sector or Wigner contraction. Hence SR05–SR06 are satisfied in CSP–C as an exact guarded companion theorem and typed nonpromotion boundary, not as a claim that the fixed classical target carries a Module B quantum state.*

Proof. None of the listed Module B coordinates is part of the classical source attachment, the ADM/constraint datum, or the finite exterior-algebra extension. The forgetful map from an inhabited Module B presentation to its Module A source exists, but Module A proves no section of that map and supplies explicit source objects outside the operator-presentable subtype. Assigning an extension to the CSP–C history without the missing data would therefore introduce an unproved source-and-state lift. Conversely, absence of those data does not prove that no richer target can supply them, so the status is not certified-empty. ☐

Corollary 10.4 (No export). *The proportionality theorem cannot be exported from its inhabited stable scalar fibres to spin, multipole, unstable, radiating, generic curved, interacting QFT or quantum-gravity sectors. In particular it is neither a premise of the CSP–C classical source theorem nor a substitute for its constraint and continuation proofs.*

11 Common-source reconstruction and target-relative finality

The preceding results can now be compared without forgetting which map has authority for which object. The structural law, metric response, canonical constraints and covariant development arise from different operations on one fixed action. Their agreement is therefore a commuting reconstruction statement, not a claim that any one marginal output reconstructs all the others.

11.1 The four faces of the source

For the fixed action $S_C^\diamond$, write

$$\mathsf{L}_C = \mathrm{Poly}(S_C^\diamond), \qquad \mathsf{T}_C = d_g S^\diamond_{C,\mathrm{matt}}, \qquad \mathsf{K}_C = \mathrm{Leg}_\Sigma(S_C^\diamond), \qquad \mathsf{E}_C = \mathrm{EL}(S_C^\diamond). \tag{86}$$

The first is a represented operation law, the second an off-shell compact-test response, the third the primary-reduced canonical system and the fourth the covariant Euler–Lagrange system. None has the type of another. The common source assertion is that all four maps have the same argument $S_C^\diamond$ and that the comparison maps below commute.

Let Γ_{A_N} be the finite-algebra solution of Theorem 8.4, with $A_N = \mathbb{C}$ for the body theorem. If $i : \Sigma \hookrightarrow M_0$ is a guarded Cauchy embedding, let $\mathrm{Ev}_i\,\Gamma_{A_N}$ be its complete canonical datum. Evaluation retains the action, target, source attachment, boundary, clock, bundle, spin and lawful-presentation coordinates.

Theorem 11.1 (Common-source commuting reconstruction). *For every guarded i and every finite N,*

$$\mathsf{L}_C = \mathfrak{p}^\diamond, \tag{87}$$

$$W_{A_N}(h) = \tfrac{1}{2}\langle T_{A_N}, h\rangle \qquad \textit{for every ordinary compact metric test } h, \tag{88}$$

$$\mathbf{\Phi}_{A_N}(\mathrm{Ev}_i\,\Gamma_{A_N}) = 0, \tag{89}$$

$$\begin{aligned} \mathrm{body}(\Gamma_{A_N}) &= \gamma_\delta, \qquad \mathrm{body}(T_{A_N}) = T_{\gamma_\delta}, \\ \mathrm{body}(\mathrm{Ev}_i\,\Gamma_{A_N}) &= \mathrm{Ev}_i\,\gamma_\delta. \end{aligned} \tag{90}$$

Lawful source comparison transports every expression in these displays and commutes with polynomial restriction, metric variation, the Legendre map, Cauchy evaluation and the body map.

Proof. Equation (87) is the exact target–source theorem 1.2. Equation (88) is the finite coefficient attachment theorem 9.2; its body is the unique ordinary total source because compact tests separate distributions. Equation (89) follows from the exact initial lift, subsidiary propagation and the covariant–canonical reconstruction of Sections 5–6. The body equalities follow coefficientwise from the distinguished augmentation. Each source comparison transports the action, its fields, coefficient maps and tests together. Differentiation, pullback, finite Taylor expansion and evaluation are natural under that simultaneous transport, so all squares commute. □

The theorem would be false if only the stress tensor were retained: distinct matter actions may have the same value of a marginal stress on one history. It would also be false if only the polynomial graph were retained: that graph contains neither constraint-satisfying data nor a metric Cauchy system. The complete source is the action together with its target and comparison ledger.

11.2 Local equivalence of covariant and canonical histories

Let $\mathcal{H}^{\mathrm{cov}}_{C,\mathrm{body}}$ be the local groupoid of source-attached covariant presentations of the ordinary body development γ_δ and let $\mathcal{H}^{\mathrm{can}}_{C,\mathrm{body}}$ be the local groupoid of complete regular Cauchy data and characteristic arrows. Both retain proper internal and Lorentz presentations; neither quotients mapping-class, boundary-charge, source-change or failure objects.

Theorem 11.2 (Source-faithful local equivalence)**.** *On the occupied ordinary body chart of γ_δ, complete Cauchy evaluation and joint Einstein/source plus Standard–Model Cauchy development define inverse local equivalences*

$$\mathcal{H}^{\mathrm{cov}}_{C,\mathrm{body}} \underset{\mathrm{Dev}}{\overset{\mathrm{Ev}}{\rightleftarrows}} \mathcal{H}^{\mathrm{can}}_{C,\mathrm{body}}. \tag{91}$$

They are fully faithful on the guarded local path groupoids and preserve the complete source, law tuple, attachment, constraints, clock profile, orientation, boundary and relational records.

Proof. The joint reconstruction theorem 6.5 gives inverse object maps up to the declared proper presentations. Differentiated Cauchy evaluation identifies presentation velocities with the complete constraint anchor, and the tame local integration theorem identifies the guarded presentation orbit with a characteristic plaque. Each local path groupoid is therefore a pair groupoid, so an ordered endpoint pair determines exactly one arrow and the object equivalence is fully faithful. Source naturality is the commuting square (57); the remaining data are explicit arrow guards. The development authority is jointly WDW–1 and WDW–4B: WDW–1 alone does not reconstruct gauge, Higgs or spinor fields from a stress tensor [3, 6]. □

Corollary 11.3 (Finite histories and the body)**.** *Finite graded histories commute with guarded Cauchy evaluation and body projection as in (90). No characteristic distribution, finite-refoliation groupoid or covariant–canonical local equivalence is proved here at nonzero Grassmann degree.*

11.3 Certificate-fibre discipline without a borrowed role count

Definition 11.4 (Frozen CSP–C role set)**.** The certificate uses the following finite, typed role set, fixed before any assignment is counted:

$$\begin{aligned}\mathfrak{R}_C = \{&R_{\mathrm{law}}, R_{\mathrm{resp}}, R_{\mathrm{src}}, R_{\mathrm{can}}, R_{\mathrm{EL}}, R_{\mathrm{grav}}, R_{\mathrm{met}}, R_{\mathrm{tr}}, R_{\mathrm{Ein}}, R_{\mathrm{body}}, R_{\mathrm{dev}},\\ &R_{\mathrm{char}}, R_{\mathrm{refol}}, R_{\mathrm{up}}, R_{\mathrm{cmp}}, R_{\mathrm{gr}}, R_{\mathrm{MB}}\}.\end{aligned}$$

Their certified occupants are respectively polynomial restriction; compact metric response; its unique tensor source; canonical Legendre/constraint system; covariant Euler–Lagrange system; premetric gravity role; licensed metric projection; typed finite transport (Levi–Civita on the metric branch); Einstein first response; corrected datum w_δ; its local development; characteristic distribution; guarded finite refoliation; upstream Moore category; downstream comparison; finite graded source/solution; and the ledger-relative guarded Module–B extension status. These are roles of maps and theorem objects, not seventeen new carriers or particles.

The certificate method of GRQM Module D is useful here only after the physical objects have been constructed. Use the already fixed role set $\mathfrak{R}_C$ of Definition 11.4 and form the candidate fibre over the complete fixed certificate

$$\mathfrak{c}_C = (\mathfrak{T}^\diamond_C, S^\diamond_C, \mathfrak{p}^\diamond, \mathbf{c}^\diamond, \text{source, boundary, regularity and comparison ledgers}). \tag{92}$$

An assignment is eligible only when its source and target types agree with the corresponding constructed map; compatibility means simultaneous commutation of all incidences in Theorem 11.1. Lawful automorphisms must preserve the entire certificate.

More explicitly, eligibility requires: the first five roles to be restrictions of the one action; R_{src} to carry complete attachment and the off-shell residual ledger; gravity, metric, transport and Einstein roles to retain their declared comparison classes; body and development roles to use w_δ and its same-source Cauchy problem; characteristic/refoliation roles to remain on the ordinary regular body chart; continuation and comparison to obey formation order and both comparison squares; the graded role to retain its finite algebra, star, constraints and body; and R_{MB} to retain its three-valued report status. Compatibility is the joint conjunction of polynomial, source, Legendre/EL, Cauchy/body, characteristic/refoliation, continuation/source-ancestry and guarded-status incidences. A failed clause is assigned to its corresponding failure fibre, not silently removed.

Proposition 11.5 (Target-relative certificate closure)**.** *On the fixed CSP–C target, every role component of an eligible live assignment is represented by the corresponding constructed role in $\mathfrak{R}_C$ up to a certificate-preserving automorphism. An alleged additional role component is either a presentation of one of those roles, a changed target/source/domain, an independently declared richer-scope extension, or an uninhabited/failure object. No numerical role count from the GRQM target is imported.*

Proof. For each displayed role, its candidate fibre consists of target-faithful presentations formed by the permitted restriction, redescription, incidence or comparison operation. The preceding theorems exhibit one eligible occupant in every fibre except that R_{MB} is occupied by the exact status not-activated, not by an operator object. Quotienting identifies precisely complete-profile-preserving presentations. The live residue is the union of quotient classes occurring in assignments that cover all seventeen roles and satisfy every compatibility incidence. The canonical assignment displayed in Definition 11.4 is such an assignment. Eligibility removes type, target, source and domain mismatches; joint compatibility removes marginal assignments that break any reconstruction, ancestry, body, continuation or status square. Thus each live class is the canonical class for its typed role. A certificate-preserving automorphism may change an internal presentation already quotiented, but fixes every role, incidence and class in the assignment; its induced action on the live assignment is trivial. A different complete profile is a changed certificate or richer scope, and a missing witness remains in the named failure fibre. This is the candidate/quotient/eligibility/compatibility method of Module D, applied to $\mathfrak{c}_C$ rather than transferring its separate eight-role result [14]. □

11.4 No-repair minimality

Theorem 11.6 (Minimal common-source packet)**.** *The following data are jointly irreducible for the result proved here:*

1. *the frozen global, representation and coefficient target;*
2. *the one action and its off-shell compact-test attachment graph;*
3. *the complete internal-exchange and boundary ledger;*
4. *the corrected coupled body and the full mixed right inverse;*
5. *the raw normal bracket with its Gauss and Lorentz terms;*
6. *the regular characteristic and guarded Cauchy-evaluation data;*
7. *the independently formed upstream seed and its downstream compatibility map;*
8. *for nonzero classical fermion occupation, the finite coefficient algebra, star-real constraint lift and coefficientwise Cauchy proof.*

Deleting any item destroys at least one asserted conclusion and cannot be repaired by a later projection, quotient, label or successful solution.

Proof. Without item 1 there is no same target. Without items 2–3 the source is a marginal insertion or has an unbalanced Ward ledger. Without item 4 the gauge–Higgs energy is added to an uncoupled scalar zero or the mixed spatial row remains unsolved. Without item 5 the off-shell deformation algebra is wrong. Without item 6 a bracket table has not been integrated to finite refoliation. Without item 7 continuation is defined by a downstream history or lacks the comparison theorem. Without item 8 the only proved analytic fermion history is the invariant zero section. Each failure changes a standing-bearing incidence, not merely its presentation, so no later skin or post-selection can restore the same theorem. □

12 Classical Standard–Model–gravity capstone

The separate constructions now have one common domain. The following theorem states the strongest endpoint without importing an operator state or global refoliation claim into the classical target.

Theorem 12.1 (CSP–C common-source realization)**.** *There exists a fixed classical target $\mathfrak{T}_C^\diamond$ with the global group, three chiral generations, one Higgs doublet, fixed represented couplings and massive scalar clock of Definition 1.1, and a single action $S_C^\diamond$, such that:*

1. *its polynomial restriction is the nonzero marked CSP–S Standard–Model law $\mathfrak{p}^\diamond$;*
2. *its nondegenerate physical interior is automatically kernel-governed, realizes the premetric gravity role, and admits licensed typed finite transport whose metric-only tangent/tensor branch is Levi–Civita;*
3. *its unique minimal first-response metric equation is $G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T^{\mathrm{closed}}_{\mu\nu}$, with the total source uniquely attached to the complete off-shell metric-response graph and closed by the summed Ward ledger;*
4. *it has an exact corrected constraint datum w_δ with nonzero color, weak and hypercharge curvature, a nowhere-zero nonconstant Higgs field, zero internal stabilizer and zero fermion body, and a source-attached globally hyperbolic development;*
5. *on one common reduced domain its Lorentz, internal, raw/improved spatial and normal constraints obey the complete off-shell Dirac-bracket algebra; the regular characteristic distribution is exactly their Hamiltonian image and is locally integrated by the proper finite-refoliation groupoid;*
6. *an independently formed clock-forward continuation category exists before projection; after the target law is adjoined as inert ancestry data, its standing-domain map intertwines with the corrected coupled development while preserving source ancestry, standing, orientation and composition. Failure and no-repair remain exact upstream coordinates;*
7. *for every finite exterior algebra A_N, the corrected body admits an exact coefficientwise classical lift. For $N \geq 2$ there are solutions with nonzero odd Dirac–Yukawa fields, a nonzero self-adjoint fermionic current and a forced nonzero even coupled response;*
8. *on the ordinary body chart of γ_δ, covariant histories and complete canonical histories are locally equivalent. Finite graded histories commute with Cauchy evaluation and body projection, but no nonzero-Grassmann characteristic or refoliation groupoid is asserted.*

The result is target-relative and local in its refoliation and clock claims. It is not an interacting quantum Standard Model, a dynamical quantum-gravity theory or an unrestricted physical apex.

Proof. The source and law compatibility is Theorem 1.2. Premetric gravity, metric licensing, the connection and Einstein response are Theorems 2.1–2.5. Source attachment and closure are Theorems 3.2 and 3.4. The corrected datum and development are Theorems 4.3 and 4.4. The algebra, split locus, characteristic equality and finite integration are Theorems 5.1, 5.2, 6.3 and 6.6. The continuation statements are Theorems 7.2, 7.6 and 7.7. The finite classical lift and its nonzero response are Theorem 8.4 and Corollary 8.5. Finally, Theorems 11.1 and 11.2 give the commuting reconstruction. All existence statements use the explicit target tuple (3), so none assumes a pre-existing intersection of the formerly separate CSP–S branches. □

12.1 Dependency-closed theorem-use ledger

Each entry states the received result; its locus; its required target intersection; and its prohibited promotion.

MA [11]. Attachment, tensor-distribution representation, Ward and finite composite closure; Sections 3 and 9; complete compact-test graph, LF, boundary/regularity and registry hypotheses; no universal source or operator.

MB [12]. Normalized-ray polarization and stable scalar matching; Section 10; full M1–M10 fibre; no activation on the classical history and no spin, unstable, generic curved, QFT or QG export.

MC [13]. Complete RPI seed, flow, Moore category and deletion; Section 7; every Definition 4.1/6.1 coordinate; no Hilbert projection here and no upstream dynamics from the inert singleton.

MD [14]. Candidate, quotient, eligibility, compatibility, live-residue, failure and automorphism method; Section 11; the frozen CSP–C certificate; no transfer of the GRQM eight-role count.

WDW–1 [3]. Source-faithful ADM prequotient and reconstruction; Sections 5 and 11; regular globally hyperbolic source target; no refoliation or SM occupancy.

WDW–2 [4]. Proper spatial reduction and residual retention; Sections 5–6; exact WDW–1 handoff/boundary class; no normal closure.

WDW–3 [5]. Off-shell HDA and guarded finite refoliation; Section 6; raw Gauss/Lorentz terms and ordinary split regular chart; no graded refoliation or global path independence.

WDW–4 [6]. Scalar-clock/SM occupancy and joint constraints; Sections 1–5; same action, bundle, source and regular branch; no replacement of w_δ by the old scalar datum.

BCRS–III [2]. Coupled gauge–Higgs–gravity history and corrected handoff; Sections 1 and 4; fixed represented branch; no poles, confinement or quantum state.

RPI [21]. Complete realized-interior role grammar; Section 7; an independently admitted regime with its full role sequence; it is not derived from $S_C^\diamond$.

Gravity [16]. Coisotropic/characteristic first-class gravity role; Section 2; regular AER class; no Einstein equation or metric primitive.

Metricity/AMetric [17, 18]. AMetric exclusion and metric licensure; Section 2; fixed nondegenerate licensing envelope; no metric authorization of upstream roles.

CovDiff [19]. Identity-symbol derivative and Levi–Civita closure; Section 2; metric-only first-jet natural selector; clock/gauge fields are not selector arguments.

Einstein [20]. Minimal first-curvature response; Sections 2 and 3; four-dimensional Lorentzian same-scope class and closed source; no numerical constants or matter dynamics.

Baer [24]. Green/symmetric-hyperbolic existence machinery; Section 8; globally hyperbolic body, finite bundles and declared regularity; no nonlinear constraint lift.

Murro–Volpe [25]. Symmetric-hyperbolic intertwining; Section 8; exact operators/comparison data; no AQFT or Hadamard-state import.

Internal graded theorem. Theorems 8.2 and 8.4; Sections 9–11; finite star algebra, full right inverse, reduced identities and compact body slab; no positive Grassmann energy, quantization or graded refoliation groupoid.

12.2 Resolution of the source-response requirements

The GRQM interfaces have unequal domains. Their exact status is part of the theorem rather than an afterthought.

Item	Status at $\mathfrak{T}_C^{\diamond}$	Result
SR01–02	positive	The RPI seed and Moore category are formed before all projections and survive their deletion.
SR03–04	positive	One complete action gives the attached distribution, finite term registry, off-shell residual ledger and closed summed Ward identity.
SR05	exact guarded companion	Module B reconstructs an operator-valued stress only on a supplied normalized-ray extension fibre. The classical construction supplies no such lift.
SR06	exact guarded companion	The inertial–active equality holds only on the stable positive-mass scalar Poincaré matching envelope; it is not exported to this curved classical history.
SR07	typed profile result	Metric/source on the standing subcategory, native-prefix record and constant compatibility/source-ancestry projections are positive and closed; Hilbert/ray and downstream failure projections are not activated. Upstream failure/no-repair remains exact.
SR08	positive target-relative closure	The frozen seventeen-role CSP–C set has explicit occupants/status, eligibility, joint compatibility, live-residue, failure and automorphism dispositions; the GRQM eight-role count is not transferred.
SR09–10	positive dependency closure	The theorem-use ledger fixes every source identity, received result, locus, hypothesis intersection and prohibited promotion; no audit grammar generates a carrier, field, solution or state.

Thus the result-bearing classical source requirements are inhabited. SR05–06 are resolved as exact partial interfaces with their positive branches unprovided at this target. If “all source requirements” is instead stipulated to mean a positive Module-B operator/scalar inhabitant over the same classical source, that stronger mixed classical–quantum requirement remains unproved and is not used in Theorem 12.1.

12.3 Resolution of the gravity and classical-realization requirements

Item	Status	Result
GR01–02	positive	Nondegeneracy forces the kernel and the regular AER comparison realizes the premetric coisotropic/characteristic gravity role.
GR03	positive	AMetric exclusion, L-metric licensing, fixation locality and no backflow are retained.
GR04	positive	Finite standing-preserving transport is constructed on the named tangent, tensor, gauge, Higgs, chiral and scalar carriers and canonically infinitesimalizes to their typed connections. The metric branch is Levi–Civita; represented matter retains the separate spin and gauge terms.
GR05	positive	The minimal first-response metric form is Einstein with a closed source.
GR06–07	positive	Source-faithful ADM reconstruction, the proper spatial quotient and global residual coordinates are present.
GR08	positive, local	The full characteristic equality and guarded finite refoliation are proved on the occupied regular chart.
GR09	positive and strengthened	One coupled classical Standard–Model body exists; the finite-algebra theorem adds nonzero odd classical fields and exact nilpotent response.
GR10–11	guarded	Global path independence, a global refoliation group, WDW operator, interacting QFT and quantum gravity are not promoted.
GR12	positive	The exact source, regularity, target, boundary, anomaly, representation and comparison coordinates are retained in the handoff.

12.4 Falsification and extension boundaries

The capstone would fail if the represented law and classical action used different coefficient data; if gauge–Higgs energy were added without the gravity–scalar correction; if a stress tensor were inserted without the complete action graph; if internal exchanges or Euler–Lagrange residuals were omitted; if the raw normal bracket lost its Gauss or Lorentz terms; if the mixed constraint column were deleted; if smooth characteristic equality were confused with a finite integration theorem; if an unrelated RPI seed were relabelled as the target source; or if a Module-B state were assigned without its full extension certificate.

The finite-algebra theorem has equally sharp limits. Its generators label coefficient directions, not particle generations. Nilpotent bilinears have zero body and no numerical positivity order. A nonzero coupled even response does not by itself prove that the metric coefficient is nonzero. Passing to an infinite exterior algebra, a completed nonlinear supermanifold, a CAR representation, a functional integral, physical poles, confinement or scattering requires new theorems.

Within these boundaries the endpoint is complete: one frozen structural Standard–Model law, one source-attached Einstein/ADM realization and one exact joint constraint/refoliation class coexist on an occupied regular branch, with a finite classical anticommuting-field strengthening and no backflow from later physical descriptions.

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